CHAPTER 10 BRIDGES

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10.1 INTRODUCTION

Bridges are defined (23 CFR 650.403) legally as structures with a centerline span of 20 feet or more. However, structures designed hydraulically as bridges are treated in this chapter regardless of length.

The decision to use a bridge rather than a large culvert should be aided by estimating construction and maintenance cost, structural, aesthetics and environmental considerations.



Photo 10.1



Photo 10.2

10.2 DESIGN CRITERIA

10.2.1 General Criteria

The following general criteria shall be used in the hydraulic analysis and design of bridge:

- The final design selection should consider the maximum backwater allowed by the National Flood Insurance Program (NFIP) unless exceeding the limit can be justified by special hydraulic conditions;
- For sites outside of Federal Emergency Management Agency (FEMA) regulation, the backwater shall not cause increased flood damage to property upstream of the crossing;
- The final design should not significantly alter the existing flow distribution in the floodplain;
- The "crest-vertical curve profile" is the preferred highway bridge crossing profile when allowing for embankment overtopping at a lower discharge and for adequate deck drainage;
- Sag vertical curves can cause deck drainage to pond and ice up on the bridges and should be avoided;
- Horizontal curve transitions cause water to flow across lanes and should not be located on a bridge because of icing and hydroplaning problems;
- Clearance or freeboard should be provided between the low girder and the design water surface to allow for the passage of ice and debris;
- The design capacity of any bridge will be the flow that will pass through the bridge with adequate freeboard and without roadway overtopping;
- Estimate all degradation and aggradation plus contraction scour and local scour for the design year and for the 500-year event. Indicate the total scour envelope with a continuous line drawn such that the structural designer may adequately design substructure components. Scour depths are to be estimated with consideration of the local geology;

- Velocities through the structure(s) will not damage either the highway facility or increase damages to adjacent property;
- Pier spacing and orientation, and abutment location shall be designed to minimize flow disruption and potential scour. Bridge piers should not be placed in the main channel area;
- Foundation design and/or scour countermeasures shall be made to avoid failure by scour. Typically, substructure components are designed to avoid failure by scour;
- Although appropriate in some debris prone streams, connecting a discrete pier column to a debris-deflecting wall can significantly increase scour depths if the channel alignment ever shifts. A debris-deflecting wall can also greatly increase the stiffness of a pier that reduces the number of available design options. More preferably, a long span bridge design reduces the number of piers and therefore, reduces the benefits derived from debris deflecting walls. It is now often more efficient for a designer to simply design a pier (and if necessary the superstructure) for increased stream loads due to debris;
- When two or more bridges are constructed in parallel over a channel, care should be taken to align the piers and to provide streamlined grading and protection for abutments. This abutment grading is to minimize expansion or contraction of flow between the two bridges;
- Commercial mining of sands and gravel in streams is common because the material is clean and well graded and the stream replenishes the supply. Borrow pits, either upstream or downstream of a highway-stream crossing, can cause or aggravate scour at the bridge. This fact should be considered when calculating bridge scour, and should be estimated by sediment transport modeling;
- Disruption of ecosystems is to be minimized. Consideration is to be given to the preservation of valuable characteristics that are unique to the floodplain and stream;
- Economic analysis of the design shall include complete life cycle costs and benefits. Factors that should be considered are construction, maintenance, operation, as well as any potential liabilities;
- Adequate right-of-way shall be provided upstream and downstream of structure for maintenance operation.

10.2.2 Specific Criteria

Overtopping Flood

Inundation of the travelway dictates the level of traffic services provided by the facility. The potential failure of the roadway embankment during overtopping should be analyzed (see FHWA Report No. RD-86/126).

Risk Evaluation

The selection of design frequency for determining the waterway opening, road grade, scour potential, riprap and other features shall consider the potential impacts to:

- interruptions to traffic;
- adjacent property;
- the environment; and
- the infrastructure of the highway.

The consideration of the potential impacts constitutes an assessment of risk for the specific site. The least total expected cost (LTEC) alternative should be developed in accordance with FHWA HEC-17 (3) when the conventional design frequency in Chapter 7, Hydrology is not used. This analysis provides a comparison between other alternatives developed in response to environmental, regulatory, and political considerations.

Backwater Increases Over Existing Conditions

The backwater increase will be defined as the difference in water surface elevations between the natural case with no bridge and the case with the proposed bridge.

The new structure will conform to FEMA regulations for sites covered by the NFIP.

For sites not covered by NFIP, the backwater increase during the passage of the 100-year flood will be limited to no more than one foot above the backwater corresponding to natural conditions that existed prior to the construction of the bridge. For sites not covered by NFIP, a greater than one foot increase in backwater is acceptable if there is adequate justification showing that the design is the only practical alternative and that the design will only cause minimal impacts. For these sites, a risk analysis (LTEC) design should be considered. Any impacted property owner must agree to the changed flood condition.

Hydraulic evaluation must include channel conditions pertaining to: i) natural channel condition prior to the construction of the existing bridge; ii) the existing bridge; and iii) the proposed bridge.

Distance to point of maximum backwater

In backwater computations, it will be found necessary in some cases to locate the point or points of maximum backwater with respect to the bridge. The maximum backwater in line with the midpoint of, the bridge occurs at point A (figure 10.1A), this point being a distance, L^* , from the waterline on the upstream side of the embankment. Where floodplains are inundated and embankments constrict the flow, the elevation of the water surface throughout the areas *ABCD* and *AEFG* will be essentially the same as at point A, where the backwater measurement was made on the models. This characteristic has been verified from field measurements made by the U.S. Geological Survey on bridges where the flood plains on each side of the main channel were no wider than twice the bridge length and hydraulic roughness was relatively low.

For crossings with exceptionally wide, rough floodplains, this essentially level ponding may not occur. Flow gradients may exist along the upstream side of the embankments due to borrow pits, ditches and cleared areas along the right-of-way. These flow gradients along embankments are likely to be more pronounced on the falling than on the rising stage of a flood. A correlation is needed between the water level along the upstream side of embankments and point A since it is difficult to obtain water surface elevations at point A in the field during floods. For the purpose of design and field verification, it has been assumed that the average water surface elevation along the upstream side of embankments, for as much as two bridge lengths adjacent to each abutment (F to G and D to C), is the same as at point A (figure 10.1B).

Normal crossings

Figure 10.1 has been prepared for determining distance to point of maximum backwater, measured normal to centerline of bridge. The curves on figure 10.1 were developed from information supplied by the U.S. Geological Survey on a number of field structures during floods. Referring to figure 10.1, the normal depth of flow under a bridge is defined here as $\overline{y} = A_{n2} / b$, where A_{n2} is the cross sectional area under the bridge, referred to normal water surface, and b is the width of waterway. A trial solution is re-

quired for determining the differential level across embankments, Δh , but from the result of the backwater computation it is possible to make a fair estimate of Δh . To obtain distance to maximum backwater for a normal channel constriction, enter figure 10.1A with appropriate values of $\Delta h / \bar{y}$ and \bar{y} and obtain the corresponding value of L^*/b . Solving for L^* , which is the distance from point of maximum backwater (point A) to the water surface on the upstream side of embankment (figure 10.1B), and adding to this the additional distance to section 3, which is known, gives the distance L_{1-3} . Then the computed difference in level across embankments is

$$\Delta h = h_1 * + h_3 * + S_O L_{1-3}$$

Should the computed value of Δh differ materially from the one chosen, the above procedure is repeated until assumed and computed values agree. Generally speaking, the larger the backwater at a given bridge the further will point A move upstream. Of course, the value of L^* also increases with length of bridge.

Eccentric crossings

Eccentric crossings with extreme asymmetry perform much like one half of a normal symmetrical crossing with a marked contraction of the jet on one side and very little contraction on the other. The measure of eccentricity is given by the parameter "e" defined by:

$$e = 1 - \frac{Q_2}{Q_3}$$
 where $Q_2 < Q_3$

where Q_2 and Q_3 are floodplain discharges inline with bridge embankments. For cases where the value of e is greater than 0.70, enter the abscissa on figure 10.1A with $\Delta h / \bar{y}$ and \bar{y} and read off the corresponding value of L^*/b as usual. Next multiply this value of L^*/b by a correction factor, ω , which is obtained from figure 10.1C. For example, suppose $\Delta h / \bar{y} = 0.20$, $\bar{y} = 10$ and e = 0.88, the corrected value would be $L^*/b = 0.84 \times 1.60$. Distance to maximum backwater is then $L^* = 1.34b$ with eccentricity.

Skewed crossings

In the case of skewed crossings, the water surface elevations along opposite banks of a stream are usually different than at point A; one may be higher and the other lower depending on the angle of skew, the configuration of the approach channel, and other factors. To obtain the approximate distance to maximum backwater L^* for skewed crossings, the same procedure is recommended as for normal crossings except the ordinate of figure 10.1 is read as L^*/b , where b is the full length of skewed bridge.



Figure 10.1 Locating the distance to maximum backwater.

Freeboard

A minimum clearance or freeboard shall be provided between the design approach water surface elevation and the low girder of the bridge. The freeboard is required to allow for wave action, ice, debris and uncertainty in estimated stage.

The minimum freeboard for a bridge should follow these guidelines:

- For a high debris stream , freeboard should be 4 feet or more.
- For low to moderate debris streams the freeboard given in the equation below should be used.

Freeboard =
$$0.089 Q^{0.3} + 0.026 V^2$$

where, Q is design discharge in (cfs); and V is the mean velocity of the design flow through the bridge in ft/s (16ft/s max.). If the mean velocity is greater than 16 fps, the bridge must be widened.

- The water surface 50 to 100 feet upstream of the face of the bridge should be the elevation to which the freeboard is added to get the bottom or low girder elevation of the bridge. The water surface elevation can be estimated by interpolating between the section at the upstream face of the bridge and the upstream approach section. If there are any hydraulic structures within 50 to100 feet from the upstream or downstream face of the bridge, these structures must be incorporated into the hydraulic analysis. If roadway or structural considerations require less freeboard, structural analysis of bouyancy and debris forces on the bridge must be performed by the bridge designer.
- A more complete analysis to determine the location of maximum backwater should be based on figure 10.1. Freeboard should be added to the water surface elevation corresonding to this location to arrive at the bottom or low-girder elevation for the bridge.
- Another important consideration with freeboard is the location of the freeboard on the structure. Requirements for locating freeboard on bridges with different profile grade configurations is given in figures 10.2 and 10.3.
- If the structure upstream girder can be made rounded or tapered to facilitate debris passage, the freeboard requirements can be reduced by one foot for the design flow. The structure must still be checked for the 100-year and 500-year events. Examples of hydraulically smooth-bottom superstructures include: rigid frame, precast slab, cast in place slab, side-by-side boxes, a solitary U-girder, etc.
- Debris deflector walls to divert the debris around a pier is recommended for all bridges on high debris streams. A debris wall detail showing acceptable dimensions is shown in Figure 10.4. An alternative to a debris wall is to extend the upstream face of the wall pier out, flush with the deck. This design does not divert the debris but does move the debris out in front of the bridge for easier removal by maintenance personnel. Basin characteristics such as snow melt, history of maintenance debris problems, as well as the timber types present in the basin, should be taken into account for the design of debris deflectors or position of the support columns and piers.

Other issues that need to be addressed when designing a bridge for debris are how quickly maintenance equipment can get to the structure to remove debris and how important the route is for emergencies. All these issues must be clearly addressed in the design documentation for the structure. For concrete rigid frames and concrete box culverts freeboard is not as important unless debris causes reduced conveyance of flow.







Figure 10.4 Debris Deflecting Wall on Upstream Face of Bridge Pier or Concrete Culvert Web Wall.

Flow Distribution

An analysis of the flow patterns at a proposed stream-crossing should be made to determine the flow distribution and to establish the location of bridge opening(s). A drawing of stream flowlines going through the bridge can be made using the flow distribution information given in HEC-RAS, WSPRO, or directly from FHWA's BRI-STARS (BRIdge STreamtube model for Alluvial River Simulation) model (Molinas, 2000). The proposed facility shall not cause any adverse change in the existing flow distribution, A range of flow distributions should be investigated for any bridge design because a bridge location might function well for one flood stage but not at other flow stages.

Relief openings in the approach roadway embankment shall be investigated if there is more than 10% of total flow in the overbank region.

Snowmelt Streams

The ordinary high water width is identified by the line of vegetation located along each bank of the stream.

On streams with flood peaks resulting from snowmelt the width of the ordinary high water should not be significantly reduced. A small reduction in the ordinary highwater width through a snowmelt bridge can be made if there is no large increase in velocities through the bridge or in backwater. The designer must be cautious of the long term effects of ponding upstream of a highway crossing due to the long duration of snowmelt floods.

10.3 HYDRAULIC DESIGN

10.3.1 Design Sequence

Prior to the scoping meeting the hydraulic designer may make an initial determination of the anticipated structure, either a CBC or bridge.

The basic sequence for the hydraulic analysis of a bridge is summarized in Figure 10.5 and consists of the following:

- 1. Determine watershed hydrology per Chapter 7, Hydrology.
- 2. Visit the site and obtain flood history from CDOT maintenance staff, bridge inspection files, and local residents. Check to see that the channel survey is adequate (see Survey Manual). Investigate upstream and downstream for conditions affecting stream stability such as man made structures, significant hydraulic features, gravel mining activities, etc.
- 3. Check for Federal Emergency Management Agency (FEMA) studies, and use the actual HEC-RAS or other model data from the study if possible. The FEMA HEC-RAS input often can be obtained from the governmental entity responsible for floodplain management or directly from a FEMA office. If FEMA data is used, the FEMA survey benchmark will need to be tied to the project survey. Often the FEMA data maybe out of date and no longer represents the actual conditions. The most updated survey should be used for sizing the bridge. An analysis of the new bridge in the outdated FEMA run will also be required to ensure that the regulatory floodway and floodplain are not adversely affected.
- 4. Complete a water surface profile analysis through the bridge reach. This analysis should include the analysis of the natural situation without any bridge and an analysis of the existing floodplain situation. The water surface profile should be determined using the USGS program WSPRO or U.S. Army Corp program HEC-RAS. The HEC-RAS program is used for most FEMA studies.





profile analysis shall be computed as discussed in Chapter 7, Hydrology. Factors which contribute to the selection of the return period include the capacity and size of the highway, whether it is located in a rural or urban area, and the expected traffic levels.

- 6. A range of bridge opening sizes smaller and larger than the existing channel should be analyzed and then compared with the existing and natural conditions to choose the optimum bridge channel width for the design flow.
- 7. Locate the bridge within the floodplain and select a skew to best fit the alignment of the main channel and floodplain. Keep skew to a minimum to reduce construction and maintenance costs. Be aware that flow patterns can change as the discharge changes.
- 8. Assess the impacts to the surrounding property and roadway for the overtopping and 100-year flood for the various alternatives identified in step 4. Comparing the proposed impacts with the existing FEMA models found in step 3 is important. Any increase in the floodplain should be avoided if possible. If not possible, the impacted area will require purchasing or other mitigation.
- 9. Make preliminary calculations for aggradation/degradation, contraction scour and local scour.
- 10. Select necessary revetment protection (i.e. rip-rap guide banks, spur dikes, etc.) for the bridge and channel. Request right-of-way (ROW) if needed for the revetment protection.
- 11. For hydraulic crossings, in general and early in preliminary design, give the preliminary channel width, elevation at excavated channel width, skew, station at centerline of channel, recurrence interval for design event, drainage area, design discharge, 100-year discharge, 500-year discharge, minimum low girder elevation, thalweg elevation, ordinary high water elevation (OHW), design high water elevation (DHW), 100-year high water elevation, 500-year high water elevation, design velocity (V), 100-year velocity, 500-year velocity and riprap dimensions to Staff Bridge on the Bridge Hydraulic Information Transmittal sheet. Examples of the Bridge Hydraulic Information to evaluate how different bridge materials and configurations can be employed in order to best span the channel. Then, from this the bridge design engineer will complete the General Layout Sheet.
- 12. For hydraulic crossings at most major structures, additional levels of hydraulics information are required after the preliminary design. Typically, the hydraulics engineer draws in elevation view the total scour envelope and differentiates the design scour depth from the 500-year scour depth. And provides the existing and proposed water surface elevations, determining the backwater associated with the profile and waterway opening, etc. These additional items of information are to be provided to the bridge designer early in the final design. The "Hydraulics Work Flowchart for Major Structures" indicates the coordination required between Hydraulics, Geological and Bridge Engineers.
- 13. After the FIR meeting and the geology report is received the final scour profile should be completed. Refer to Section 10.4.1 for a more detailed discussion of bridge scour methods and requirements. The scour depth should be prvided to the structural engineer and the geologist for final bridge design.
- 14. Complete all documentation, the Bridge Hydraulics Report and the bridge hydraulic information sheets for the plans. The scour depths will be shown on the bridge layout plan sheet. Figures 10.6 and 10.7 provide copies of the Bridge Hydraulic Information Transmittal Sheets for spillthrough and vertical-wall abutments and examples of bridge hydraulic information plans are given at the end of this chapter.

These general design steps need to be followed for every bridge.

Bridge Hydraulic Information Transmittal Sheet for Spillthrough Abutments Here is the structure opening and hydraulic information required for the bridge across on SH at/near **PROJECT INFORMATION** Date: Construction Project Number: P.E. Project No.: To: From: Project Name: **BRIDGE INFORMATION** Existing Structure Number: Station at Centerline of Channel: Skew: Minimum Low Girder Elevation: Design Year Event: year recurrence. Excavated Channel Width At Elevation: ft Net Channel Width ft $T = __in.$ $D_{50} = in.$ ft Thalweg Elev.: _____ft HYDRAULIC INFORMATION D.A. = $Q_{(100)} = __cfs$ cfs cfs $Q_{(500)} =$ sq. miles $Q_{(Design)} =$ OHW = ft $DHW_{(Design)} =$ ft $HW_{(100)} =$ ft $HW_{(500)} =$ ft $V_{(Design)} =$ $V_{(100)} = ___fps$ $V_{(500)} =$ _____ cfs fps Please submit to Staff Bridge the information required by CDOT Drainage Design Manual so they may proceed with design. Bridge Layout requested: yes no Comments: _____D__ -------

Figure 10.6 Transmittal of Bridge Hydraulic Information Sheet for Spillthrough Abutments.

Bridge Hydraulic Information Transmittal Sheet For Vertical-Wall Abutments

Here is the structure opening and hydraulic information required for the bridge across ______ on SH ______ at/near _____.

PROJECT INFORMATION

Date:	Construction Project Number:
To:	P.E. Project No.:
From:	Project Name:

BRIDGE INFORMATION

Existing Structure Number:	
Station at Centerline of Channel:	
Skew:	
Minimum Low Girder Elevation:	
Design Year Event:	year recurrence.



HYDRAULIC INFORMATION

D.A. = $\sq.$ miles	$Q_{(Design)} = \cfs$	$Q_{(100)} = \cfs$	$Q_{(500)} = \cfs$		
OHW = ft	$DHW_{(Design)} =ft$	$HW_{(100)} =ft$	$HW_{(500)} =ft$		
	$V_{(Design)} = \cfs$	$V_{(100)} =fps$	$V_{(500)} = \underline{fps}$		
Please submit to Staff Bridge the information required by CDOT Drainage Design Manual so they may proceed with design. Bridge Layout requested: yes no Comments:					

Figure 10.7 Transmittal of Bridge Hydraulic Information Sheet for Vertical-wall Abutments.

10.3.2 Hydraulic Performance of Bridges

The stream-crossing system is subject to either free-surface flow or pressure flow, through one or more bridge openings with possible embankment overtopping. These hydraulic complexities should be analyzed using computer programs such as WSPRO, HEC-RAS, or BRI-STARS.

The hydraulic variables and flow types are defined in figures 10.8 and 10.9. Backwater (h_l) is measured relative to the normal water surface elevation which is without the effect of the bridge at the approach cross-section (Section 1). It is the result of contraction and re-expansion head losses and head losses due to bridge piers. Backwater can also be the result of a "choking condition" in which critical depth is forced to occur in the contracted opening with a resultant increase in depth and specific energy upstream of the contraction. This is illustrated in Figure 10.9C.

Type I consists of subcritical flow throughout the approach, bridge, and exit cross sections and is the most common condition encountered in practice.

Type IIA and IIB both represent subcritical approach flows which have been contracted resulting in the occurrence of critical depth in the bridge opening. In Type IIA the critical water surface elevation in the bridge opening is lower than the undisturbed normal water surface elevation. In the Type IIBflows are choked and the critical water surface elevation is higher than the normal water surface elevation and a hydraulic jump immediately downstream of the bridge contraction is possible.

Type III flow is supercritical approach flow and remains supercritical through the bridge contraction. Such a flow condition is not subject to backwater unless it chokes and forces the occurrence of a hydraulic jump upstream of the contraction.

A subcritical analysis should be used for designing a bridge on a steep supercritical stream because of the unstable nature of steep streams. For supercritical streams there can be standing waves and debris flows that can cause much greater flow depths than the supercritical water surface profile might show. True supercritical flow rarely occurs in natural streams. A subcritical analysis will show a much greater depth and thus be more conservative. In addition, an increase in freeboard for steep channels should be considered.



Figure 10.8 Bridge Hydraulic Definition Sketch.



Figure 10.9. Bridge flow types.

10.3.3 Dual Bridges

Arrangement

Dual bridges of essentially identical design that are placed parallel and only a short distance apart are commonly encountered in highway systems. The backwater produced by dual bridges is naturally larger than that for a single bridge, yet less than the value which would result by considering the two bridges separately. As the combinations of dual bridges encountered in the field show large variations, it was necessary to restrict model tests to the simplest arrangement; namely, identical parallel bridges crossing a stream normal to the flow (see sketch in figure 10.10). The tests were made principally with 45° wingwall abutments, but also included a limited number of the spillthrough type, both having embankment slopes of 1:1. The distance between bridges was limited by the range permissible in the model which was 10 feet or $L_d/l = 11$.



Figure 10.10 Determination of terms used in dual bridges.

Backwater determination

The method of testing consisted of establishing normal flow conditions, then placing one bridge constriction in the flume and measuring the backwater, h_1^* . A second bridge constriction, identical to the first, was next placed downstream and the backwater for the combination, h_d^* , was measured upstream from the first bridge. The ratio, h_d^*/h_1^* , thus obtained, is plotted with respect to the parameter, L_d/l , on figure 10.10, where l is the width of bridge and L_d is the distance from the upstream face of the first bridge to the downstream face of the second bridge. The curve was established from tests made with and without piers. The ratio, h_d^*/h_1^* , which is assigned the symbol η , increases as the bridges are moved apart, apparently reaching a limit and then decreases as the distance between the bridges is further increased. The range of the model was sufficient to explore only the rising portion of the curve but most cases in practice will fall within this range. With bridges in close proximity to one another, the flow

To determine backwater for dual bridges meeting the above requirements, it is necessary first to compute the backwater, h_1 *, for a single bridge. The backwater for the dual combination, measured upstream from the first bridge (figure 10.10), is then:

$$h_d^* = h_1^* \eta$$

Drop in water surface across embankments

In the case of dual bridges, the designer may wish to know the water surface elevation on the downstream side of the roadway embankment of the first bridge, or the water surface elevation on the downstream side of the embankment of the second bridge. Fluctuations in the water surface between bridges, due to turbulence and surging, caused the measurements to be so erratic that it was thought inadvisable to include the results here. A characteristic to be noted in this connection, however, is that the water surface between bridges usually stands above normal stage (See sketch in figure 10.10).



Figure 10.11 determination of drop for the dual bridge combinations.

The water surface downstream from the second bridge, on the other hand, was quite stable permitting accurate measurements. The procedure for determining the water surface level immediately downstream from the second bridge embankment at section 3B (see sketch in figure 10.10) consists of first computing h_1^* and h_3^* for the upstream. For convenience, the sum $h_1^* + h_3^*$ for the single bridge is assigned the symbol ψ_h . Likewise the sum $h_d^* + h_{3B}^*$ for the two-bridge combination is represented by the symbol ψ_{h3B} . The ratio of the second head differential to the first carries the symbol ξ , or

$$\xi = \frac{h_d^* + h_{3B}^*}{h_1^* + h_3^*} = \frac{\psi_{h_{3B}}}{\psi_{h}}$$

The ratio ξ has been plotted with respect to L_d/l on figure 10.11. To obtain the drop in level ψ_{h3B} for the dual bridge combination, it is only, necessary to multiply ψ_h for the single bridge by the factor ξ from figure 10.11. The difference in water surface elevation between the upstream side of the first bridge embankment and the downstream side of the second should then be:

or

$$\Delta h_{3B} = \psi_{h_{3B}} + S_0 L_{I-3B}$$
$$\Delta h_{3B} = \psi_h \xi + S_0 L_{I-3B}$$

Should the water surface level on the downstream side of the second bridge embankment at section 3B be desired relative to normal stage:

$$h_{3B}^* = \psi_{h_{3B}} - h_d^*$$

The left end of the curves on figures 10.10 and 10.11 are shown as broken lines since no data were taken to definitely establish their positions in this region.

The abutment grading and placement of revetment between the upstream and downstream bridges shall be streamlined in such a manner not to cause flow expansion and contraction between the bridges.

10.3.4 <u>Computer Programs</u>

HEC-RAS

The U.S. Army Corps of Engineers' HEC-RAS model uses a variation of the momentum method in the special bridge routine when there are bridge piers. This model is the descendant of the HEC-2 model, which historically has been used for the majority of flood insurance studies performed under the NFIP. HEC-RAS has been improved over HEC-2 to better model bridges and culverts. When importing HEC-2 data into HEC-RAS, the bridge data needs to be updated to reflect the improvements in bridge analysis in HEC-RAS over HEC-2.

WSPRO

WSPRO combines step-backwater analysis with bridge backwater calculations. This method allows for pressure flow through the bridge, embankment overtopping, and flow through multiple openings and culverts. The WSPRO program was developed especially for bridge hydraulics and gives a more accurate hydraulics analysis of bridges than HEC-RAS.

2 -Dimensional Modeling

The water surface profile and velocities in a section of river are often predicted using a computer model. In practice, most analysis is performed using one-dimensional methods such as the standard step method found in WSPRO or HEC-RAS. While one-dimensional methods are adequate for many applications, these methods cannot provide a detailed determination of the cross-stream water surface elevations, flow velocities or flow distribution. Instead floodplains with large flows in the overbanks should be analyzed using two-dimensional models that also calculate variations laterally across the floodplain.

Two-dimensional models are more complex and require more time to set up and, depending on complexity, may require significantly more field data and effort to calibrate and use.

The USGS has developed a two-dimensional finite element model for the FHWA that is designated as **FESWMS**. This model has been developed to analyze flow at bridge crossings where complicated hydraulic conditions exist. This two - dimensional modeling system is flexible and may be applied to many types of steady and unsteady flow problems including bridge crossings with large overbank flows, floodplains with large variations in roughness, multiple channels and flow around islands. Where the flow is essentially two-dimensional in the horizontal plane a one-dimensional analysis may lead to costly over-design or possibly improper design of hydraulic structures and improvements.

BRI-STARS

A compromise between the simplified one-dimensional and the complex two- and three-dimensional models is the quasi-2 dimensional model **BRI-STARS** (BRIdge Stream Tube model for Alluvial River Simulation). This model was developed for the FHWA to analyze the sediment transport process that take place at bridge sites including aggradation/degradation, contraction scour, and local pier and abutment scour. In cases where sediment activity may be important for the design of abutment foundation and the pier pile depths, BRI-STARS model can be used to compute potential scour depths by considering all three types of sediment processes. Since the simulation uses actual design hydrographs rather than ultimate scour quantities, in cases where short-duration design events are experienced, more realistic scour quantites than those predicted by HEC-18 equations are obtained. As its name implies, BRI-STARS utilizes streamtubes to define lateral distribution of flow and sediment at a given cross section. Streamtubes are imaginary tubes bounded by streamlines that carry a constant discharge along their length. By the use use streamtubes, BRI-STARS can simulate the variation of flow not only in the direction of flow but also across the channel (two-dimensional flow). With the inclusion of bottom

variation, the model provides three dimensional channel information through time. Sediment balance is maintained along each tube separately to also define lateral variation of channel erosion/deposition activity. As a result, while one part of the channel is scouring other parts may be depositing. With the selection of a single tube, the model operates as a one-dimensional sediment routing model. With the selection of no sediment transport, the model becomes a one-dimensional rigid-bed model, capable of automatically computing sub-critical, super-critical and a combination of both flow regimes.

10.4 BRIDGE SCOUR

10.4.1 Introduction

A hydraulic analysis of a bridge requires an assessment of the proposed bridge's vulnerability to scour. Because of the extreme hazard and economic hardships posed by a catastrophic bridge collapse, special considerations must be given to the scour and foundation analysis of any new bridge. Since the area of scour prediction and analysis is new and constantly changing the hydraulic engineer should always be aware of and use the most current scour predicting methodologies.

Designers should consult FHWA Publications HEC-18 "Evaluating Scour At Bridges" and HEC-20 "Stream Stability at Highway Structures" for a more thorough treatise on scour and scour prediction methodologies.

A complete analysis of stream stability requires a multilevel solution procedure involving hydraulics, bridge, and geology staff. The evaluation and design of a highway stream crossing or encroachment should begin with a qualitative assessment of stream stability. This involves application of geomorphic concepts to identify potential problems and alternative solutions. This analysis should be followed with a quantitative analysis using basic hydrologic, hydraulic, and sediment transport engineering concepts. Such analyses should include evaluation of flood history, channel hydraulic conditions (water surface profile analysis) and basic sediment transport analyses (watershed sediment transport, incipient motion analysis, and scour calculations). An analysis of this type is adequate for most locations in Colorado. If not, a more complex quantitative analysis based on detailed mathematical modeling and/or physical hydraulic models should be considered. This multilevel approach is presented in HEC-20.

Additional geotechnical data needs aside from the typical abutment and channel borings shall be identified by the Hydraulics Engineer.

10.4.2 Scour Types

Bridge scour shall be evaluated as interrelated components. The major components of scour are:

- General scour (aggradation and degradation);
- Plan form change (lateral channel movement);
- Contraction scour; and
- Local scour (pier and abutment).

Aggradation and Degradation

This is long term bed elevation changes due to nature or the activities of man within a watershed. History of stream bed elevation changes at existing bridge sites is documented in the inspection records maintained by the Bridge Inspection Unit, Staff Bridge Branch. Published flood studies will often have information regarding the tendency of streams to aggrade and/or degrade in certain areas.

Plan Form Changes

Plan form changes are morphological changes such as meander migration or bank widening. The lateral movement of meanders can threaten bridge approaches as well as increase scour by changing flow patterns approaching a bridge opening. The comparison of past and present aerial photographs are informative to the behavior of a river.

Channel shifting can cause a significant change in the distribution of flow in the main channel upstream from a bridge. The change in flow pattern may alter the angle of attack on bridge piers and abutments which will cause additional local scour.

Contraction Scour

Contraction scour is the removal of streambed material caused by the bridge constricting the natural channel flow. This contraction generally causes flow to accelerate, increasing the flows erosive strength. Contraction scour is directly additive to aggregation, degradation, and local scour.

Local Scour

Local scour occurs around piers, abutments, the ends of guide banks, and any other obstructions to flow. Local scour is the result of the turbulence and local velocity vector changes caused by obstructions.

10.4.3 Scour Analysis

- Typically, the design-frequency scour depth is used to design the bridge abutments (pier caps, abutment retaining walls, etc); and the 500-year scour depth is used to design the bridge foundation.
- Scour depths shall be evaluated for the most severe scour conditions.
- The discharge occurring right before overtopping of the roadway is often the condition producing the most critical bridge scour. If the roadway is not overtopped for storms less than the 100 years, the 100-year flood should be taken as the worse condition for scour.
- For all designs, scour should not cause failure of the bridge structure for the 500-year flood.
- Prior to the completion of the final geology report, a multi-disciplinary team of hydraulics, geotechnical, and structural engineers shall meet after the borings are taken to assess the validity of scour depth calculations.
- The presence of riprap will only mitigate or lessen the amount of scour. Scour is typically calculated in the absence of riprap. For design, riprap cannot be used to completely eliminate scour. Riprapped guide banks are acceptable to mitigate abutment scour. With a guide bank, the abutment scour will be moved upstream from the bridge to the upstream toe of the guide bank. For a guide bank to work effectively, the riprap on the bank must be maintained at all times. The predicted scour depth should dictate the elevation and amount of riprap to be used. (see Chapter 17 and HEC- 11).
- Under certain situations, a flood less than the 500-year flood could cause the worse case scour conditions. This situation might occur when the overtopping flood is less than the 500-year flood. The overtopping flood should be evaluated along with the 500-year scour. The worse case scour condition with no relief should use the 500-year discharge for the bridge foundation design.

10.4.4 Data Requirements

Bed Material

Obtain channel bed and foundation material information for all bridge crossings upstream and downstream. Bed material should be used to analyze the potential for armoring and reduced scour.

Geometry

Obtain existing stream and flood plain cross sections, stream profile, site plan and the stream's present, and where possible, historic geomorphic plan form. Also, analyze the bridge site with respect to such things as other bridges in the area, tributaries to the stream or close to the site, bed rock controls, man made controls (dams, old check structures, river training works, etc.), and downstream confluences with other streams. Locate any "headcuts" due to natural causes or such things as gravel mining operations. Any data related to plan form changes such as meander migration and the rate at which they may be occurring is useful.

Historic Scour

Obtain any scour data on other bridges or similar facilities along the stream. If the project involves the replacement of an existing structure, scour data on the old structure can be important. Scour holes often fill after floods so that finding historic scour depths is difficult. Geology boring logs can show some of this information.

CDOT Bridge Branch, bridge inspection records are a good source of historic data for obtaining aggradation or degration at bridge sites.

Hydrology

Identify the character of the stream hydrology; i.e., perennial, intermittent as well as whether it is subject to sudden peaks floods or to slow long duration floods resulting from general frontal rainstorms or snowmelt.

Stream Morphology

Classify the geomorphology of the site; i.e., such things as whether it is a floodplain stream, crosses a delta, or crosses an alluvial fan; youthful, mature or old age. From this information we can assess whether changes in the sediment transport of the stream are going to be quick or gradual.

The form and shape of the stream path created by its erosion and deposition characteristics comprise its morphology. A stream can be braided, straight, or meandering, or it can be in the process of changing from one form to another as a result of natural or manmade influences. A historical study of the stream morphology at a proposed stream-crossing site is mandatory. This study shall also include an assessment of any long-term trends in aggradation or degradation. Braided streams and alluvial fans shall especially be avoided for streamcrossing sites whenever possible. Bridges should not be located on meander bends because of lateral movement by the river.

10.4.5 <u>Assessing and Plotting Scour</u>

The procedures and guidelines outlined in HEC- 18 should be used to compute and assess bridge scour. HEC-18 includes several examples of scour calculations and a procedure to plot scour depths.

A plot of scour depths corresponding to the design flow and the 500-year discharge shall be included in the design plans. Scour is usually plotted as part of the Bridge Hydraulic Information Transmittal sheet and the Bridge General Layout Sheet.

10.4.6 Scour Resistant Materials

Caution is necessary in determining the scour resistance of bed materials and the underlying strata. With noncohesive material, the passage of a single flood may result in the predicted scour depths. Conversely, in scour resistant material the maximum predicted depth of scour may not be realized during the passage

of a particular flood; however, some scour resistant material will be lost. Commonly, this material is replaced with more easily scoured material. Thus, at some later flood the predicted scour depth maybe reached. Serious scour has been observed to occur in materials commonly perceived to be scour resistant such as consolidated soils and glacial till, as well as bedrock streams and streams with gravel and boulder beds.

Where bedrock is above the calculated scour depth, an evaluation shall be made of the bedrock's scour resistance by a multidisciplinary design team consisting of the hydraulic engineer, the geotechnical engineer, and the structural engineer. The mutidisciplinary design team shall determine the resistance to scour considering the following:

- Experience in the project area;
- Uniformity of the bedrock material;
- Type of foundation and its effect on the bedrock. Blasting for excavation of spread footers and driving piling may fracture the bedrock and should be avoided;
- Evaluation of undisturbed core samples considering:
 - Rock quality designation;
 - Unconfined compressive strength; and
 - Orientation and condition of natural jointing or fractures in the core sample;
- Relative duration of the scouring flood. A 500-year snowmelt flood may last for weeks, while a rainfall flood may last only several hours and large basin rainfall floods may last days;
- Depth of bedrock to channel invert and frequency of bedrock exposure to scour and air. Wet-dry cycles in shale can reduce it to highly erodible particles.

10.4.7 <u>Armoring</u>

Armoring occurs because a stream or river is unable, during a major flood, to move the more coarse material comprising either the bed or, if some bed scour occurs, its underlying material. Scour may occur initially but later become arrested by armoring before the full scour potential is reached again for a given flood magnitude. When armoring occurs the coarser bed material will tend to remain in place or quickly redeposit, form a layer of riprap-like armor on the stream bed or in the scour holes and thus limiting further scour for a particular discharge. This armoring effect can decrease scour hole depths which were predicted based on formulae developed for sand or other fine material channels. When a larger flood occurs than used to cause the previous scour hole depths, resultant scour will probably penetrate deeper until armoring again occurs at some lower threshold.

Armoring may also cause bank widening. Bank widening encourages rivers or streams to seek a more unstable, braided regime. Such instabilities may pose serious problems for bridges as they encourage stream migration. Bank widening also spreads the approach flow distribution which results in a more severe bridge opening contraction.

10.4.8 <u>Preventive/Protection Measures</u>

Based on an assessment of potential scour provided by the Hydraulic Engineer, the structural designers can incorporate design features that will prevent or mitigate scour damage at piers. Spread footings should be used only where the stream bed is extremely stable below the footing or where the spread footing is founded at a depth below the maximum scour.

Rock riprap can be used, if sufficient size and density is available, to armor abutment fill slopes and the area around the base of piers. Riprap design information is presented in Chapter 17.

Whenever possible, clearing of vegetation upstream and downstream of the toe of the embankment slope should be avoided. Embankment overtopping may be incorporated into the design but should be located well away from the bridge abutments and superstructure. Guide banks are recommended to align the approach flow with the bridge opening and to prevent scour around the abutments. Guide banks, embankments, and abutments shall be protected by adequately sized rock riprap or with other type revetments approved by CDOT.

10.5 DESIGN ISSUES

10.5.1 Introduction

Streams are dynamic natural systems which, as a result of the encroachment caused by elements of a stream-crossing system, will respond in a way that may well challenge even an experienced hydraulic engineer. The complexities of the stream response to encroachment demand that:

- 1. Hydraulic engineers must be involved from the outset in the choice of alternative stream crossing locations, and
- 2. At least some of the members of the engineering design team must have extensive experience in the hydraulic design of stream-crossing systems. Hydraulics engineers should also be involved in the solution of stream stability problems at existing structures.

This section discusses qualitatively some of the design issues which contribute to the overall complexity of spanning a stream with a stream-crossing system. A much more thorough discussion of design philosophy and design considerations is found in the AASHTO Highway Drainage Guidelines, "Hydraulic Analyses for the Location and Design of Bridges".

10.5.2 Location of Stream Crossing

All bridges crossing streams must have two signs showing the name of the stream. Although many factors, including nontechnical ones, enter into the final location of a stream-crossing system, the hydraulics of the proposed location must have a high priority. Hydraulic considerations in selecting the location include floodplain width and roughness, flow distribution and direction, stream type (braided, straight, or meandering), stream regime (aggrading, degrading, or equilibrium), and stream controls. Bridge skew should be minimized provided it does not change regime or flow patterns. The hydraulics of a proposed location also affect environmental considerations such as aquatic life, wetlands, sedimentation, and stream stability. Finally, the hydraulics of a particular site determine whether or not certain national objectives such as wise use of flood plains, reduction of flooding losses, and preservation of wetlands can be met.

10.5.3 <u>Coordination Permits/Approvals</u>

The interests of other government agencies must be considered in the evaluation of a proposed stream-crossing system, and cooperation and coordination with these agencies, especially water resources planning agencies, must be undertaken. Coordination with FEMA is required when a:

proposed crossing encroaches on a regulatory floodway and would require an amendment to the floodway map,

proposed crossing encroaches on a floodplain where a detailed FEMA study has been performed but no floodway has been designated and the maximum one foot increase in the base flood would be exceeded,

community is expected to enter into the regular program within a reasonable period and detailed floodplain studies are underway, and

community is participating in the emergency program and the base flood elevation in the vicinity of insurable buildings is increased by more than one foot.

Whenever practicable, the stream-crossing system shall avoid encroachment into the FEMA regulated floodway (see Chapter 2). When this is not feasible, modification of the floodway itself shall be considered. If neither of these alternatives is feasible, FEMA regulations for "floodway encroachment where demonstrably appropriate" shall be met.

Designers of stream-crossing systems must be cognizant of relevant local, State, and Federal laws and permit requirements. Permits for construction activities in navigable waters are under the jurisdiction of the U.S. Army Corps of Engineers. Applications for Federal permits may require environmental impact assessments under the National Environmental Policy Act of 1969. In Colorado provisions of Senate Bill 40 need to be addressed on any stream crossing. (see Chapter 2 for more information)

10.5.4 Deck Drainage

Improperly drained bridge decks can cause numerous problems including corrosion, icing, and hydroplaning. Ideally bridges shall be placed on crest vertical profile grades and bridges on sags vertical curves should be avoided. A superelevation transition on a bridge is not acceptable because of cross flow problems.

Whenever possible, bridge decks should be watertight and all deck drainage should be carried to the ends of the bridge. Drains at the end of the bridge should have sufficient inlet capacity to carry all of the minor drainage. A curb roll is required from the bridge ends to the end of the guard rail. At the end of .this curb roll an inlet and pipe (preferred design) or well depressed rundown with a transition from the curb roll is required to convey the drainage down the fill slope.

Where it is necessary to intercept deck drainage at intermediate points along the bridge, the design of the interceptors shall conform to the HEC-21, "Design Of Bridge Deck Drainage" procedures.

Even short span bridges should provide storm drains at both ends of the bridge to minimize flow onto the the bridge. Combination curb opening and grated inlets should be used.

10.5.5 <u>Waterway Enlargement</u>

There are situations where roadway and structural constraints dictate the vertical positioning of a bridge and result in a small vertical clearance between the low chord and the channel flowline or overbank. Significant increases in span length provide small increases in effective waterway opening in these cases.

It is possible to increase the effective area by excavating a flood channel through the reach affecting the hydraulic performance of the bridge. There are, however, several factors that must be accommodated when this action is taken:

- The flow line of the new enlarged channel should be set above the stage elevation of the ordinary highwater.
- The flood channel must extend far enough up and downstream of the bridge to establish the desired flow regime through the affected reach.
- The flood channel must be stabilized to prevent erosion and scour.

10.5.6 <u>Auxiliary Opening</u>

The need for auxiliary waterway openings, or relief openings as they are commonly termed, arises on streams with wide floodplains. The purpose of openings on the floodplain is to pass a portion of the flood flow in the floodplain when the stream reaches a certain stage. It does not provide relief for the principal waterway opening in the sense that an emergency spillway at a dam does, but has predictable capacity during flood events.

Basic objectives in choosing the location of auxiliary openings include:

- Maintenance of flow distribution and flow patterns;
- Accommodation of relatively large flow concentrations on the floodplain;
- Avoidance of flood plain flow along the roadway embankment for long distances; and
- Accommodation of Colorado Division of Wildlife requests for minimal flows for wildlife.

The technological weakness in modeling auxiliary openings is in the use of one-dimensional models to analyze two-dimensional flow. The development of 2-D models is a major step toward more adequate analysis of complex streamcrossing systems.

The most complex factor in designing auxiliary openings is determining the division of flow between the two or more structures. If incorrectly proportioned, one or more of the structures may be overtaxed during a flood event. The design of auxiliary openings should usually be generous to guard against that possibility.

10.5.7 Bridge Rehabilitation

Often an existing bridge over a drainageway only needs widening and rehabilitation. For these types of bridges, the hydraulic engineer must consider all the same design criteria as for a completely new structure and come to a decision whether it is cost effective to rehabilitate the existing bridge or replace it with a new bridge based on the hydraulics.

The most important issues that should be addressed are:

- Capacity of the existing bridge;
- Overtopping depths;
- Scour and foundation condition;
- Freeboard;
- Service life of the existing bridge;
- Flood history; and
- The importance of the highway.

The decision to replace or rehabilitate a bridge can be made based on engineering judgement including a detailed cost-benefit analysis depending on the complexity of the situation. Any hydraulic analysis on rehabilitated bridges should be well documented and coordinated with the Structural Engineer or Geologist. All rehabilitated bridges over drainageways should have a detailed hydraulic analysis performed.





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