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# CONSTRUCTION AND DESIGN SOIL PROPERTY CORRELATION

APPLIED RESEARCH &
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16. Abstract

The main goal of this research study was to develop correlations among R-value, resilient modulus, and soil's basic properties for available AASHTO soil types in databases in Colorado. In this study, an extensive database of systematically conducted resilient modulus and R-value tests along with the basic soil properties for Colorado soils was established. The R-value test database was developed by acquiring and analyzing many soil reports with R-value tests performed by CDOT. A multiple regression analysis was performed to correlate the R-value as the dependent variable with the fundamental soil properties as the independent variables. The resilient modulus database was developed by evaluating the test results of over 200 previously conducted tests. The MEPDG-adopted generalized resilient modulus model was used in the statistical analysis and correlations were developed for estimating the resilient modulus model parameters $k_i$ based on the soil index properties. Because the previously conducted resilient modulus tests did not have all of the needed accompanying soil index test results, AASHTO soil type A-2-4 was selected as the target soil type for a more detailed and systematic testing and regression analysis. Thirty specimens of A-2-4 soil type were collected and tested for resilient modulus, R-value, and their basic physical properties through tests such as the sieve analysis, hydrometer testing, standard Proctor test, and Atterberg limits. A multiple regression analysis and stepwise regression analysis were performed to correlate the experimentally measured resilient modulus values with the fundamental soil properties; and the verified regression models for the A-2-4 soil type were shown in this study to provide superior performance over the existing published models.

Implementation

Based on the results of this research study, several regression models were developed to be used for estimation of the R-value for AASHTO soil types A-1-a, A-1-b, A-2-4, A-2-6, A-2-7, A-4, A-6, and A-7-6. In addition, regression models for prediction of resilient modulus for AASHTO soil types A-2-4 were presented in this study. The same testing program and regression methodology can be applied to other soil types of interest.

17. Keywords
Resilient modulus, R-value, Colorado soils, A-2-4 soil, regression models, basic soil properties.

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# Construction and Design Soil Property Correlation

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# EXECUTIVE SUMMARY

Proper structural design of pavement systems requires knowing the resilient modulus of the soil as this parameter is a proven predictor of the stress-dependent elastic modulus of soil materials under traffic loading. In addition, the R-value test is conducted using a device called a stabilometer, where the material's resistance to deformation is expressed as a function of the ratio of the transmitted lateral pressure to that of the applied vertical pressure. Both tests are expensive and time consuming; however, establishing accurate and reliable correlations between the test results and the soil's physical properties, in lieu of laboratory testing, can save a considerable amount of time and money in the analysis and quality control process. For these reasons, correlations are typically used for estimating the resilient modulus and R-value for soils. The variability of a given soil type in different regions and states requires developing modified and specific correlations for each state based on statistical analysis of the statewide soil data collected. The main goal of this research study was to develop correlations among R-value, Resilient modulus, and soil's basic properties for available AASHTO soil types in databases in Colorado.

In this research study, an extensive database of systematically conducted R-value tests was developed by acquiring and analyzing paper copies of many soil reports with R-value tests performed by CDOT. A multiple regression analysis was performed to correlate the R-value as the dependent variable with the fundamental soil properties as the independent variables. The prediction models for the R-values of available AASHTO soil types had adjusted $R^2$ values ranging from 0.529 to 0.944. The results showed that 98, 90, 73, 64, 57, 80, 48, and 46% of the R-values obtained from the prediction equations for A-1-a, A-1-b, A-2-4, A-2-6, A-2-7, A-4, A-6, and A-7-6, respectively, fell within ±20% of the laboratory R-values.

In this research study, Ground Engineering Consultants, as the main source for resilient modulus test data in Colorado, was contracted to collect and compile detailed reports of the resilient modulus and the associated basic soil properties for Colorado soils. This task included identifying historical resilient modulus, gradation, particle size analysis, R-value, maximum dry density, optimum moisture content, and Atterberg limits data, which were collected at Ground Engineering's Commerce City laboratory. The resilient modulus data for 203 test samples were obtained but given that the tests were mainly conducted for evaluation of the resilient modulus data, a large portion of the tests did not have the accompanying soil index tests. AASHTO soil types A-1-b, A-2-4, A-4, and A-6 were present in the developed database for further regression analysis.

The generalized resilient modulus model that was developed through NCHRP Project 1-28A was adopted in this study. This model is widely accepted and applicable to all soil types. A multiple regression analysis was performed to correlate the resilient modulus value as the dependent variables with the fundamental soil properties as the independent variables. All the fundamental soil properties available in the established database were treated as potential independent variables and numerous combinations of soil properties (independent variables) were used in the regression analysis. The adjusted $R^2$ values obtained for the prediction models for the $k_{1\text{-}3}$ coefficients, using the generalized resilient modulus model, ranged from 0.488 to 0.903. The prediction models showed that 87, 57, and 20% of the resilient modulus values obtained from the prediction equations for A-1-b, A-4, and A-6, respectively, fell within ±20% of the laboratory resilient modulus values. It was determined that two important independent variables, being the percent silt and clay content, were not available in the established database of resilient modulus tests.

AASHTO soil type A-2-4 was the most prevalent in the established database, and upon the recommendation of the study panel, was selected as the target soil type for a more detailed and systematic testing and regression analysis. To determine the physical sample properties, the following standard laboratory tests were conducted for each sample of A-2-4 soil: (a) Grain size distribution (sieve and hydrometer analyses) following ASTM D422-63, Atterberg limits (liquid limit, *LL* and plastic limit, *PL*) following ASTM D4318, modified Proctor test to determine the optimum moisture content and maximum dry unit weight following ASTM D1557, and R-value test according to AASHTO T190. A multiple regression analysis and stepwise regression analysis were performed to correlate the experimentally-measured resilient modulus values for A-2-4 soil type with the fundamental soil properties; and the MEPDG-adopted generalized resilient modulus model was used in our statistical analysis. The adjusted $R^2$ for the prediction model was 0.827, which is very high. The majority of the predicted resilient modulus values for A-2-4 soil were within +/- 50% of the measured resilient modulus value and the proportion of points where the predicted resilient modulus value was within +/- 20% of the measured resilient modulus value was 65.7%. Using the developed prediction models in this study can be expected to save a considerable amount of time and money in testing and analyzing soil properties. In this study, in addition to the development of models for prediction of resilient modulus and R-value from basic soil properties, direct correlations between the resilient modulus and R-value were explored for A-2-4 soil type and four models were developed. Higher quality models require information about soil basic properties beyond the R-value and considering the higher accuracy of resilient modulus models for the A-2-4 soil type, we suggest using the resilient modulus models which incorporate basic soil properties and the stress conditions instead of estimating the resilient modulus solely based on the R-value number.

## Implementation Statement

Based on the results of this research study, several regression models were developed to be used for estimation of the R-value for AASHTO soil types A-1-a, A-1-b, A-2-4, A-2-6, A-2-7, A-4, A-6, and A-7-6. In addition, regression models for prediction of resilient modulus for AASHTO soil types A-1-b, A-2-4, and A-4 were presented in this study. Two important independent variables of percent silt and clay content were not available in the established database of resilient modulus tests for A-1-a and A-4 soils; therefore, we suggest using the regression models for A-1-b and A-4 soils with caution. The systematic testing in this study on A-2-4 soil type resulted in high quality regressions and similar testing program and regression analysis methodology can be applied to other soil types of interest. We suggest performing the hydrometer testing for all soil types of interest since the percent silt and clay content in the soils were identified as important independent variables from our regression analysis and the literature. In addition, for each soil AASHTO soil type, we recommend testing at least 30 samples using the same testing program to obtain the same basic soil properties for all studied soil specimens. The existing CDOT database lacked the exudation pressure and moisture content information for soil specimens that were tested for R-values, and the reported R-value was the final value corresponding to 300 psi exudation pressure. We suggest recording the moisture content and the exudation pressure for each specific R-value test performed by CDOT in its database. Section 10 “Conclusions and Implementation” presents a summary of the research work.

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# 1. Introduction

The aim of this research study was to develop correlations for estimating the resilient modulus and R-values of available Colorado soils in established databases from basic physical soil properties. Using such correlation equations would reduce the time and cost for testing and analyzing the material properties for each pavement project. In addition, the need to leave the construction field trailer to perform additional R-value or resilient modulus testing to verify the quality of construction materials also could be reduced if not totally eliminated.

This project report consists of the following main parts: (a) completed synthesis report of correlations among R-value, Resilient Modulus, and basic (index) soil properties; (b) statistical analysis of the CDOT soil archive database for R-values; (c) statistical analysis of the established resilient modulus database from Ground Engineering Consultants for all available soil types, and (d) A-2-4 soil testing and statistical analysis of the obtained test results.

# 2. Synthesis on R-value and its correlations with basic soil properties

## 2.1 R-value testing procedure

The R-value test, which is used to measure the strength of the subgrade, subbase, and base course materials in pavements, is used by many transportation agencies as the quantifying parameter in evaluating the subgrade and base course for pavement design as well as a criterion for acceptance of aggregates for pavement systems. The R-value test is conducted using a device called the stabilometer, which expresses the material’s resistance to deformation as a function of the ratio of the transmitted lateral pressure to the applied vertical pressure. The test can be performed according to AASHTO T190, ASTM D2844, and Colorado CP-L3101 procedures.

R-values can range from zero to 100 (low soil strength to high soil strength). The R-value test is time consuming, requires specific equipment and trained personnel, and is not commonly available at commercial testing facilities. Establishing accurate and reliable correlations between the soil’s

R-value and physical properties can save a considerable amount of time and money in testing and analyzing the properties of construction materials.

The R-value testing process consists of four required steps: (a) compacting a 4-inch specimen using a kneading compactor; (b) loading the compacted specimen in a steel mold until enough moisture is squeezed out of the specimen to light up five out of six bulbs on an exudation indicator device and recording the exudation pressure; (c) soaking the specimen for 24 hours; and (d) testing the specimen in a Hveem stabilometer to measure the R-value. The R-value is defined as:

$$\text{R-value} = 100 - \frac{100}{\frac{2.5}{D_2}\left(\frac{P_v}{P_h} - 1\right) + 1} \tag{1}$$

where $P_v$ and $P_h$ are the vertical and horizontal (lateral) pressures applied/experienced by the soil specimen, respectively, and $D_2$ is the number of turns of the screw to inject oil into the chamber. The lateral pressure depends on the stiffness of the soil. For example, for a soil specimen with an R-value of 100, the soil will not deform under the vertical load. In contrast, an R-value of zero indicates that the soil will deform like a liquid in the same amount laterally and vertically.

Reporting the R-value for each soil specimen requires testing at least three specimens at three different moisture contents and exudation pressures. The R-value is obtained by extrapolating at an exudation pressure of 300 psi. Figure 1 shows the typical R-value test results for a soil specimen.

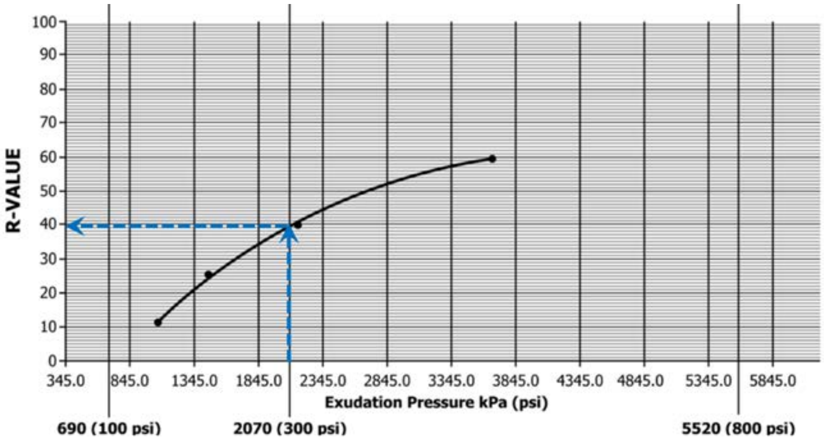


**Figure 1. Typical presentation of R-value results as a function of exudation pressure (Modified after ASTM D2844)**

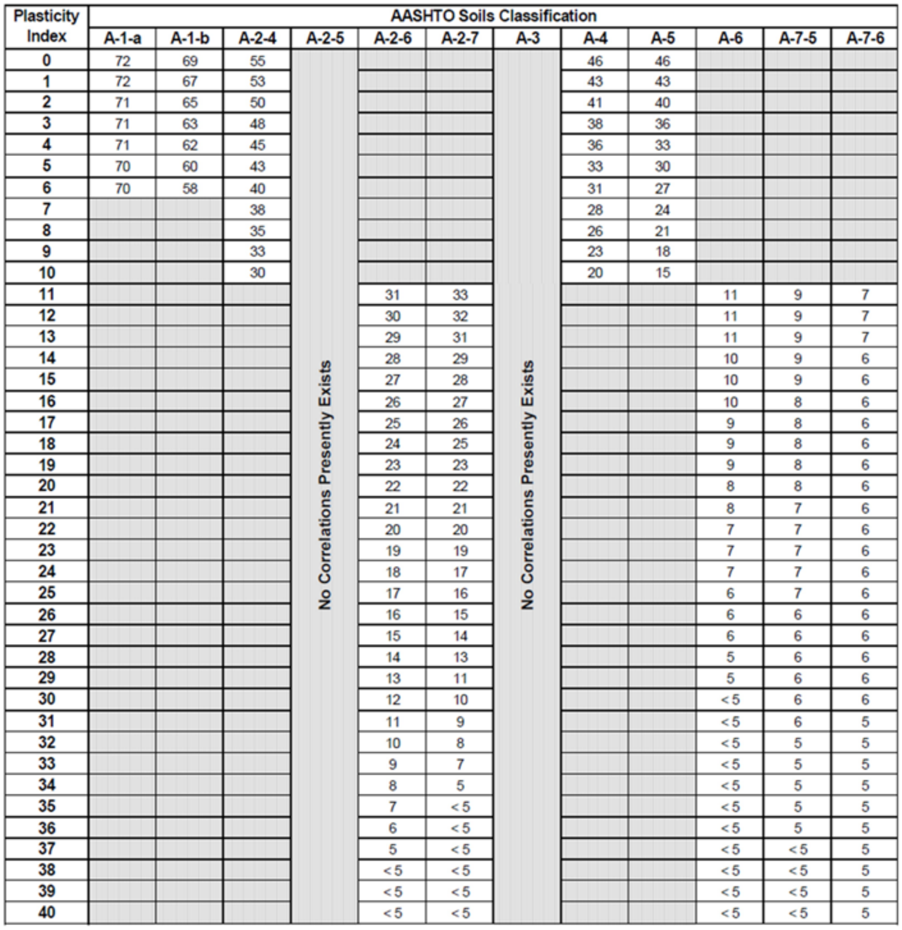
### 2.2 Correlations between R-value and soil index properties

The New Mexico Department of Transportation (NMDOT) developed a chart for estimating R-values for soils from different AASHTO classifications based on the plasticity index (PI) values at 60% reliability (i.e., 60% chance of having estimated values from the charts being equal or greater than the actual R-values). The estimation chart is presented in Table 1.

Since the R-value indicates the soil strength and stiffness, it cannot be properly represented by the PI of the soil only. Lenke et al. (2006) performed R-value tests for 15 collected specimens from different sites and compared the actual R-values against the estimated R-values obtained using the NMDOT R-value chart shown in Table 2 and reported that the best fit line was an $R^2$ value of 0.5837. The closer $R^2$ was to 1.0, the better the regression model. This average level coefficient of determination confirmed that the PI alone cannot be a good indicator of the R-value for soils. In addition, Lenke et al. (2006) proposed correlations with the following three ASTM field tests: (a) Clegg hammer, (b) dynamic cone penetrometer, and (c) soil stiffness gauge (GeoGauge). The AASHTO soil classification system considers granular materials with a maximum of 35% passing of sieve No. 200 (75 µm) and fine-grained soils with passing greater than 35%. However, Lenke et al. (2006) proposed a dividing limit of 20% for passing of sieve No. 200. In other words, soils with more than 20% fines were considered as fine-grained soils while soils with less than 20%

fines were considered as coarse-grained soils. Separate correlations also were proposed for the fine- and coarse-grained soils to relate the R-value with the three field tests. Table 2 presents a summary for the coefficient of determination for the actual versus estimated R-values.

**Table 1. R-value estimation chart (60% reliability) (after Lenke et al., 2006)**



| Plasticity Index | AASHTO Soils Classification | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A-1-a | A-1-b | A-2-4 | A-2-5 | A-2-6 | A-2-7 | A-3 | A-4 | A-5 | A-6 | A-7-5 | A-7-6 |
| 0 | 72 | 69 | 55 | No Correlations Presently Exists | | | No Correlations Presently Exists | 46 | 46 | | | |
| 1 | 72 | 67 | 53 | | | | | 43 | 43 | | | |
| 2 | 71 | 65 | 50 | | | | | 41 | 40 | | | |
| 3 | 71 | 63 | 48 | | | | | 38 | 36 | | | |
| 4 | 71 | 62 | 45 | | | | | 36 | 33 | | | |
| 5 | 70 | 60 | 43 | | | | | 33 | 30 | | | |
| 6 | 70 | 58 | 40 | | | | | 31 | 27 | | | |
| 7 | | | 38 | | | | | 28 | 24 | | | |
| 8 | | | 35 | | | | | 26 | 21 | | | |
| 9 | | | 33 | | | | | 23 | 18 | | | |
| 10 | | | 30 | | | | | 20 | 15 | | | |
| 11 | | | | | 31 | 33 | | | | 11 | 9 | 7 |
| 12 | | | | | 30 | 32 | | | | 11 | 9 | 7 |
| 13 | | | | | 29 | 31 | | | | 11 | 9 | 7 |
| 14 | | | | | 28 | 29 | | | | 10 | 9 | 6 |
| 15 | | | | | 27 | 28 | | | | 10 | 9 | 6 |
| 16 | | | | | 26 | 27 | | | | 10 | 8 | 6 |
| 17 | | | | | 25 | 26 | | | | 9 | 8 | 6 |
| 18 | | | | | 24 | 25 | | | | 9 | 8 | 6 |
| 19 | | | | | 23 | 23 | | | | 9 | 8 | 6 |
| 20 | | | | | 22 | 22 | | | | 8 | 8 | 6 |
| 21 | | | | | 21 | 21 | | | | 8 | 7 | 6 |
| 22 | | | | | 20 | 20 | | | | 7 | 7 | 6 |
| 23 | | | | | 19 | 19 | | | | 7 | 7 | 6 |
| 24 | | | | | 18 | 17 | | | | 7 | 7 | 6 |
| 25 | | | | | 17 | 16 | | | | 6 | 7 | 6 |
| 26 | | | | | 16 | 15 | | | | 6 | 6 | 6 |
| 27 | | | | | 15 | 14 | | | | 6 | 6 | 6 |
| 28 | | | | | 14 | 13 | | | | 5 | 6 | 6 |
| 29 | | | | | 13 | 11 | | | | 5 | 6 | 6 |
| 30 | | | | | 12 | 10 | | | | <5 | 6 | 6 |
| 31 | | | | | 11 | 9 | | | | <5 | 6 | 5 |
| 32 | | | | | 10 | 8 | | | | <5 | 5 | 5 |
| 33 | | | | | 9 | 7 | | | | <5 | 5 | 5 |
| 34 | | | | | 8 | 5 | | | | <5 | 5 | 5 |
| 35 | | | | | 7 | <5 | | | | <5 | 5 | 5 |
| 36 | | | | | 6 | <5 | | | | <5 | 5 | 5 |
| 37 | | | | | 5 | <5 | | | | <5 | <5 | 5 |
| 38 | | | | | <5 | <5 | | | | <5 | <5 | 5 |
| 39 | | | | | <5 | <5 | | | | <5 | <5 | 5 |
| 40 | | | | | <5 | <5 | | | | <5 | <5 | 5 |

**Table 2. Performance of suggested field-based estimation methods (Lenke et al., 2006)**

| Method of Estimation | Coefficient of Determination ($R^2$) for estimated versus actual R-value |
|---|---|
| NMDOT Chart | 0.5837 |
| Clegg Hammer | 0.9477 |
| Dynamic Cone Penetrometer | 0.9636 |
| GeoGauge with sand interface | 0.9797 |

The R-value also has been directly related to the soil index properties. Table 3 provides the correlation developed by the Arizona Department of Transportation (Hashiro, 2005). In this correlation, the R-value is estimated based on the PI of the soil and the percent passing of sieve No. 200. The scope of this correlation was very limited and only the PI and the percent of fines (PF) were considered as the two independent variables. For non-plastic soils, this correlation relies solely on the percent passing of sieve No. 200, which is likely insufficient as the sole predictor of the strength and stiffness of soils. In other words, because of the dependence of the model on the soil's PI, the effectiveness of applying this model to non-plastic soils is questionable. The Arizona DOT Materials Design Manual proposed the following $\log_{10}$ model:

$$\log_{10}(\text{R-value}) = b_0 + b_1(\text{PI}) + b_2(\text{PF}) \tag{2}$$

where $b_i$ terms are regression coefficients.

Miller (2009) conducted statistical analysis to establish correlations between the R-value of soil specimens from six different districts in Idaho and the soil's basic property data (i.e., soil classification, Atterberg limits, and PF). The distribution of R-values clearly showed some relation to the PF and PI of the soils because they are used in the Unified Soil Classification System (USCS) to differentiate the soil classes. Miller (2009) observed a widespread R-value for high plasticity soils (i.e., clayey soils) and higher R-value ranges were observed for coarser soils with lower percent of fines. The observations from Miller's work can be summarized as follows: (a) coarse-grained soils with 12 percent fines or less typically had R-values greater than 40; (b) soils with a PI greater than 50 generally had R-values less than 20; (c) non-plastic and low-plasticity soils had R-values that were spread across a wide range; and (d) soils with resistivity exceeding 8,000 ohm-cm almost always had R-values greater than 60. In their study, four soil attributes were considered as the potential independent variables: (1) the USCS classification code, which is the number indicating the soil's classification based on the USCS classification system; (2) the liquid limit; (3) the PI; and (4) the PF. Among these four parameters, the liquid limit did not add any significant information to the regression models and was later dropped. The model presented in equation 3 reported the highest $R^2$ for all the soils combined, which outperformed the other regression models.

**Table 3. R-value at 300 psi exudation pressure (re-produced from Hashiro, 2005)**

| Plasticity Index | Percent Passing Sieve No. 200 (Percent fines) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 | 66 | 69 | 72 | 75 | 78 | 81 | 84 | 87 | 90 | 93 | 96 |
| **0** | 100 | 96 | 92 | 88 | 85 | 81 | 78 | 75 | 72 | 69 | 66 | 63 | 61 | 58 | 56 | 54 | 52 | 49 | 47 | 45 | 44 | 42 | 40 | 39 | 37 | 35 | 34 | 33 | 31 | 30 | 29 | 28 | 27 |
| **1** | 96 | 92 | 89 | 85 | 81 | 78 | 75 | 72 | 69 | 66 | 64 | 61 | 58 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 31 | 30 | 29 | 28 | 27 | 26 |
| **2** | 92 | 89 | 85 | 82 | 78 | 75 | 72 | 69 | 66 | 64 | 61 | 59 | 56 | 54 | 52 | 50 | 49 | 46 | 44 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 31 | 30 | 29 | 28 | 27 | 26 | 25 |
| **3** | 89 | 85 | 82 | 79 | 75 | 72 | 69 | 67 | 64 | 61 | 59 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 |
| **4** | 86 | 82 | 79 | 76 | 72 | 70 | 67 | 64 | 61 | 59 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 41 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 |
| **5** | 82 | 79 | 76 | 73 | 70 | 67 | 64 | 62 | 59 | 57 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 41 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 |
| **6** | 79 | 76 | 73 | 70 | 67 | 64 | 62 | 59 | 57 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 41 | 39 | 37 | 36 | 35 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 |
| **7** | 76 | 73 | 70 | 67 | 64 | 62 | 59 | 57 | 55 | 52 | 50 | 48 | 46 | 44 | 43 | 41 | 39 | 38 | 36 | 35 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 |
| **8** | 70 | 70 | 67 | 65 | 62 | 59 | 57 | 55 | 52 | 50 | 48 | 46 | 44 | 43 | 41 | 39 | 38 | 36 | 35 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 |
| **9** | 68 | 67 | 65 | 62 | 60 | 57 | 55 | 53 | 50 | 48 | 46 | 45 | 43 | 41 | 39 | 38 | 35 | 35 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 19 |
| **10** | 65 | 65 | 62 | 60 | 57 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 39 | 38 | 36 | 33 | 33 | 32 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 19 | 18 |
| **11** | 63 | 62 | 60 | 57 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 40 | 38 | 36 | 35 | 32 | 32 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 |
| **12** | 60 | 60 | 58 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 40 | 38 | 36 | 35 | 34 | 31 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 | 17 |
| **13** | 58 | 58 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 40 | 38 | 37 | 35 | 34 | 32 | 30 | 30 | 29 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 | 17 | 16 |
| **14** | 56 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 40 | 38 | 37 | 35 | 34 | 32 | 31 | 29 | 29 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 |
| **15** | 53 | 53 | 51 | 49 | 47 | 45 | 43 | 42 | 40 | 38 | 37 | 35 | 34 | 32 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 |
| **16** | 51 | 51 | 49 | 47 | 45 | 43 | 42 | 40 | 38 | 37 | 35 | 34 | 33 | 31 | 30 | 29 | 26 | 26 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 |
| **17** | 49 | 49 | 47 | 45 | 44 | 42 | 40 | 38 | 37 | 35 | 34 | 33 | 31 | 30 | 29 | 28 | 25 | 25 | 24 | 22 | 22 | 22 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 | 14 |
| **18** | 48 | 47 | 45 | 44 | 42 | 40 | 39 | 37 | 35 | 34 | 33 | 31 | 30 | 29 | 28 | 27 | 24 | 24 | 23 | 22 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 15 | 15 | 14 | 14 | 13 |
| **19** | 46 | 46 | 44 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 31 | 30 | 29 | 28 | 27 | 26 | 24 | 23 | 23 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 |
| **20** | 44 | 44 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 23 | 23 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 |
| **21** | 42 | 42 | 40 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 22 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 |
| **22** | 41 | 41 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 21 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 |
| **23** | 39 | 39 | 37 | 36 | 34 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 20 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 |
| **24** | 38 | 37 | 36 | 35 | 33 | 32 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 19 | 19 | 19 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 |
| **25** | 36 | 36 | 35 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 19 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 |
| **26** | 35 | 35 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 |
| **27** | 33 | 33 | 32 | 31 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 19 | 17 | 17 | 16 | 16 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 | 9 |
| **28** | 32 | 32 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 19 | 18 | 17 | 17 | 15 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 | 9 | 9 |
| **29** | 31 | 31 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 | 16 | 16 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 | 9 | 9 | 9 |
| **30** | 30 | 30 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 | 17 | 15 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 |
| **31** | 29 | 29 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 | 14 | 13 | 12 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 |
| **32** | 27 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 14 | 14 | 14 | 13 | 12 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 |
| **33** | 26 | 26 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 |
| **34** | 25 | 25 | 24 | 23 | 22 | 21 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 | 13 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 |
| **35** | 24 | 24 | 23 | 22 | 22 | 21 | 20 | 19 | 18 | 17 | 17 | 16 | 15 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 |
| **36** | 23 | 23 | 22 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 15 | 15 | 14 | 14 | 13 | 12 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 |
| **37** | 23 | 23 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 |
| **38** | 22 | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 |
| **39** | 22 | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 |
| **40** | 21 | 20 | 19 | 18 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 10 | 10 | 10 | 10 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 |
| **42** | 19 | 19 | 18 | 17 | 16 | 16 | 15 | 14 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 9 | 10 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 5 | 6 | 5 | 5 |
| **44** | 18 | 17 | 16 | 16 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 |
| **46** | 17 | 16 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 10 | 9 | 9 | 8 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 |
| **48** | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 7 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 |
| **50** | 14 | 14 | 13 | 12 | 12 | 11 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 |
| **52** | 13 | 13 | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 |
| **54** | 12 | 12 | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 |
| **56** | 11 | 11 | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 5 | 6 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 |
| **58** | 10 | 10 | 10 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 |
| **60** | 10 | 9 | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| **62** | 9 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| **64** | 8 | 8 | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| **66** | 8 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 |
| **68** | 7 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| **70** | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| **72** | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| **74** | 6 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 1 |
| **76** | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |

$$\text{R-Value} = b_0 + b_1(uscs) + b_2(PI) + b_3(PI \times PF)^{1/3} \tag{3}$$

Miller (2009) also presented several regression models for all the soil groups and for the non-plastic soil groups. The following equations present examples of the established regressions in Miller's study:

R-value regression model for all soil data:

$$\text{R-Value} = 55.91 + 1.1(uscs) - 0.41(PI) - 2.49(PI \times PF)^{1/3} \ (R^2 = 0.6353) \tag{4}$$

R-value regression model for all soil data with resistivity (RES) values:

$$\text{R-Value} = 20.15 + 2.27(uscs) + 0.51(PF) - 2.68(PF / uscs) + 0.48\sqrt[3]{RES} \ (R^2 = 0.4965) \tag{5}$$

R-value regression model for only non-plastic soils:

$$\text{R-Value} = 64.60 + 0.78(uscs) - 0.15(PF) + 0.51(PF / uscs) - 0.18\sqrt[3]{RES} \ (R^2 = 0.2064) \tag{6}$$

R-value regression model for only non-plastic soils based on maximum dry unit weight ( $\gamma_{d\max}$ ):

$$\text{R-Value} = 63.95 + 0.54(uscs) - 0.31(PF) + (PF / uscs) + 0.03\gamma_{d\max} \ (R^2 = 0.3160) \tag{7}$$

## 3. Synthesis on correlations of resilient modulus with soil basic properties

### 3.1 Resilient modulus test procedure

Resilient modulus ($M_r$), an important mechanical property of soil, is used for analysis and design of pavements. $M_r$ can properly describe the stress-dependent elastic modulus of soil materials under traffic loading. The Mechanistic Empirical Pavement Design Guide (MEPGD) requires the resilient modulus of soil and aggregate materials for the structural design of the layers. Several studies have shown the influence of $M_r$ on the thickness of the base course and the asphalt layers (Darter et al. 1992; Nazzal and Mohammad, 2010). Successful implementation of the MEPGD requires a comprehensive and accurate $M_r$ database for the local subgrade soils and base course materials. Three common approaches for estimating the resilient modulus are (a) conducting repeated load triaxial tests; (b) back-calculating the values from in situ tests such as Dynaflect and falling weight deflectometer (FWD); and (c) correlating the values with the soil's physical properties.

For laboratory measurement of resilient modulus, the repeated load triaxial test is conducted based on AASHTO T307: Determining the Resilient Modulus of Soils and Aggregate Materials. The resilient modulus is defined as the ratio of the repeated deviator stress (cyclic stress in excess of confining pressure, $\sigma_d$) to the recoverable resilient (elastic) strain ($\varepsilon_r$) in repeated dynamic loading, as defined below:

$$M_r = \frac{\sigma_d}{\varepsilon_r} \tag{8}$$

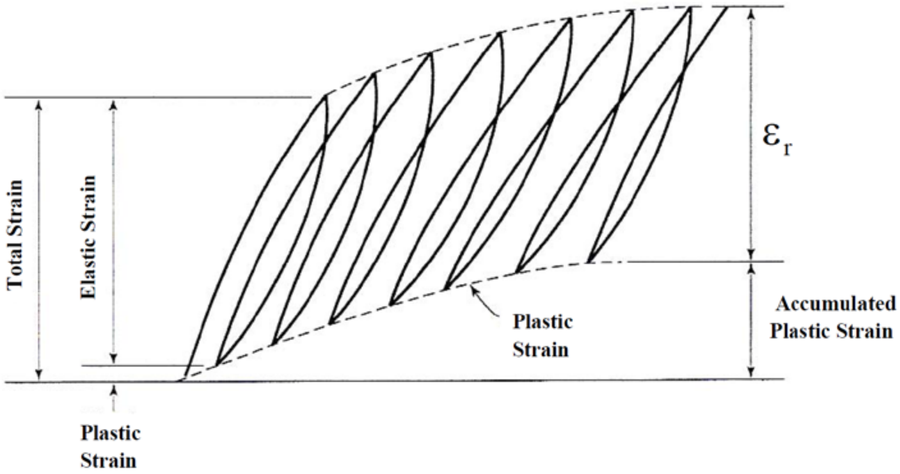


**Figure 2. Elastic and plastic strains during cyclic tests (after Coleri, 2007)**

The resilient modulus can be determined from the established correlations between laboratory or in-situ measurements of the resilient modulus and the soil's physical properties. Many researchers and transportation agencies, including CDOT, have studied the relations between the resilient modulus and the soil's properties to save some of the time and costs associated with laboratory testing. Examples include Carmichael and Stuart (1978), Drumm et al. (1991), Chang et al., (1994), Santha (1994), Von Quintus and Killingsworth (1998), George (2004), Titi et al. (2006), Malla and Joshi (2007), and Nazzal and Mohammad (2010).

Factors that influence the resilient modulus of subgrade soils include the soil's physical condition, stress level, soil group, loading conditions (i.e., confining stress and deviator stress), percent of fines, clay content, plasticity characteristics, particle size distribution, specific gravity, and organic content. Several past studies reported the interrelations between the above-mentioned variables and the resilient moduli of fine-grained and coarse-grained (granular) soils. In many cases, the relation is different for fine- and coarse-grained soils. For example, as the deviator stress increases, the resilient modulus of fine-grained soils decreases while the resilient modulus of coarse-grained soils increases. Also, the resilient modulus of fine-grained soils does not depend on the confining pressure while the increase in confining pressure for coarse-grained soils can significantly increase the resilient modulus. The effects of stress and moisture content on the resilient modulus values can be significant. For example, Li and Selig (1994) reported that for a fine-grained subgrade soil, the change in the stress state and moisture content can result in resilient modulus values ranging from 2,000-20,000 psi. It is therefore essential to understand the factors affecting the resilient modulus of subgrade soils. Factors that have significant effects on the magnitude of the resilient modulus can be grouped into the following categories: (a) stress state (the confining stress and deviatoric stress); (b) soil group and its structure; and (c) soil physical state.

Many different constitutive models were proposed to relate the resilient modulus to the deviator stress for coarse- and fine-grained soils. Those models include the bilinear model (Thompson and Robnett 1976); the semi-log model (Fredlund et al., 1977); the hyperbolic model (Drumm et al., 1990); and the octahedral model (Shackel, 1973). Some of the proposed models included bulk stress only for granular soils or deviator stress only for cohesive soils or both bulk stress and deviator stress. The bulk stress model is $M_r = k_1 P_a \left(\theta / P_a\right)^{k_2}$ where $\theta$ is the bulk stress (sum of the principal stresses), $P_a$ is the atmospheric stress (101.4 kPa), and $k_1$, $k_2$ are the material physical property parameters (Malla and Joshi, 2007). This model does not depend on the shear stress. For cohesive soils, the deviator stress model $M_r = k_1(\sigma_d)^{k_2}$, where $\sigma_d$ is the deviator stress was proposed. This model was found inappropriate for cohesive soils at greater depth and higher traffic loads as the effect of confining stress was ignored. The MEPDG adopted the generalized resilient modulus model that was developed through NCHRP Project 1-28A. This model is widely accepted and applicable to all types of subgrade materials. The resilient modulus model is as follows:

$$\frac{M_r}{P_a} = k_1 \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \tag{9}$$

where $\tau_{oct}$ is the octahedral shear stress and $k_1$, $k_2$, and $k_3$ are material model parameters (material constants). In resilient modulus tests on cylindrical specimens, $\sigma_1$ is the major principal stress, $\sigma_3$ is the minor principal stress, and $\sigma_2$ is the intermediate principal stress, which is the same as the minor principal stress (i.e., $\sigma_2 = \sigma_3$ ). The bulk stress, $\theta = \sigma_1 + 2\sigma_3$ and the octahedral shear stress is equal to $\tau_{oct} = \sqrt{2} / 3(\sigma_1 - \sigma_3)$ .

### 3.2 Correlations between resilient modulus and soil properties

Several past studies developed the relationships between the soil properties and the *k* parameters in Equation 15) (e.g., Shongtao and Zollars, 2002; Titi et al., 2006; Archilla et al. 2007). Coefficient $k_1$ is directly proportional to the resilient modulus and therefore should be positive; coefficient $k_2$ is the exponent of the bulk stress and should be positive since increasing the bulk stress increases the resilient modulus; and coefficient $k_3$ should be negative since increasing the shear stress typically softens the materials and reduces the resilient modulus.

Earlier studies that attempted to find correlations between the $k_{1-3}$ parameters and a wide range of soil groups and conditions reported poor correlations while studies that confined the scope of the correlations to specific soil types (i.e., fine-grained, plastic coarse-grained soils, non-plastic coarse-grained soils) reported good correlations (Titi et al., 2006). Yau and Von Quintus (2002) developed several correlation models for different soil types from the LTPP-FHWA database. Equations 10 through 12 present their models for predicting $k_{1-3}$ for fine-grained soils:

$$k_1 = 1.3577 + 0.0106(\%clay) - 0.0437w_c \tag{10}$$

$$k_2 = 0.5193 - 0.0073P4 + 0.0095P40 - 0.0027P200 - 0.003LL - 0.0049w_{opt} \tag{11}$$

$$\begin{aligned} k_3 &= 1.4258 - 0.0288P4 + 0.0303P40 - 0.0521P200 + 0.0251(\%silt) + 0.0535LL \\ &-0.0672w_{opt} - 0.0026\gamma_{opt} + 0.0025\gamma_s - 0.6055(w_c / w_{opt}) \end{aligned} \tag{12}$$

where P4 = percentage passing No. 4 sieve; P40 = percentage passing No. 40 sieve; $w_c$ = moisture content of the specimen (%); $w_{opt}$ = optimum moisture content of the soil (%); $\gamma_s$ = dry density of the sample (kg/m$^3$); and $\gamma_{opt}$ = optimum dry density (kg/m$^3$). Note that the variability of a given soil type in different regions and states requires developing modified and specific correlations for different states based on statistical analysis of the collected statewide soil data. Therefore, the models developed from the LTPP-FHWA database may not be equally reliable for all states.

Nazzal and Mohammad (2010) conducted a study that involved collecting Shelby tube samples of subgrade soils from 10 different pavement projects throughout Louisiana covering the A-4, A-6, A-7-5, and A-7-6 soil groups. They performed a laboratory testing program that involved resilient modulus tests as well as physical property tests on the collected samples. They collected 90 soil specimens and tested those specimens to produce data for establishing the correlations. The best model in their study was selected based on the statistical analysis. First, the significance of the independent variables of different models was examined and then the multi-collinearity and possible correlations among the independent variables were evaluated. The t-test was used to examine the significance of each of the independent variables used in their study. The probability associated with the t-test was designated with a p-value. A p-value less than 0.05 indicated that, at 95% confidence level, the independent variable was significant in explaining the variation of the dependent variable. Multi-collinearity was detected using the variance inflation factor (VIF). A VIF greater than 10 indicated that weak dependencies may be affecting the regression estimates. The adequacy of the models was assessed using the coefficient of determination, $R^2$, and the square root of the mean-square errors (RMSE). The RMSE represents the standard error of the regression model. The $R^2$ represents the proportion of variation in the dependent variable that is accounted for by the regression model and ranges from 0 to 1. When $R^2$ is equal to 1, the entire observed points lie on the suggested least-squares line, which means a perfect correlation exists. Nazzal and Mohammad (2010) proposed the following prediction models for the $k_{1-3}$ parameters.

$$\begin{aligned}&\ln(k_1) = 1.334 + 0.0127(P200) + 0.016(LL) - 0.036(\gamma_{opt})\\&-0.011(MCCL) + 0.001(MCDD\max P)\ (\mathrm{R}^2 = 0.61;\ \mathrm{RMSE}{=}0.23)\end{aligned} \tag{13}$$

$$\begin{aligned}&k_2 = 0.722 + 0.0057(LL) - 0.00454(MCDD\max PI)^{0.641} + 0.00324(MCDDP)^{1.28}\\&-0.875(P200)\ (\mathrm{R}^2 = 0.74;\ \mathrm{RMSE}{=}0.1)\end{aligned}$$

(14)

$$\begin{aligned}&k_3 = -7.48 + 0.235\left(\frac{\gamma_s}{mc}\right) + 0.038(LL) - 0.0008(MCPI) + 0.033(\gamma_{opt})\\&-0.016(MCDDP)\ (\mathrm{R}^2 = 0.66;\ \mathrm{RMSE}{=}0.49)\end{aligned} \tag{15}$$

where

$$MCCL = (w_c - w_{opt}).\mathrm{clay}\% \tag{16}$$

$$MCDD\max P = P200\,\frac{w_c - w_{opt}}{w_{opt}}\,\frac{\gamma_s}{\gamma_{opt}} \tag{17}$$

$$MCDD\max PI = PI.\frac{w_c - w_{opt}}{w_{opt}}\,\frac{\gamma_s}{\gamma_{opt}} \tag{18}$$

$$MCDDP = \frac{P200.\gamma_s}{w_c} \tag{19}$$

$$MCPI = PI\,\frac{w_c - w_{opt}}{w_{opt}} \tag{20}$$

where clay% is the percentage of the clay in the soil (%), $\gamma_s$ =dry unit weight of the sample (pcf); and $\gamma_{opt}$ =optimum dry unit weight (pcf).

Nazzal and Mohammad (2010) found that index properties such as the liquid limit, PI, and percent passing No. 200, were influential variables in their developed models. The most significant variable affecting the $k_1$ parameter was found to be the MCCL variable, which represented the percent of clay and moisture content properties.

In a similar study, Titi et al. (2006), performed extensive statistical analysis on soil specimens from Wisconsin. This study involved performing 136 repeated loading triaxial tests for determination of resilient modulus values for Wisconsin subgrade soils. The soils were grouped as non-plastic coarse-grained soils, plastic coarse-grained soils, and fine-grained soils. They reported significant improvement in the quality of the established correlations between $k_{1-3}$ and the basic soil properties by grouping the soils in the aforementioned categories.

Based on statistical analysis of the investigated non-plastic coarse-grained soils, the resilient modulus model parameters ($k_{1\text{-}3}$) was estimated from the basic soil properties using the following equations (Titi et al., 2006):

$$k_1 = 809.547 + 10.568P4 - 6.112P40 - 578.337(\frac{w_c}{w_{opt}})(\frac{\gamma_s}{\gamma_{opt}}) \qquad (21)$$

$$k_2 = 0.5661 + 0.006711P40 - 0.02423P200 + 0.05849(w_c - w_{opt}) + 0.001242w_{opt}\gamma_{opt} \qquad (22)$$

$$k_3 = -0.5079 - 0.041411P40 + 0.14820P200 - 0.1726(w_c - w_{opt}) - 0.01214w_{opt}\gamma_{opt} \qquad (23)$$

For the plastic coarse-grained soils, the resilient modulus model parameters were proposed to be estimated from the following equations (Titi et al., 2006):

$$k_1 = 8642.873 + 132.643P200 - 428.067(\%silt) - 254.685PI + 197.23\gamma_d - 381.4(\frac{w_s}{w_{opt}}) \tag{24}$$

$$k_2 = 2.3250 - 0.00853P200 + 0.02579LL - 0.06224PI - 1.7338(\frac{\gamma_s}{\gamma_{opt}}) + 0.20911(\frac{w_s}{w_{opt}}) \tag{25}$$

$$k_3 = -32.5449 + 0.7691P200 - 1.1370(\%silt) + 31.5542(\frac{\gamma_s}{\gamma_{opt}}) - 0.4128(w_s - w_{opt}) \tag{26}$$

Finally, for the fine-grained soils, the following equations were proposed (Titi et al., 2006):

$$k_1 = 404.166 + 42.933PI + 52.26\gamma_s - 987.353(\frac{w_c}{w_{opt}}) \tag{27}$$

$$k_2 = 0.25113 - 0.0292PI + 0.5573(\frac{w_c}{w_{opt}}).(\frac{\gamma_s}{\gamma_{opt}}) \tag{28}$$

$$k_3 = -0.20772 + 0.23088PI + 0.00367\gamma_s - 5.4238(\frac{w_c}{w_{opt}}) \tag{29}$$

## 4. Synthesis on correlations of R-value and the resilient modulus

Yeh and Su (1989) established a correlation for Colorado soils between the resilient modulus and the soil's R-value, as follows:

$$M_r(psi) = 3500 + 125(\text{R-value}) \tag{30}$$

This correlation shows a direct relation between the R-value and the resilient modulus. However, there was no indication of the performance of their model. The graphical presentation of the tested $M_r$ and R-value data points clearly indicated a non-linear relation between the two properties and that the anticipated $R^2$ value for this model may not exceed the value of 0.5. Their results relied on testing a very limited number of soil specimens (six fine-grained clay specimens and 13 mostly granular specimens).

CDOT’s 2019 Pavement Design Manual includes a correlation for estimating $M_r$ from the R-value. Equation 31 provides an estimate of the $M_r$ value and is only valid for R-values obtained by experiments following the AASHTO T 190 procedure. According to the CDOT Manual, if the R-value of the existing subgrade or embankment material is estimated to be greater than 50, a FWD analysis or a resilient modulus test using AASHTO T 307 should be performed.

$$M_r(psi) = 3438.6(\text{R-value})^{0.2753} \tag{31}$$

The above equation has been used for estimating the resilient modulus and first appeared in the AASHTO 1993 Pavement Design Guide. This equation should only be used for R-values of 50 or less. Therefore, formulating a reliable and more comprehensive correlation between the resilient modulus and the R-values for Colorado soils was of great value.

CDOT’s design manual also lists typical $M_r$ values for embankments and subgrade soils. The listed values are for soils at the optimum moisture content and therefore the maximum dry density condition. The listed values are appropriate for use in the preliminary pavement design; and for the final pavement design, it is required that the $M_r$ values used are either obtained from laboratory testing or correlation using equation 31.

One of the best approaches for relating $M_r$ to R-value is to review the concept from the soil mechanics perspective and establish a theoretical framework for the relationship between the two properties. Chua and Tenison (2003) developed a framework for this relation and their work is summarized here.

**Table 4. Resilient modulus for embankments and subgrade soils (CDOT design Manual)**

| AASHTO Soil Group | Resilient Modulus ($M_r$) at optimum moisture content (psi) | |
|---|---|---|
| | Flexible Pavements | Rigid Pavements |
| A-1-a | 19700 | 14900 |
| A-1-b | 16500 | 14900 |
| A-2-4 | 15200 | 13800 |
| A-2-5 | 15200 | 13800 |
| A-2-6 | 15200 | 13800 |
| A-2-7 | 15200 | 13800 |
| A-3 | 15000 | 13000 |
| A-4 | 14400 | 18200 |
| A-5 | 14000 | 11000 |
| A-6 | 17400 | 12900 |
| A-7-5 | 13000 | 10000 |
| A-7-6 | 12800 | 12000 |

Considering a cylindrical specimen subjected to the triaxial state of stress, the radial strain for a soil specimen, $\varepsilon_r$, can be calculated as follows:

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right] \tag{32}$$

where $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ are the radial, tangential, and vertical stresses applied to the specimen, respectively, $\nu$ is the Poisson's ratio for the soil, and $E$ is the elastic modulus of the soil. The vertical strain is given by:

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right] \tag{33}$$

The volumetric strain for the test specimen in the stabilometer test can be calculated as follows

$$(\varepsilon_r - \varepsilon_z)\frac{\pi D^2}{4} L = CD_2 \tag{34}$$

where $D$ is the diameter of the specimen and $C$ is the conversion used to calculate the amount of fluid injected into the chamber by turning the screw one turn. $D_2$ is the number of turns of the screw on the stabilometer device. By substituting the stresses from equations 38 and 39 into Equation 40, the following equation was obtained.

$$\frac{L}{D_2}(\sigma_z - \sigma_r) = \frac{E}{(1-\nu)} C.\frac{4}{\pi D^2} \tag{35}$$

Using notation of $P_v$ for $\sigma_z$ and $P_h$ for $\sigma_r$, Equation 41 can be re-arranged as follows:

$$E = \frac{\pi D^2}{4C}(1-\nu)\frac{R}{100-R} P_h \tag{36}$$

Chua and Tenison (2003) reported that there is a minimum elastic modulus for the materials, $E_0$ . This value is assumed to be 2,000 psi for clay and granular subgrade and 7,500 psi for granular course materials. They also replaced the horizontal pressure, $P_h$, in Equation 42 with the product of vertical pressure and the at-rest lateral earth pressure coefficient.

$$E = \frac{\pi D^2}{4C}(1-\nu)\frac{R}{100-R}(1-\sin\varphi')\sqrt{OCR} \times (1 + \frac{\Delta\sigma_{ex}}{P_v})P_v + E_0 \tag{37}$$

where $P_v$ is the last applied vertical pressure (160 psi), $\Delta\sigma_{ex}$ is the exudation pressure, $OCR$ is the soil's over-consolidation ratio, and $\varphi'$ is the soil's angle of internal friction. For cohesion-less materials, $\Delta\sigma_{ex}$ should be set to zero because the residual stress from the exudation stress would have been relieved when the specimen is removed from the preparation mold (Chua and Tenison (2003).

## 5. Statistical analysis background

We performed multiple regression analyses to correlate the R-value and resilient modulus values as the dependent variables and the fundamental soil properties as the independent variables. All the fundamental soil properties present in databases were treated as potential independent variables. Numerous combinations of soil properties (independent variables) were used in our regression analyses. The general multiple linear regression model is expressed as:

$$\text{R-value or Resilient Modulus} = A_0 + A_1 x_1 + A_2 x_2 + \ldots + A_k x_k + \varepsilon \qquad (38)$$

where $A_0$ is the intercept of the regression plane, $A_i$ is the regression coefficient, $x_i$ is the independent variable (soil parameter or combination of soil properties), and $\varepsilon$ is the random error.

To assess the models, two performance indices of coefficient of determination, $R^2$ and root mean square error (RMSE) were considered with the following equations:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y - y')^2}{\sum_{i=1}^{N} (y - \tilde{y})^2} \qquad (39)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y - y')^2} \qquad (40)$$

where $y$ and $y'$ are the predicted and measured dependent variables, respectively, $\tilde{y}$ is the mean of the $y$ values, and $N$ is the total number of data points. The predictive equation will be excellent if $R^2 = 1$ and RMSE= 0. The adjusted $R^2$, a modified version of $R^2$, also was evaluated. The adjusted

$R^2$ was adjusted for the number of independent variables in the model and was increased only if the new variable possibly could improve the model more than might be expected.

To perform the regressions, MATLAB scripts were written that first sorted the data by the soil group and then produced models using different combinations of the independent variables that were available in the databases. Each unique combination of variables was tested, starting with the individual variables and adding more independent variables until a maximum of 12 variables were explored.

## 5.1 Multicollinearity testing

Some linear regressions can suffer from multicollinearity, which means that the independent variables are more strongly correlated with each other than with the dependent variable. For example, the liquid limit and plastic limit are indicators of the PI of a soil. However, if a model includes the liquid limit, plastic limit, and PI as independent variables, it will artificially give too much weight to the PI because it is effectively also included in the plastic limit and liquid limit variables. To control for this, we used the variance inflation factor (VIF), which is defined as the diagonal of the inverse of the coefficient matrix. It is typically suggested that when VIF is greater than 8, some multicollinearity problems may exist. In our study, the VIF therefore was required to be less than 8 for a model to be considered.

## 5.2 Significance testing

For significance testing of the model, we used an F-test to ensure a linear relationship between the independent variables and the dependent variable (i.e., R-value or resilient modulus). The hypotheses were as follows:

$H_0$: all the coefficients for the model are zero

$H_a$: at least one of the coefficients is not zero

The F-test statistic is:

$$F_0 = \frac{SS_R / p}{SS_E / (n - p - 1)} \tag{41}$$

where $SS_R$ is the sum of the squared errors due to the regression, $SS_E$ is the sum of squares due to the errors, *n* is the number of observations, and *p* is the number of independent variables included. For a model to be considered, $H_0$ must be rejected, that is, $F_0 < \alpha$. For all parts of our study, $\alpha = 0.05$.

For our significance testing of the individual independent variables, a similar hypothesis test was used. In this case, the hypotheses were as follows:

$H_0$: the coefficient of this variable is equal to zero

$H_a$: the coefficient is not equal to zero

The test statistic is:

$$t_0 = \frac{\widehat{\beta_i}}{\sqrt{\hat{\sigma}^2 C_{ii}}} \tag{42}$$

where $C_{ii}$ is the diagonal element of $(X^{/}X)^{-1}$ corresponding to $\widehat{\beta_i}$ (estimator of $\beta_i$), $\hat{\sigma}$ is an estimator for the standard deviation of errors, X (*n*,*p*) is the matrix of all levels of the independent variables, $X^{/}$ is the diagonal X matrix, *n* is the number of observations, and *p* is the number of independent variables. For a model to be considered, $H_0$ must be rejected (i.e. $t_0 < \alpha$).

In order to determine which one of the considered models was the best for each soil group, we used the three models with the highest $R^2$ adjusted values. We used this statistic, rather than just the $R^2$, because the $R^2$ is expected to increase with the addition of independent variables even if it does not predict the $R^2$ value better than a previous model. The $R^2$ adjusted therefore was adjusted by eliminating the independent variables that were not contributing to the regression and thus retaining those that were more suited for determining the most effective model without being unnecessarily complicated.

## 6. R-Value prediction models based on soil index properties for Colorado soils

### 6.1 Development of the CDOT database for R-Value regression analysis

The CDOT soil archive database includes the following soil properties information: soil classification, gradation, Atterberg limits, specific gravity, absorption, and proctor test results (optimum moisture content and the maximum dry unit weight), as well as the R-value for the soil specimen for an exudation pressure of 300 psi. This database addresses the following AASHTO soil groups: A-1-a, A-1-b, A-2-4, A-2-6, A-2-7, A-3, A-4, A-6, and A-7-6. The reported R-values in CDOT's soil archive database are the final extrapolated values from at least three R-value tests for each soil specimen. In general, the R-value of a specimen is strongly affected by the change in the moisture content, especially for cohesive soils; and an increase in the moisture content generally reduces the R-value for cohesive soils. To increase the possibility of achieving robust correlations, we obtained paper copies of many soil reports with R-value tests performed by Mr. Jon Grinder and his predecessor from 2000-2008 as the basis for establishing a "revised" database by manually entering the three and more R-value test results. We acknowledge Mr. Grinder's excellent testing and reporting practices as well as his assistance in understanding the collected information.

Please note that the existing CDOT database lacked the exudation pressure and moisture content information for soil specimens that were tested for R-values, and the reported R-value was the final value corresponding to 300 psi exudation pressure. In contrast, the revised database includes the moisture content and the exudation pressure for each specific R-value test performed and documented in the form of hard copy reports.

The newly entered/added data in the CDOT database by our team (the revised database) was used to investigate the direct relationships between the soil properties and the R-values. Table 5 is a summary of our statistical analysis of the soil groups in the revised CDOT database. The distribution of R-values for the revised dataset is shown for each soil group. Note that, as expected, a wide spread of R-values was observed for high plasticity soils and higher R-value ranges were observed for coarser soils.

Our research included dividing the available data by AASHTO soil type and exploring the summary statistics related to their R-values, the results of which can be seen in Table 5. It is important to note that some soil types had far more data available than others, with the number of observations ranging from 77 to 625.

The revised CDOT database consists of the following input parameters for every soil specimen: (a) specific gravity, (b) absorption (%), (c) optimum dry unit weight (pcf), (d) optimum moisture content (%), (e) percent passing sieve No. 4, (f) percent passing sieve No. 10, (g) percent passing sieve No. 40, (h) percent passing sieve No. 200, (i) liquid limit (%), (j) plastic limit (%), (k) exudation pressure (psi), and (l) moisture content (%).

**Table 5. Statistical data of R-values for each AASHTO soil group**

| Soil Group | Mean | Median | Minimum | Maximum | Std Deviation | Std Error | Number of Observations |
|---|---|---|---|---|---|---|---|
| A-1-a | 79.47 | 81 | 14 | 88 | 9.16 | 0.115 | 162 |
| A-1-b | 75.46 | 78 | 2 | 89 | 11.03 | 0.146 | 612 |
| A-2-4 | 65.13 | 72 | 5 | 92 | 18.87 | 0.289 | 625 |
| A-2-6 | 51.31 | 52.5 | 4 | 89 | 21.09 | 0.411 | 398 |
| A-2-7 | 34.68 | 31 | 5 | 84 | 19.58 | 0.564 | 77 |
| A-3 | 74.53 | 75 | 25 | 80 | 6.50 | 0.087 | 67 |
| A-4 | 57.72 | 63 | 8 | 89 | 21.91 | 0.379 | 108 |
| A-6 | 28.69 | 25 | 2 | 89 | 16.91 | 0.589 | 439 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A-7-6 | 25.41 | 18 | 0 | 86 | 20.65 | 0.812 | 211 |

An extensive database of systematically conducted R-value tests provided by the Colorado Department of Transportation was analyzed for establishing relationships between the R-value and the basic soil properties. Reporting of R-value for each soil specimen requires testing at least three specimens at three different moisture contents and exudation pressures. The final reported R-value is obtained by interpolating at exudation pressure of 300 psi. The range and distribution of R-values for each AASHTO soil group is presented in Figure 3.

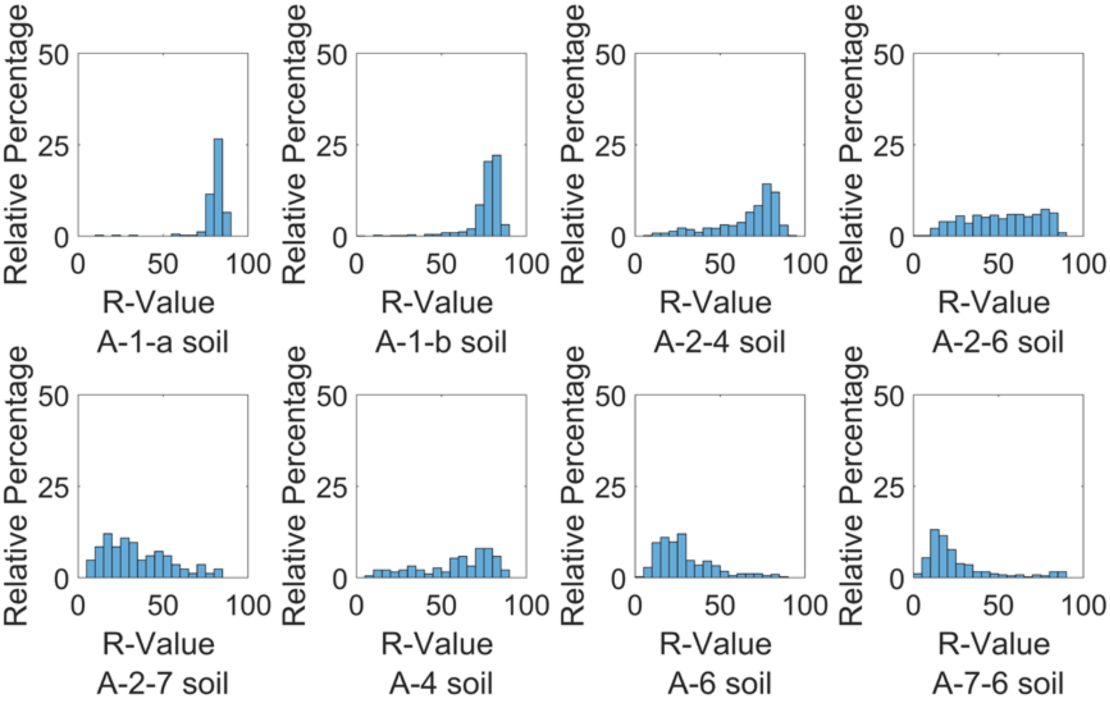


**Figure 3**. **Histogram of R-values for all exudation pressures**

Note that each point in Figure 3 has an associated exudation pressure (EP) based on the specimen condition and the pressure is not necessarily 300 psi. Figure 4 shows the histograms for the R-values corresponding to EP=300 psi. Typically, the higher the EP, the greater the R-value. The R-value is strongly affected by the change in the moisture content (exudation pressure), especially for cohesive soils. An increase in the moisture content generally reduces the R-value for the cohesive soils. Since the exudation pressure has such a large effect on the R-value, it should be included in the regression analysis as an independent variable.

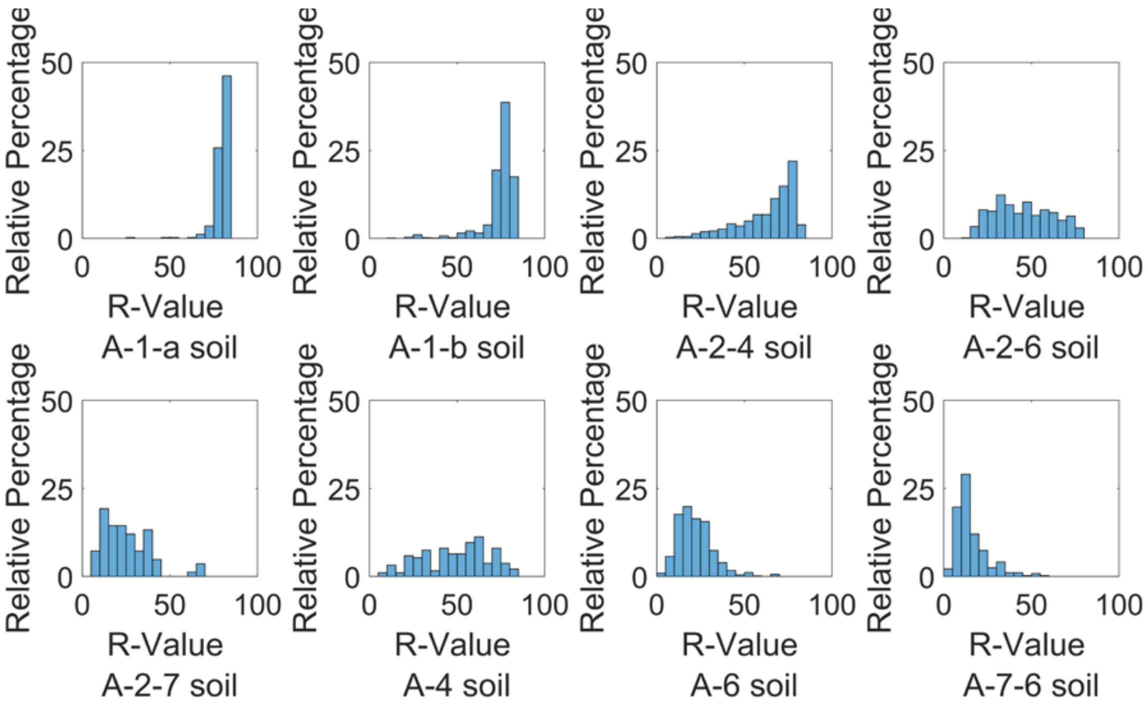


**Figure 4. Histogram of R-values for exudation pressure of 300 psi**

The basic soil properties were selected based on their availability, their effect on the R-value, and a thorough examination of the literature that suggested several combined variables.

Table 6 presents the ranges of the soil properties that were used as independent variables in the regression analysis. The independent variables are as follows: specific gravity (SpG), absorption (Abs), maximum dry density (MDD), optimum moisture content (OM), in-situ moisture content (MC), liquid limit (LL), plastic limit (PL), plasticity index (PI), percent passing of #4 sieve ($P_{\#4}$), percent passing of #10 sieve ($P_{\#10}$), percent passing of #40 sieve ($P_{\#40}$), percent passing of #200 sieve ($P_{\#200}$), difference in moisture content ($MC_{diff}$ = MC-OM), moisture content differential ratio multiplied by plasticity index (MCDRPI = $(MC_{diff})/OM \times PI$), and exudation pressure (EP).

**Table 6. Limits of soil properties values used in R-Value regressions**

| Property [units] | Limits | A-1-a soil | A-1-b soil | A-2-4 soil | A-2-6 soil | A-2-7 soil | A-4 soil | A-6 soil | A-7-6 soil |
|---|---|---|---|---|---|---|---|---|---|
| SpG | Min | 2.35 | 1.92 | 2.12 | 2.12 | 2.35 | 1.95 | 1.00 | 2.25 |
| | Max | 2.75 | 3.13 | 2.92 | 2.77 | 2.63 | 2.76 | 2.92 | 2.68 |
| Abs[%] | Min | 0.38 | 0.38 | 0.20 | 0.50 | 0.58 | 0.50 | 0.58 | 0.55 |
| | Max | 8.0 | 8.8 | 9.1 | 7.4 | 4.6 | 7.9 | 5.7 | 7.0 |
| MDD [pcf] | Min | 106 | 103 | 100 | 105 | 99.7 | 98.8 | 93.6 | 84.9 |
| | Max | 139 | 139 | 138 | 140 | 122 | 134 | 131 | 125 |
| OM [%] | Min | 5.88 | 5.78 | 5.97 | 6.03 | 10.2 | 7.03 | 8.36 | 10.5 |
| | Max | 13.9 | 16.5 | 19.3 | 17.5 | 22.7 | 19.8 | 21.6 | 34.9 |
| MC [%] | Min | 5.50 | 1.66 | 3.55 | 2.69 | 7.83 | 7.98 | 7.35 | 6.09 |
| | Max | 29.5 | 85.3 | 70.3 | 15.2 | 20.9 | 48.7 | 23.6 | 40.2 |
| LL | Min | 17 | 17 | 17 | 23 | 41 | 14 | 23 | 41 |
| | Max | 32 | 32 | 39 | 40 | 87 | 37 | 40 | 76 |
| PL | Min | 14 | 13 | 9 | 11 | 16 | 11 | 5 | 14 |
| | Max | 29 | 29 | 34 | 25 | 28 | 53 | 26 | 29 |
| PI | Min | 1 | 1 | 1 | 11 | 17 | 1 | 11 | 13 |
| | Max | 6 | 6 | 10 | 25 | 71 | 10 | 27 | 49 |
| $P_{\#4}$ [%] | Min | 9 | 37 | 33 | 19 | 57 | 64 | 57 | 66 |
| | Max | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $P_{\#10}$ [%] | Min | 7 | 31 | 23 | 17 | 47 | 59 | 51 | 61 |
| | Max | 50 | 99 | 100 | 100 | 99 | 100 | 100 | 100 |
| $P_{\#40}$ [%] | Min | 5 | 8 | 6 | 9 | 16 | 49 | 45 | 47 |
| | Max | 30 | 50 | 100 | 83 | 56 | 100 | 100 | 100 |
| $P_{\#200}$ [%] | Min | 0.5 | 0.7 | 1.60 | 3.8 | 8.3 | 36 | 36 | 36 |
| | Max | 15 | 25 | 35 | 35 | 34 | 100 | 94 | 99 |
| $MC_{diff}$ | Min | -5.44 | -7.57 | -8.61 | -5.68 | -8.81 | -5.94 | -4.92 | -14.6 |
| | Max | 21.6 | 77.7 | 59.3 | 2.48 | 6.57 | 12.7 | 9.99 | 18.2 |
| MCDRPI | Min | -1.21 | -2.43 | -3.41 | -8.22 | -13.7 | -2.53 | -7.93 | -18.5 |
| | Max | 8.87 | 1.34 | 3.6 | 4.04 | 40.1 | 6.48 | 17.6 | 47.4 |
| EP [psi] | Min | 105 | 47 | 74 | 47 | 115 | 99 | 100 | 113 |
| | Max | 808 | 816 | 860 | 808 | 815 | 800 | 882 | 816 |

## 6.2 R-value models for individual soil groups

Our MATLAB code was programmed to run regressions using every possible combination of variables, starting with single-variable correlations and continuing up to 12 variables at a time. This upper bound was chosen because there was not much noticeable improvement beyond eight variables and the upper limit of 12 variables was considered reasonable. After the MATLAB code identified the most viable models for each soil group, a review was conducted to ensure the accuracy and suitability of the resulting models. If a model did not meet the criteria presented in section 5, it was not considered in this review. We selected the three top models using the $R^2$ adjusted to ensure that the model was not overfitted and that each selected independent variable contributed to the final R-value.

Tables 7 through 14 present summaries of the regression analysis results in which the models for the R-values from the basic soil properties were obtained for each available AASHTO soil group. Three models are presented for each soil group and the $R^2$ and $R^2$ adjusted values are reported for each model. Examination of Tables 7 through 14 shows that these models are consistent with the natural behavior of the soils.

Figures 5 through 12 are graphical comparisons between the R-values predicted by the three models and the actual measured R-values from the revised CDOT database. For qualitative assessment of the accuracy and performance of each model, the plot of the predicted versus the measured R-values could be used (Figures 5 through 12). When these points were close to the y=x line (shown in the figures in blue), the model was considered reliable in predicting the R-value.

### 6.2.1. A-1-a soil

**Table 7. Correlations between R-value and basic soil properties for A-1-a soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -293.91 | -280.07 | -153.70 |
| Specific Gravity | 133.81 | 130.41 | 90.13 |
| Absorption [%] | 4.45 | 3.95 | - |
| Liquid Limit | - | -0.195 | -0.192 |
| Plasticity Index | -1.16 | -0.968 | - |
| $P_{no.\ 10}$ [%] | - | - | -0.192 |
| $P_{no.\ 200}$ [%] | -1.08 | -1.15 | -0.94 |
| Exudation Pressure [psi] | 0.0147 | 0.0148 | 0.0154 |
| MC [%] | 3.93 | 3.94 | 3.912 |
| MCDRPI = $(MC_{diff})/OM \times PI$ | -9.85 | -9.91 | -10.02 |
| $R^2$ | 0.9537 | 0.9542 | 0.9517 |
| $R^2$ adjusted | 0.9442 | 0.9431 | 0.9418 |

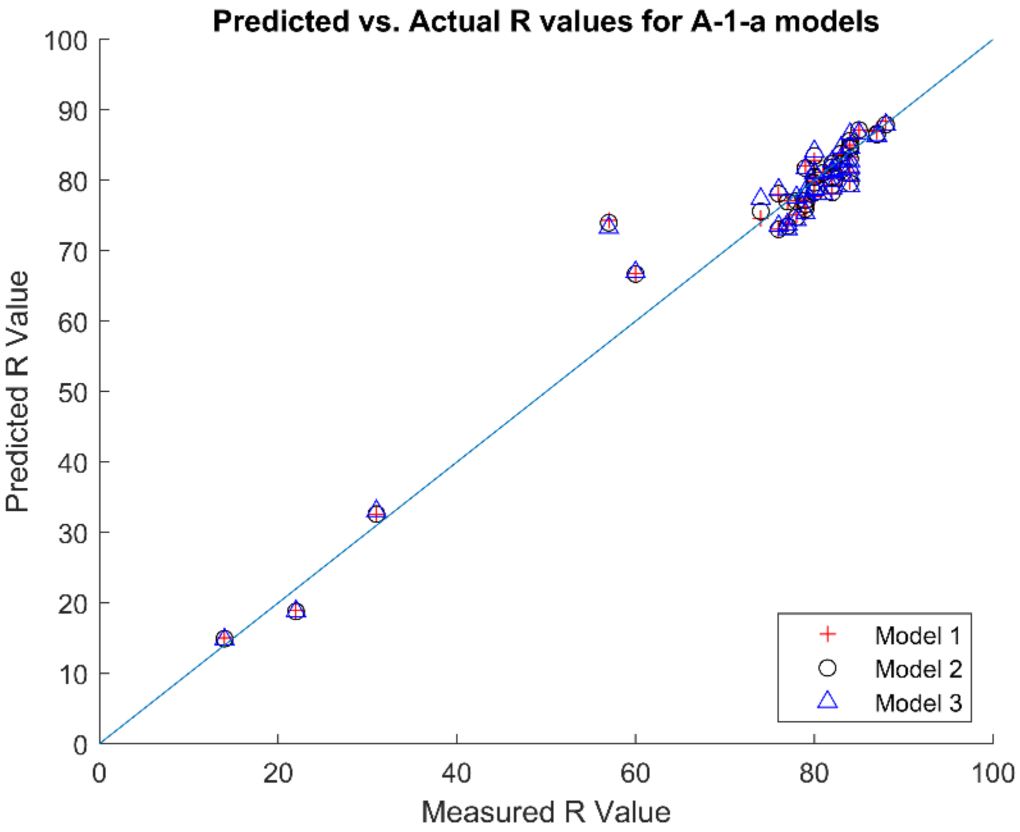


Figure 5. Predicted versus measured R-values for A-1-a soil

### 6.2.2. A-1-b soil

**Table 8. Correlations between R-value and basic soil properties for AASHTO A-1-b soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -12.711 | 31.070 | 23.610 |
| MDD [lb/ft$^3$] | 0.772 | 0.520 | 0.537 |
| Liquid Limit | -2.232 | - | - |
| Plastic Limit | -0.633 | - | - |
| Plasticity Index | - | -2.696 | -1.328 |
| $P_{\text{No. 40}}$ [%] | -0.633 | -0.814 | -0.834 |
| $P_{\text{No. 200}}$ [%] | -0.570 | - | - |
| Exudation Pressure [psi] | 0.0231 | 0.0251 | 0.0245 |
| MCDRPI = $(MC_{diff})/OM \times PI$ | -8.893 | -7.940 | - |
| (MC-OM)/OM | - | - | -36.887 |
| $R^2$ | 0.5517 | 0.5341 | 0.5338 |
| $R^2$ adjusted | 0.5289 | 0.5157 | 0.5172 |

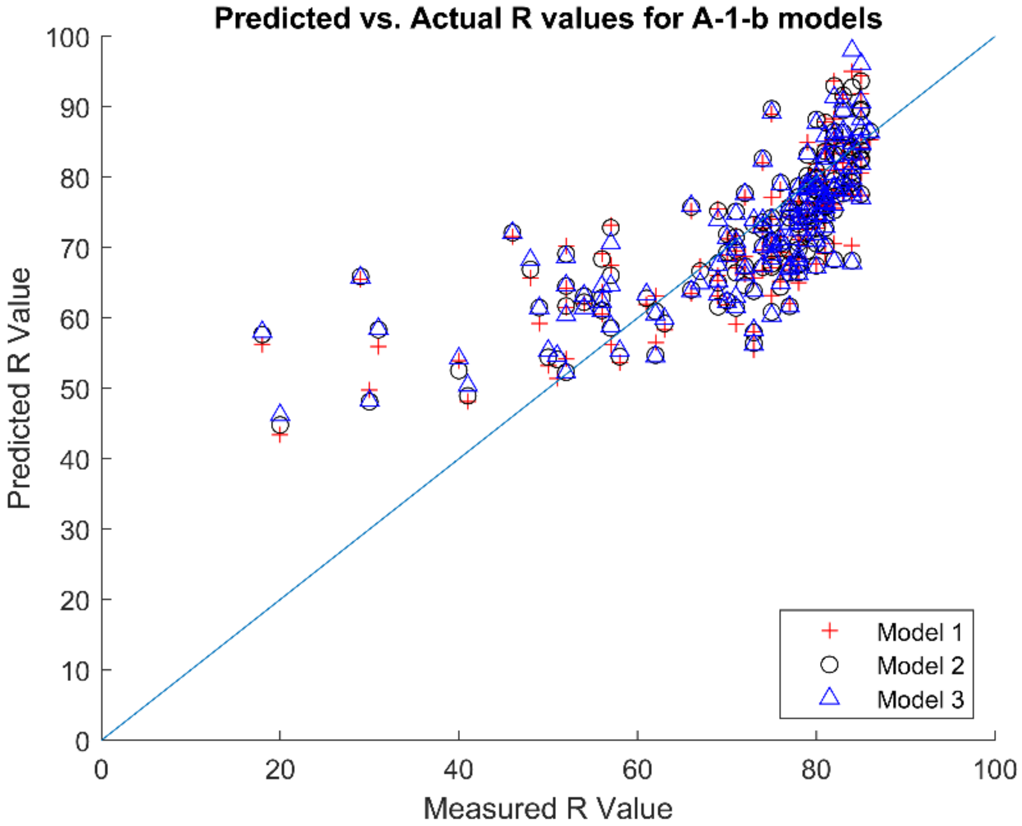


**Figure 6. Predicted versus measured R-values for A-1-b soil**

### 6.2.3. A-2-4 soil

**Table 9. Correlations between R-value and basic soil properties for AASHTO A-2-4 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 150.639 | 150.639 | 65.736 |
| Specific Gravity | -28.820 | -28.820 | - |
| $\omega_{opt}$ [%] | -3.83 | 3.225 | -3.464 |
| Liquid Limit | 0.730 | 0.730 | - |
| Plastic Limit | - | - | 0.723 |
| Plasticity Index | -1.233 | -1.233 | - |
| $P_{No. 10}$ [%] | 0.170 | 0.170 | 0.275 |
| $P_{No. 200}$ [%] | -0.999 | -0.999 | -1.156 |
| Exudation Pressure [psi] | 0.0422 | 0.0422 | 0.0485 |
| MC [%] | - | -7.055 | - |
| $MC_{diff}$ | -7.055 | - | -9.086 |
| MCDRPI = ($MC_{diff}$)/OM×PI | - | - | 4.739 |
| $R^2$ | 0.6277 | 0.6277 | 0.6093 |
| $R^2$ adjusted | 0.6180 | 0.6180 | 0.6023 |

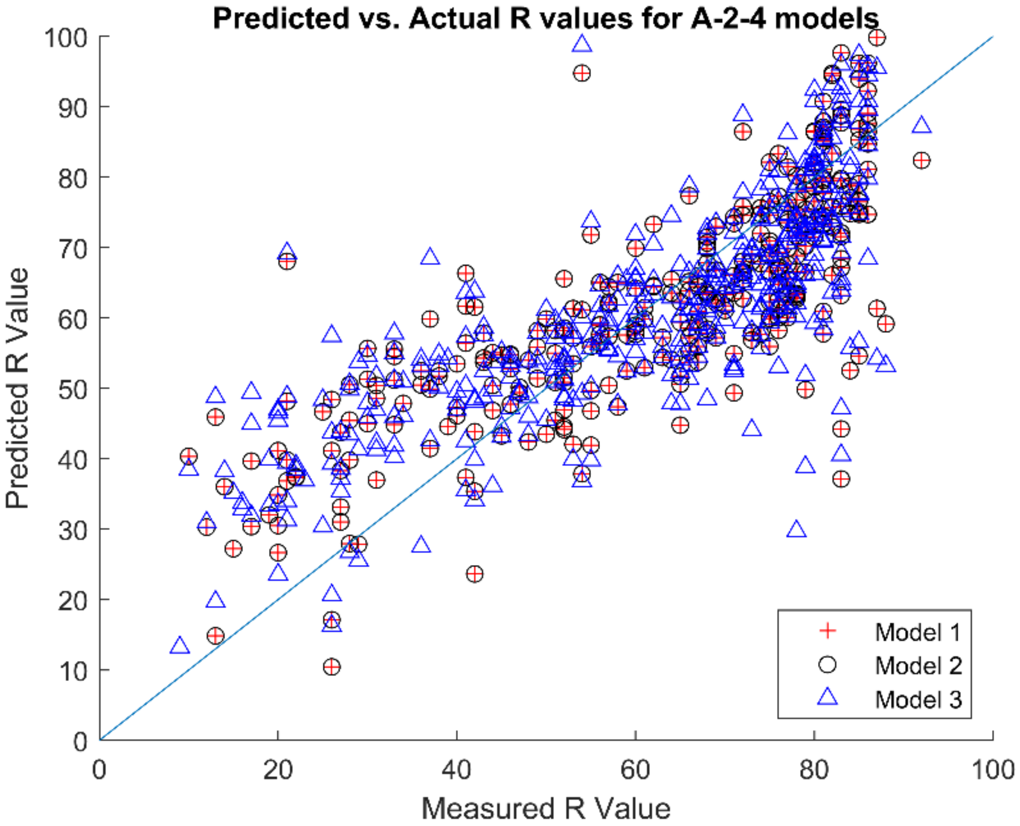


**Figure 7. Predicted versus measured R-values for A-2-4 soil**

### 6.2.4. A-2-6 soil

**Table 10. Correlations between R-value and basic soil properties for AASHTO A-2-6 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -179.143 | -179.143 | -244.611 |
| Absorption [%] | 4.167 | 4.167 | 3.982 |
| MDD [lb/ft$^3$] | 1.527 | 1.527 | 1.817 |
| OM [%] | - | - | 7.089 |
| Liquid Limit | -0.469 | 1.910 | 1.864 |
| Plastic Limit | 2.379 | - | - |
| Plasticity Index | - | -2.379 | -2.333 |
| $P_{No.\ 40}$ [%] | 0.582 | 0.582 | 0.604 |
| $P_{No.\ 200}$ [%] | -1.846 | -1.846 | -1.889 |
| Exudation Pressure [psi] | 0.0616 | 0.0616 | 0.623 |
| $MC_{diff}$ | -6.199 | -6.199 | - |
| MC [%] | - | - | -6.044 |
| $R^2$ | 0.6932 | 0.6932 | 0.6931 |
| $R^2$ adjusted | 0.6844 | 0.6844 | 0.6842 |

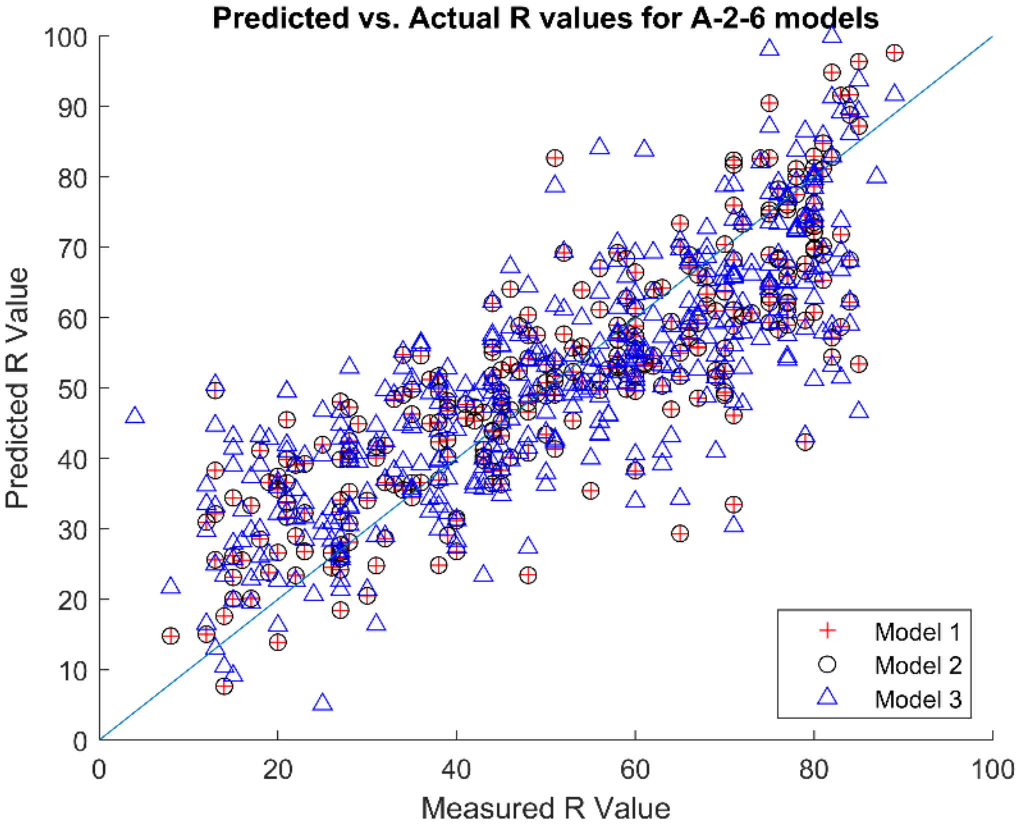


**Figure 8. Predicted versus measured R-values for A-2-6 soil**

### 6.2.5. A-2-7 soil

**Table 11. Correlations between R-value and basic soil properties for AASHTO A-2-7 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -35.954 | -41.692 | 325.163 |
| Specific Gravity | - | - | -121.411 |
| Liquid Limit | - | 0.120 | - |
| Plastic Limit | 1.152 | 1.186 | 2.166 |
| $P_{No.\ 10}$ [%] | 0.969 | 0.997 | - |
| $P_{No.\ 40}$ [%] | -1.663 | -1.774 | - |
| $P_{No.\ 200}$ [%] | 1.118 | 1.186 | 0.953 |
| Exudation Pressure [psi] | 0.0411 | 0.0409 | 0.0207 |
| MC [%] | - | - | -4.999 |
| $MC_{diff}$ | -2.859 | -2.899 | - |
| $R^2$ | 0.7082 | 0.7107 | 0.7027 |
| $R^2$ adjusted | 0.6832 | 0.6813 | 0.6735 |

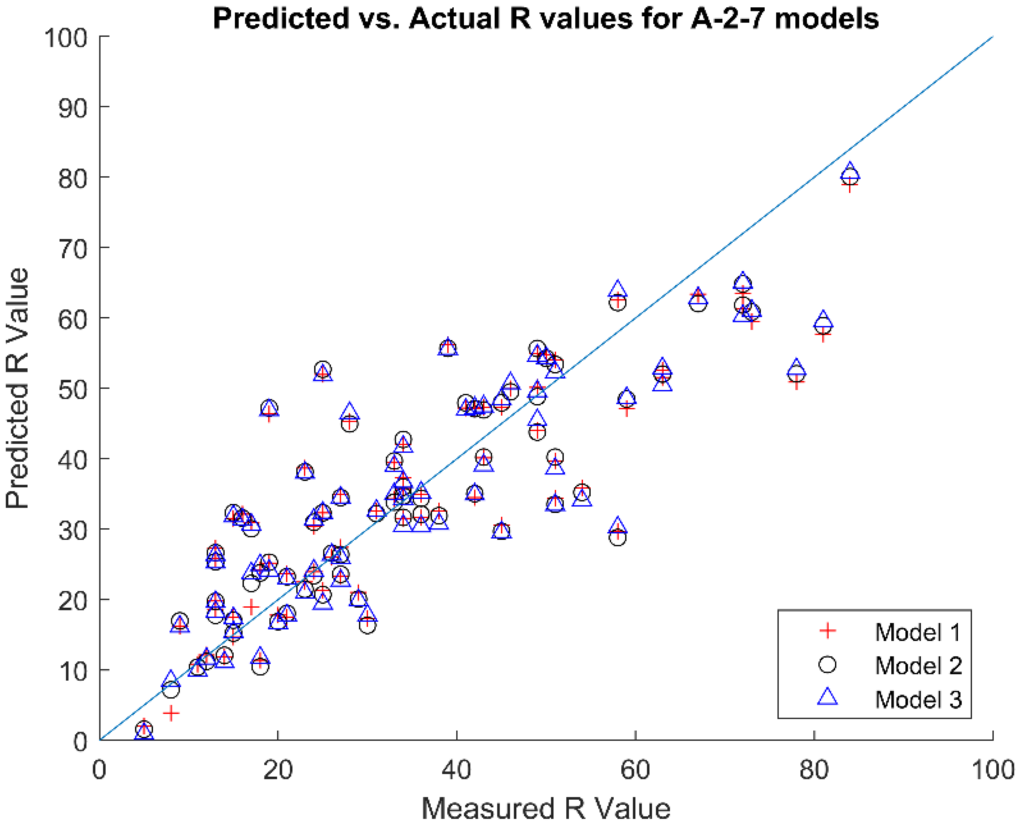


**Figure 9. Predicted versus measured R-values for A-2-7 soil**

### 6.2.6. A-4 soil

**Table 12. Correlations between R-value and basic soil properties for AASHTO A-4 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -427.651 | -427.651 | -481.091 |
| Specific Gravity | -156.304 | -156.304 | -163.462 |
| MDD [lb/ft$^3$] | 4.472 | 4.472 | 4.601 |
| Liquid Limit | -1.967 | 7.413 | - |
| Plastic Limit | 9.381 | - | 8.061 |
| Plasticity Index | - | -9.381 | - |
| $P_{No.\ 10}$ [%] | 3.316 | 3.316 | 3.545 |
| $P_{No.\ 200}$ [%] | -1.986 | -1.986 | -1.769 |
| Exudation Pressure [psi] | 0.0281 | 0.0281 | 0.0304 |
| MCDRPI = $(MC_{diff})/OM \times PI$ | -23.262 | -23.262 | -23.262 |
| $R^2$ | 0.9105 | 0.9105 | 0.9038 |
| $R^2$ adjusted | 0.8907 | 0.8907 | 0.8857 |

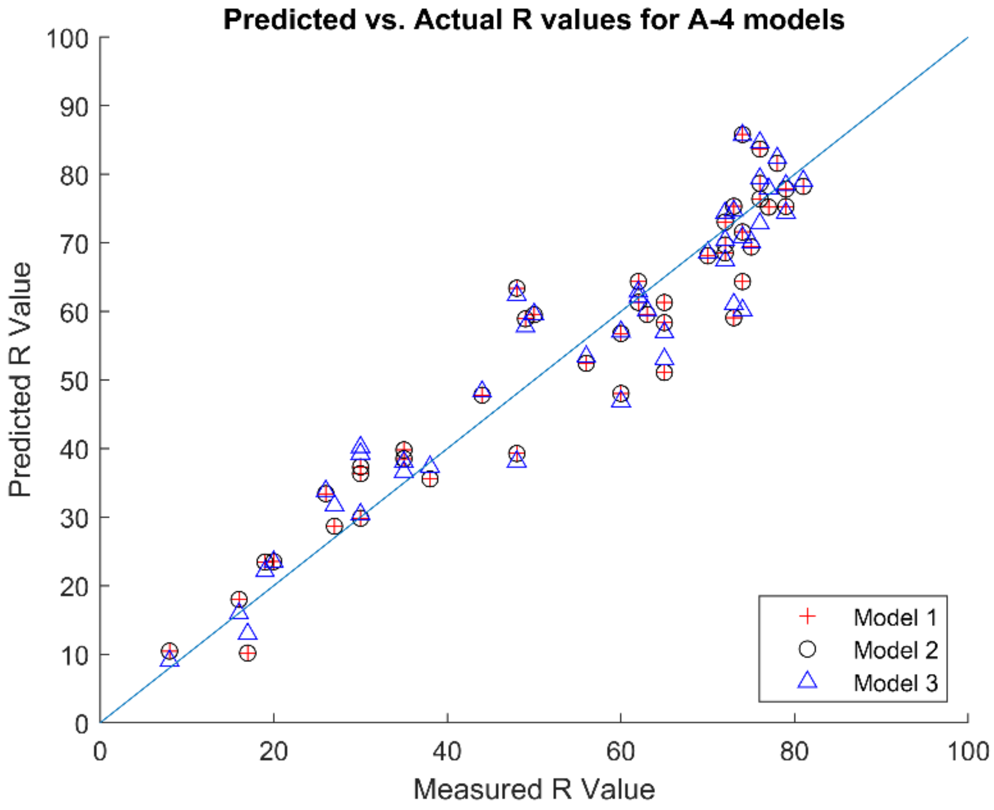


Figure 10. Predicted versus measured R-values for A-4 soil

### 6.2.7. A-6 soil

**Table 13. Correlations between R-value and basic soil properties for AASHTO A-6 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 29.865 | 34.034 | 195.394 |
| Absorption [%] | -3.508 | -3.094 | - |
| OM [%] | 5.013 | 5.322 | -1.448 |
| Liquid Limit | - | -0.463 | 1.211 |
| Plasticity Index | -0.951 | -0.580 | -1.396 |
| $P_{No. 10}$ [%] | - | - | -0.297 |
| $P_{No. 40}$ [%] | - | - | 0.591 |
| $P_{No. 200}$ [%] | - | - | -0.295 |
| Exudation Pressure [psi] | 0.0430 | 0.0433 | 0.0117 |
| MC [%] | -4.452 | -4.443 | - |
| MC/OM | - | - | -167.838 |
| MCDRPI = $(MC_{diff})/OM \times PI$ | - | - | 5.290 |
| $R^2$ | 0.6971 | 0.6987 | 0.6684 |
| $R^2$ adjusted | 0.6881 | 0.6879 | 0.6614 |

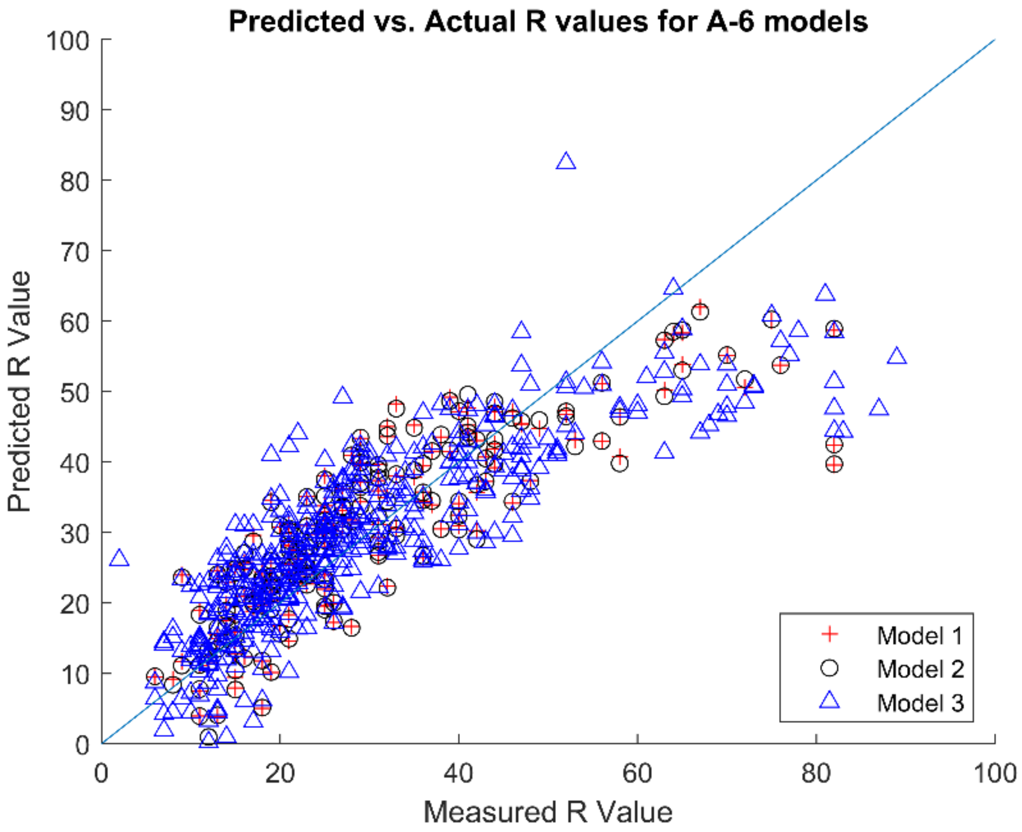


**Figure 11. Predicted versus measured R-values for A-6 soil**

### 6.2.8. A-7-6 soil

**Table 14. Correlations between R-value and basic soil properties for AASHTO A-7-6 soil**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 0.920 | - | 12.020 |
| Absorption [%] | 5.379 | 6.036 | 4.631 |
| MDD [lb/ft$^3$] | - | 0.603 | - |
| Liquid Limit [%] | -0.557 | - | - |
| Plastic Limit [%] | - | - | 1.302 |
| Plasticity Index [%] | - | -1.099 | -0.635 |
| $P_{No. 4}$ [%] | 0.473 | 0.353 | - |
| $P_{No. 10}$ [%] | - | - | 0.385 |
| Exudation Pressure [psi] | 0.0137 | 0.0157 | 0.0169 |
| $MC_{diff}$ | -4.265 | - | -2.546 |
| MCDRPI = $(MC_{diff})/OM \times PI$ | 0.897 | 2.988 | - |
| MC/OM | - | -54.334 | - |
| (MC-OM)/OM | - | -70.510 | - |
| MC.MDD | - | - | -0.0199 |
| $R^2$ | 0.8583 | 0.8605 | 0.8526 |
| $R^2$ adjusted | 0.8444 | 0.8442 | 0.8355 |

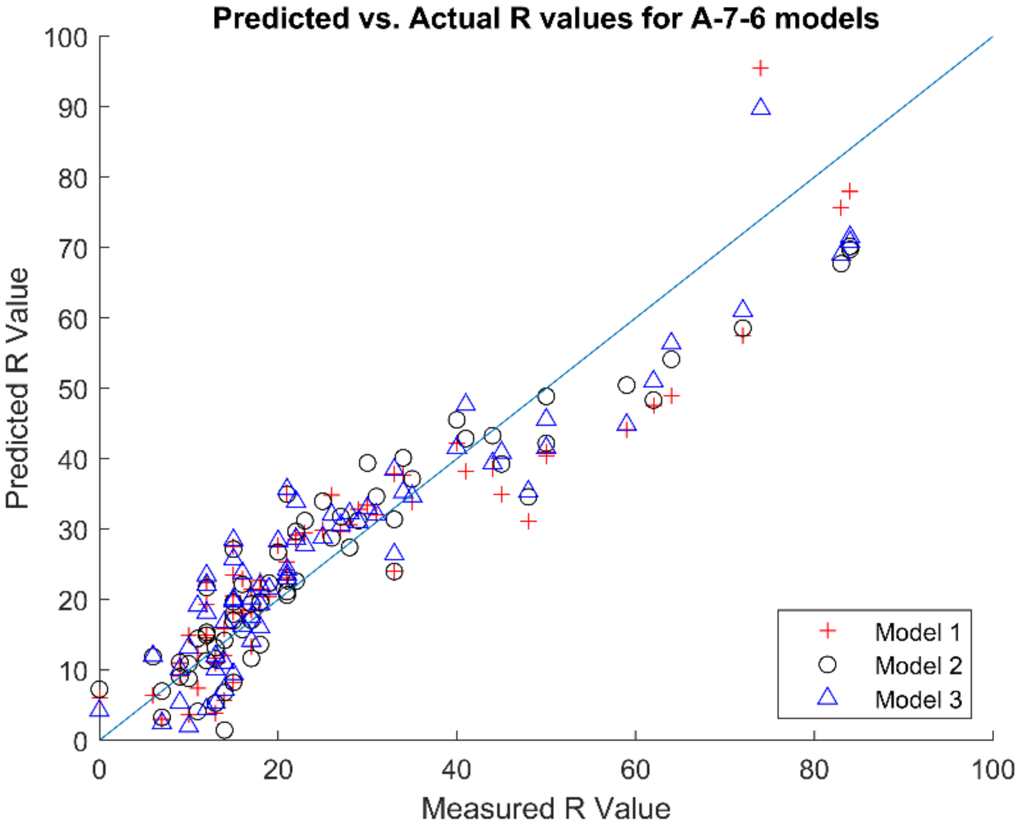


**Figure 12. Predicted versus measured R-values for A-7-6 soil**

To evaluate the merit of the possible regressions, the $R^2$ and adjusted $R^2$ values were calculated for each regression model. The following regression models presented in equations 43 through 50 were selected as the top ranked ones with the highest adjusted $R^2$ values to estimate the R-value based on the basic soil properties. To calculate the final R-value for any soil type, the value of 300 psi should be assumed for the EP value and the knowledge of the moisture content associated with the exudation pressure of 300 psi is required.

Soil type: A-1-a

R-value = -293.9 + 134 SpG + 4.45 Abs – 1.16 PI – 1.08 $P_{\#200}$ +0.0147 EP + 3.93 MC – 9.85 MCDRPI {$R^2$ = 0.954, Adj. $R^2$= 0.944, RMSE = 3.83} (43)

Soil type: A-1-b

R-value = -12.711 + 0.772 MDD – 2.232 LL +2.99 PL – 0.633 $P_{\#40}$ -0.570 $P_{\#200}$ + 0.0231 EP – 8.893 MCDRPI {$R^2$ = 0.552, Adj. $R^2$= 0.529, RMSE = 9.38} (44)

Soil type: A-2-4

R-value = 150.64 – 28.82 SpG – 3.83 OM + 0.730 LL - 1.233 PI + 0.170 $P_{\#10}$ – 0.999 $P_{\#200}$ + 0.0422 EP –7.055 $MC_{diff}$ {$R^2$ = 0.628, Adj. $R^2$= 0.618, RMSE = 12.44}
(45)

Soil type: A-2-6

R-value = 179.143 + 4.167 Abs + 1.527 MDD – 0.469 LL + 2.379 PL + 0.582 $P_{\#40}$ – 1.846 $P_{\#200}$ + 0.0616 EP – 6.199 $MC_{diff}$ {$R^2$ = 0.693, Adj. $R^2$= 0.684, RMSE = 11.83}
(46)

Soil type: A-2-7

R-value = -35.954 + 1.152 PL + 0.969 $P_{\#10}$ – 1.663 $P_{\#40}$ + 1.118 $P_{\#200}$ + 0.0411 EP – 2.859 $MC_{diff}$ {$R^2$ = 0.708, Adj. $R^2$= 0.683, RMSE = 11.02} (47)

Soil type: A-4

R-value = -427.651 – 156.304 SpG + 4.472 MDD – 1.967 LL + 9.381 PL + 3.316 $P_{\#10}$ – 1.986 $P_{\#200}$ + 0.0281 EP – 23.262 MCDRPI {$R^2$ = 0.911, Adj. $R^2$= 0.891, RMSE = 7.13}
(48)

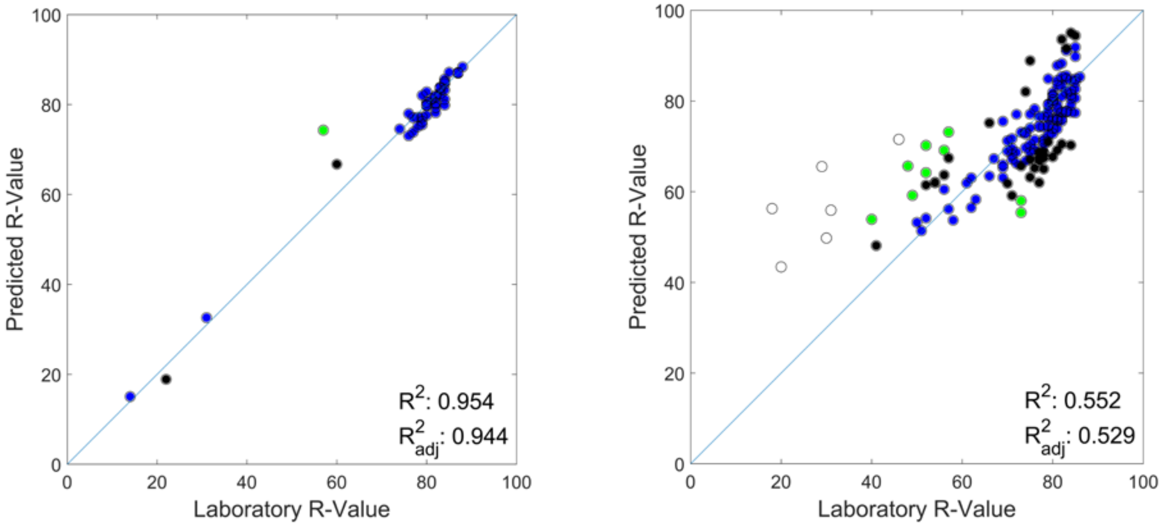
Soil type: A-6

R-value = 29.865 – 3.508 Abs + 5.013 OM – 0.951 PI +0.0430 EP – 4.452 MC {$R^2$ = 0.697, Adj. $R^2$= 0.688, RMSE = 9.22} (49)

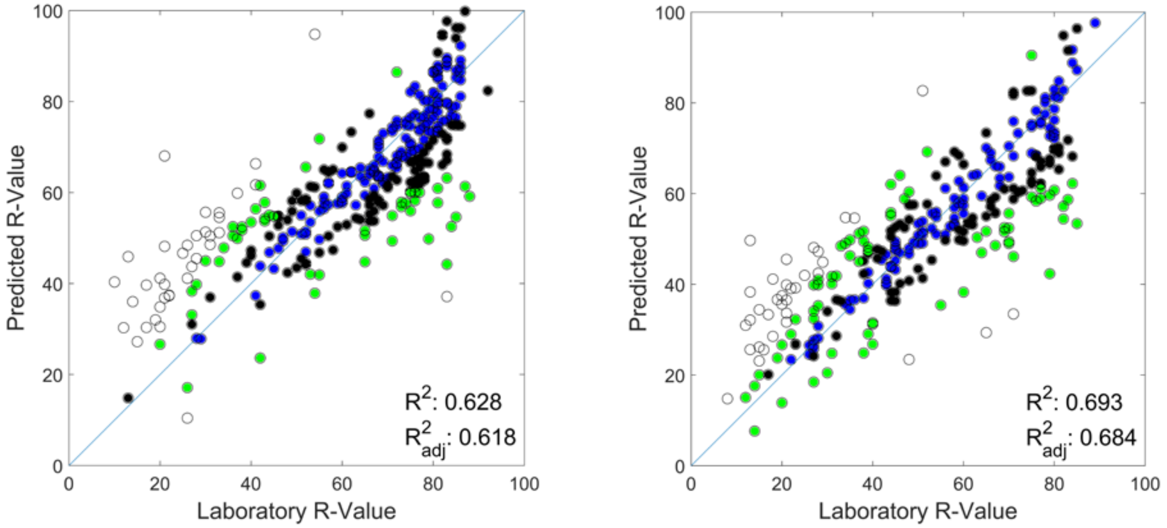
Soil type: A-7-6

R-value = 0.920 + 5.379 Abs – 0.557 LL + 0.473 $P_{\#4}$ + 0.0137 EP – 4.265 $MC_{diff}$ + 0.897 MCDRPI {$R^2$ = 0.858, Adj. $R^2$= 0.844, RMSE = 8.02} (50)

The plots of the R-values predicted by the above equations and the laboratory R-values are presented in Figures 13-14. The diagonal line represents the case where the predicted value was equal to the laboratory value. A comparison of the percentage of data points where the predicted R-value was within ±10%, ±20%, and ±50% of the laboratory measured R-value is shown in Table 15. The predicted values that were within ±10% of the laboratory value are shown in the blue-filled circles, those within ±20% of the laboratory value are shown in the black-filled circles, and those within ±50% of the laboratory value are shown in the green-filled circles. All other cases are plotted in hollow circles.

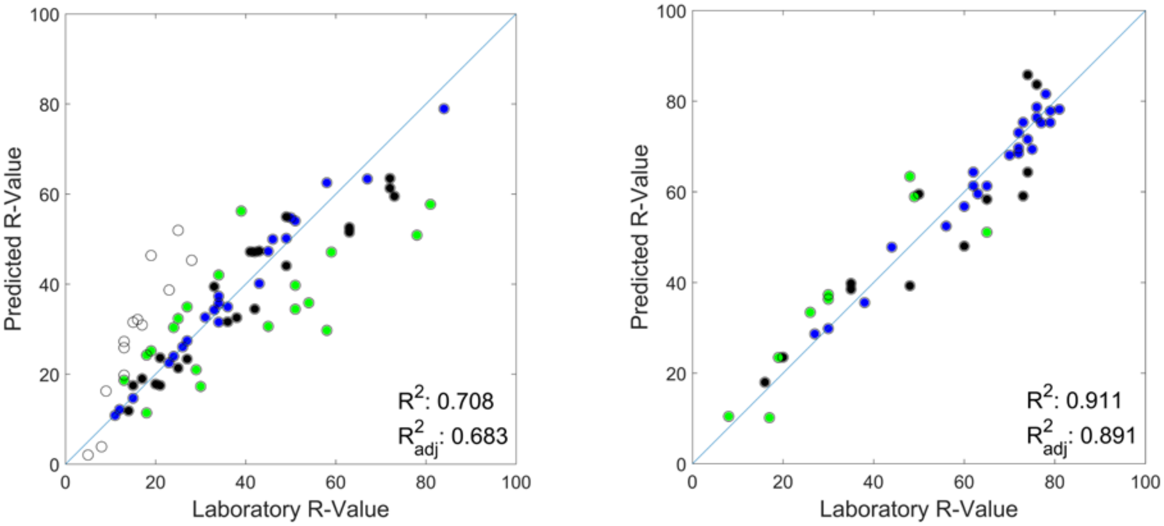


(a) A-1-a soil (b) A-1-b soil



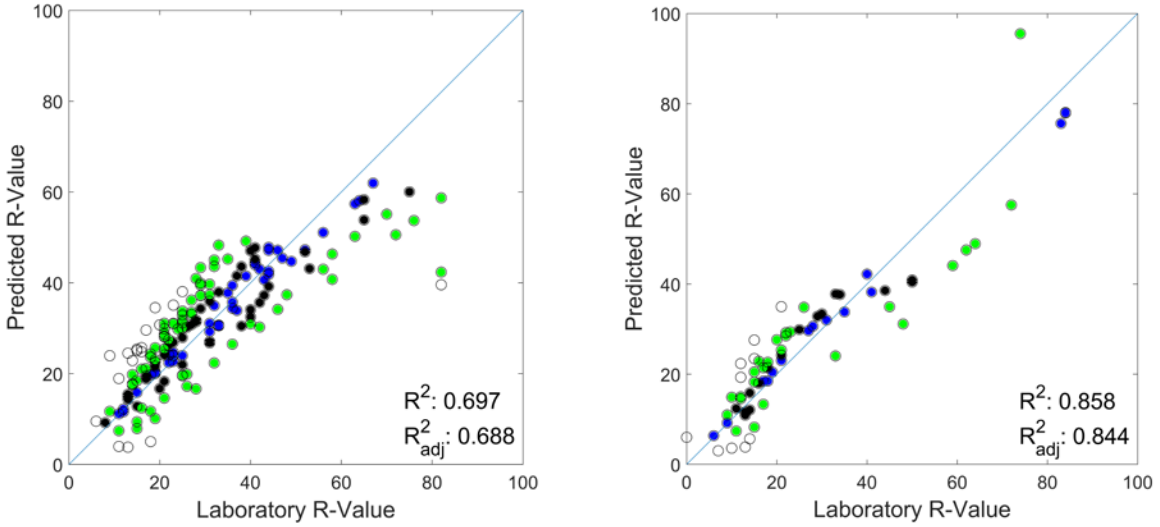
(c) A-2-4 soil (d) A-2-6 soil

**Figure 13. Predicted versus measured values for soil types A-1-a to A-2-6**



(a) A-2-7 soil

(b) A-4 soil



(c) A-6 soil

(d) A-7-6 soil

**Figure 14. Predicted versus measured values for soil types A-2-7 to A-7-6**

**Table 15. Predicted R-values within certain percentages of the laboratory R-values**

| | A-1-a soil | A-1-b soil | A-2-4 soil | A-2-6 soil | A-2-7 soil | A-4 soil | A-6 soil | A-7-6 soil |
|---|---|---|---|---|---|---|---|---|
| ±10% | 93% | 68% | 43% | 35% | 29% | 53% | 24% | 22% |
| ±20% | 98% | 90% | 73% | 64% | 57% | 80% | 48% | 46% |
| ±50% | 100% | 96% | 89% | 88% | 82% | 100% | 87% | 84% |

## 6.3 Alternative R-value equations without moisture content information

The regression models presented in equations 43 through 50 require the knowledge of the moisture content associated with the exudation pressure of 300 psi for the accurate estimation of the final R-value for the soil. Such information can be obtained from the R-value test already conducted for the representative soil specimens at the site. The regression analysis revealed a strong dependency of the R-value with the moisture content of the soil. In cases where the R-value test result for a soil specimen is not available to provide an estimate for the moisture content associated with the exudation pressure of 300 psi, another set of regressions were identified in this study to provide an estimate for the R-value. The following section presents the alternative regression models, which do not require any prior information about the moisture content for the soil at the exudation pressure of 300 psi.

Soil type: A-1-a

$$\begin{aligned} R-value &= -1016.8676 - 21.1913 * Abs + 7.1937 * MDD + 37.5803 * OM \\ &-5.1423 * LL + 0.738 * PI - 1.935 * P_{\#10} + 3.035 * P_{\#40} \end{aligned} \tag{51}$$

Soil type: A-1-b

$$R-value = 193.785 - 39.483 * SpG - 1.917 * PL + 0.0396 * EP \tag{52}$$

Soil type: A-2-4

$$R-value = 133.698 - 22.206 * SpG - 0.749 * LL - 0.958 * P_{\#200} + 0.0627 * EP \tag{53}$$

Soil type: A-2-6

$$R-value=-183.827+1.590*MDD+3.797*OM+0.739*LL \\ -2.404*PI+0.185*P_{\#4}-1.368*P_{\#200}+0.0775*EP \tag{54}$$

Soil type: A-2-7

$$R-value=-47.628+1.654*LL-1.899*PI+0.428*P_{\#10}+0.0507*EP \tag{55}$$

Soil type: A-4

$$R-value=-692.704+6.402*Abs+3.448*MDD+5.631*PL \\ +2.834*P_{\#4}+0.521*P_{\#40}-1.732*P_{\#200}+0.0839*EP \tag{56}$$

Soil type: A-6

$$R-value=7.234-6.588*Abs+3.794*OM-1.917*PL-2.658*PI+0.946*P_{\#4} \\ -0.741*P_{\#40}+0.0731*EP \tag{57}$$

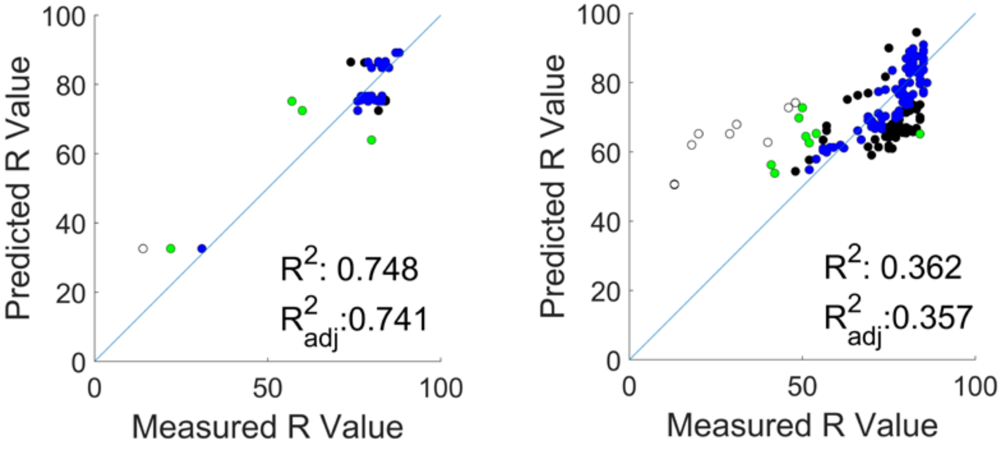
Soil type: A-7-6

$$R-value=-318.529+107.764*SpG-4.612*LL+3.967*PL+0.975*P_{\#10} \\ -0.565*P_{\#200}+0.0423*EP \tag{58}$$

The plots of the R-values predicted by equations 51 through 58 and the laboratory R-values are presented in Figures 15-16. The diagonal line represents the case where the predicted value was equal to the laboratory value. A comparison of the percentage of data points where the predicted R-value was within ±10%, ±20%, and ±50% of the laboratory measured R-value is shown in Table 16. The predicted values that were within ±10% of the laboratory value are shown in the blue-filled circles, those within ±20% of the laboratory value are shown in the black-filled circles, and those within ±50% of the laboratory value are shown in the green-filled circles . All other cases are plotted in hollow circles.

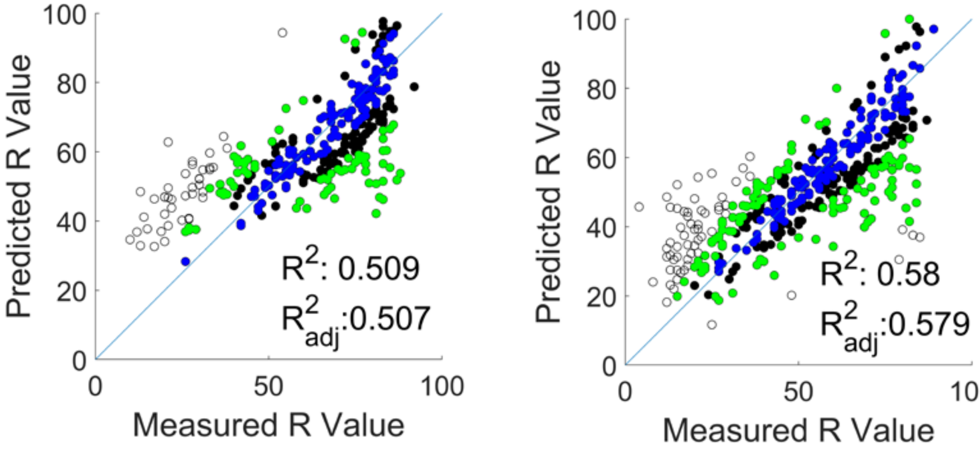
**Table 16. Predicted R values using equations 51-58**

| | A-1-a | A-1-b | A-2-4 | A-2-6 | A-2-7 | A-4 | A-6 | A-7-6 |
|---|---|---|---|---|---|---|---|---|
| +/- 10% | 71.43% | 53.10% | 35.85% | 30.90% | 15.58% | 48.89% | 26.44% | 27.94% |
| +/- 20% | 83.33% | 88.28% | 64.47% | 56.78% | 31.17% | 71.11% | 45.40% | 42.65% |
| +/- 50% | 97.62% | 93.79% | 87.11% | 85.18% | 75.32% | 93.33% | 83.33% | 69.12% |



(a) A-1-a soil

(b) A-1-b soil



(c) A-2-4 soil

(d) A-2-6 soil

**Figure 15. R value for soil types A-1-a through A-2-6 using equations 51-54**

## 7. Resilient modulus prediction models based on soil index properties

### 7.1 Database of resilient modulus for Colorado soils obtained from Ground Engineering

One of the project tasks was to establish a comprehensive dataset of existing resilient modulus data and basic soil properties for Colorado soils. Ground Engineering Consultants, as the main source for resilient modulus test data in Colorado, with the suggestion of the study panel, was contracted to collect and compile detailed reports of the resilient modulus and associated basic soil properties for Colorado soils. Their work also included identifying historical resilient modulus, gradation, particle size analysis, R-value, maximum dry density, optimum moisture content, and Atterberg limits data, which were collected at Ground Engineering's Commerce City laboratory. We obtained the resilient modulus data for 203 test samples.

The values of the *k* coefficients were calculated using the constitutive model presented in Equation 9. The NCHRP Design Guide (NCHRP 2003, 2004) recommends soil tests with a high $R^2$ for the constitutive model to develop regressions because these *k* values are the most accurate ones to produce high-quality $M_r$ models. Once the three *k* values for each sample were calculated, the regression equations were developed using MATLAB for various possible combinations of independent variables. The independent variables were as follows: dry unit weight ($\gamma_d$)(pcf), in-situ moisture content ($MC$), maximum dry density ($MDD$) (pcf), optimum moisture content ($OM$), percent passing of 2- inch sieve ($P_{2in}$), percent passing of 1-inch sieve ($P_{1in}$), percent passing of ½-inch sieve ($P_{1/2in}$), percent passing of 3/8-inch sieve ($P_{3/8in}$), percent passing of #4 sieve ($P_{\#4}$), percent passing of #10 sieve ($P_{\#10}$), percent passing of #16 sieve ($P_{\#16}$), percent passing of #50 sieve ($P_{\#50}$), percent passing of #100 sieve ($P_{\#100}$), and percent passing of #200 sieve ($P_{\#200}$), liquid limit (LL), and plasticity index (PI).

**Table 17. Range of k coefficients for all specimens used in resilient modulus regressions**

| AASHTO soil type | Variable | nObs | Mean | Standard Deviation | Min | Max |
|---|---|---|---|---|---|---|
| A-1-b | $k_1$ | 9 | 1140.9 | 450.10 | 540.03 | 2030.8 |
| | $k_2$ | 9 | 0.481 | 0.292 | 0.105 | 1.061 |
| | $k_3$ | 9 | -1.42 | 1.56 | -3.89 | 0.45 |
| A-2-4 | $k_1$ | 56 | 1603.4 | 894.90 | 0.68 | 4914.3 |
| | $k_2$ | 56 | 0.365 | 0.643 | -3.875 | 0.985 |
| | $k_3$ | 56 | -2.40 | 1.33 | -4.27 | 0.89 |
| A-4 | $k_1$ | 29 | 1622.1 | 755.68 | 501.08 | 3529.3 |
| | $k_2$ | 29 | 0.312 | 0.307 | -0.356 | 0.792 |
| | $k_3$ | 29 | -3.62 | 0.89 | -5.58 | -1.48 |
| A-6 | $k_1$ | 57 | 1376.5 | 515.26 | 533.87 | 2588.1 |
| | $k_2$ | 57 | 0.184 | 0.195 | -0.264 | 0.534 |
| | $k_3$ | 57 | -3.66 | 0.99 | -5.92 | -1.19 |

Table 17 lists the range of each soil property used as an independent variable. Only the AASHTO soil types with sufficient data are listed in table 17. Please note that several of the resilient modulus tests compiled by Ground Engineering Consultants either lacked gradation, moisture content, or unit weight information which resulted in a lower number of usable data points for the regression analysis. Soil groups A-1-b, A-2-4, A-4, and A-6 had sufficient number of soil tests to be used for statistical analysis but still lacked the information on the percentage of clay and silt for the soil.

**Table 18. Limit of Soil Property Values Used in Resilient Modulus Regression**

| AASHTO group | A-1-b soil | | A-2-4 soil | | A-4 soil | | A-6 Soil | |
|---|---|---|---|---|---|---|---|---|
| Number of samples | 9 | | 56 | | 29 | | 57 | |
| Limits | Min | Max | Min | Max | Min | Max | Min | Max |
| Dry Unit weight $\gamma_d$ [pcf] | 116.4 | 136 | 110.9 | 136.9 | 95.5 | 122 | 93.9 | 123.7 |
| MC [%] | 6.7 | 10.5 | 5.9 | 14.6 | 10.1 | 19.9 | 11 | 23.3 |
| MDD [pcf] | 122.5 | 137 | 111 | 137.6 | 100.4 | 128.4 | 100.3 | 130.2 |
| OM [%] | 6.7 | 9 | 5.5 | 13.2 | 9.2 | 19 | 9 | 22.7 |
| $P_{2in}$ [%] | 95 | 95 | 87 | 100 | 100 | 100 | 100 | 100 |
| $P_{1in}$ [%] | 90 | 100 | 79 | 100 | 100 | 100 | 100 | 100 |
| $P_{1/2in}$ [%] | 84 | 100 | 73 | 100 | 92 | 100 | 97 | 99 |
| $P_{3/8in}$ [%] | 77 | 99.7 | 70 | 100 | 85 | 100 | 95 | 100 |
| $P_{\#4}$ [%] | 63 | 96 | 66 | 100 | 76 | 100 | 90 | 100 |
| $P_{\#10}$ [%] | 52 | 74 | 60 | 99.8 | 66 | 99.9 | 84 | 99.5 |
| $P_{\#16}$ [%] | 40 | 60 | 53 | 99.4 | 62 | 98.6 | 77 | 98 |
| $P_{\#50}$ [%] | 15 | 40 | 33 | 81 | 50 | 92.1 | 60 | 88 |
| $P_{\#100}$ [%] | 6 | 32 | 24 | 61 | 43 | 100 | 54.2 | 100 |
| $P_{\#200}$ [%] | 2.9 | 24.9 | 11.3 | 35 | 35.2 | 82.1 | 37 | 83.5 |
| LL | 13 | 23 | 15 | 29 | 20 | 32 | 25 | 41 |
| PI | 3 | 6 | 1 | 10 | 6 | 10 | 11 | 20 |

## 7.2 Prediction models

The resilient modulus model parameters $k_{1\text{-}3}$ were first determined for all the soil tests adopting the resilient modulus model (Equation 9). These parameters then were considered as dependent variables and were individually correlated to the basic soil properties. Various fundamental soil properties were considered as the independent variables and were used in regression analysis. The multiple linear regression model presented in Equation 38 was used, and the top model based on $R^2$ adjusted was identified. Tables 19 through 30 list the top three regression models for the k parameters for each soil type along with the identified independent variables.

### 7.2.1. A-1-b soil

**Table 19. $k_1$ correlations for A-1-b soil group**

| **Variable** | **Model 1** | **Model 2** | **Model 3** |
|---|---|---|---|
| Intercept | 1,123.7 | 4,807.8 | 29,487.2 |
| $\gamma_d$ | | | -133.22 |
| % passing 1" sieve | -33.69 | | |
| % passing sieve #10 | 72.01 | | -60.92 |
| % passing sieve #100 | | | 29.50 |
| % passing sieve #200 | -42.29 | 74.42 | |
| In Situ Moisture | | -395.78 | -470.61 |
| Optimum Moisture | | | -576.76 |
| Liquid Limit | | -86.62 | |
| $R^2$ | 0.999 | 0.999 | 0.948 |
| $R^2$ adjusted | 0.999 | 0.998 | 0.862 |

**Table 20. $k_2$ correlations for A-1-b soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 0.1149 | 3.531 | -6.769 |
| $\gamma_d$ | | | 0.0768 |
| In-situ Moisture | | 0.0812 | |
| Optimum Moisture | | | 0.6838 |
| % passing 1” sieve | | -0.0470 | |
| % passing ¾” sieve | | | -0.2049 |
| % passing ½” sieve | | | 0.1348 |
| % passing sieve #4 | | 0.00939 | |
| % passing sieve #100 | 0.0833 | | |
| % passing sieve #200 | -0.0611 | | |
| Plasticity Index | -0.134 | | |
| $R^2$ | 0.999 | 0.999 | 0.928 |
| $R^2$ adjusted | 0.999 | 0.997 | 0.833 |

**Table 21. $k_3$ correlations for A-1-b soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 6.284 | 37.753 | -63.616 |
| In-situ Moisture | 1.109 | | |
| $\gamma_d$ | | -0.228 | |
| Max. Dry Density | | | 0.417 |
| % passing ½" sieve | | | 0.142 |
| % passing sieve #50 | -0.605 | -0.532 | |
| % passing sieve #100 | | | -0.273 |
| Plasticity Index | 0.876 | | |
| Liquid Limit | | 0.436 | |
| $R^2$ | 0.999 | 0.999 | 0.638 |
| $R^2$ adjusted | 0.999 | 0.999 | 0.422 |

### 7.2.2. A-2-4 soil

**Table 22. $k_1$ correlations for A-2-4 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 3218.3 | -20,452.8 | -1,324.7 |
| $\gamma_d$ |  | 122.28 |  |
| In-situ Moisture |  |  | -217.433 |
| % passing 2” sieve | -102.91 |  |  |
| % passing 1.5” sieve |  | 69.535 |  |
| % passing ¾” sieve |  |  | 40.942 |
| % passing sieve #4 | 74.275 |  |  |
| % passing sieve #40 |  | -103.81 | -29.568 |
| % passing sieve #100 |  | 138.59 | 51.388 |
| % passing sieve #200 |  | 65.517 |  |
| Plasticity Index | 353.846 |  | 200.792 |
| $R^2$ | 0.998 | 0.911 | 0.873 |
| $R^2$ adjusted | 0.994 | 0.822 | 0.803 |

### Table 23. $k_2$ correlations for A-2-4 soil group

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 7.160 | 9.725 | -4.466 |
| % passing 2” sieve | -0.0753 | -0.130 |  |
| % passing 1” sieve |  |  | 0.0971 |
| % passing 3/8” sieve |  | 0.0505 |  |
| % passing sieve #16 | 0.0198 |  | -0.0367 |
| % passing sieve #50 |  |  | 0.0317 |
| Plasticity Index | -0.155 | -0.187 | 0.202 |
| Liquid Limit |  |  | -0.210 |
| $R^2$ | 0.999 | 0.999 | 0.858 |
| $R^2$ adjusted | 0.999 | 0.999 | 0.756 |

### Table 24. $k_3$ correlations for A-2-4 soil group

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 1.402 | 21.540 | -9.725 |
| $\gamma_d$ |  | -0.0967 |  |
| In-situ Moisture |  |  | 0.323 |
| Max Dry Density |  |  | 0.0879 |
| % passing 2” sieve | 0.0131 |  |  |
| % passing 1.5” sieve |  | -0.123 | -0.0754 |
| % passing sieve #100 | -0.168 |  |  |
| Plasticity Index |  | -0,0355 |  |
| $R^2$ | 0.961 | 0.881 | 0.859 |
| $R^2$ adjusted | 0.923 | 0.810 | 0.799 |

### 7.2.3. A-4 soil

**Table 25. $k_1$ correlations for A-4 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 13,475.5 | 14,810.0 | 6,403.6 |
| In-situ Moisture |  | -281.869 | -1,387.55 |
| Optimum Moisture |  |  | 916.329 |
| % passing ½” sieve | -91.386 |  |  |
| % passing 3/8” sieve |  | -108.434 |  |
| % passing sieve #10 |  |  | -56.651 |
| % passing sieve #200 |  | 59.994 |  |
| Plasticity Index | -408.657 | -242.091 |  |
| Liquid Limit |  |  | 297.241 |
| $R^2$ | 0.995 | 0.970 | 0.915 |
| $R^2$ adjusted | 0.986 | 0.850 | 0.847 |

**Table 26. $k_2$ correlations for A-4 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 2.622 | -0.913 | 1.946 |
| In-situ Moisture |  | 0.0551 | 0.0776 |
| % passing ½” sieve | -0.00816 |  |  |
| % passing 3/8” sieve |  | 0.0115 |  |
| % passing sieve #16 |  |  | -0.00867 |
| % passing sieve #200 |  | -0.0165 |  |
| Plasticity Index | -0.215 | 0.0611 |  |
| Liquid Limit |  |  | -0.0686 |
| $R^2$ | 0.984 | 0.971 | 0.809 |
| $R^2$ adjusted | 0.954 | 0.856 | 0.714 |

**Table 27. $k_3$ correlations for A-4 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -3.974 | -16.008 | 19.860 |
| $\gamma_d$ | -0.0866 | | |
| Max Dry Density | | | -0.164 |
| In-situ Moisture | | 0.0739 | |
| Optimum Moisture | | | -0.538 |
| % passing 3/8" sieve | 0.133 | 0.135 | |
| % passing sieve #40 | | | 0.0289 |
| % passing sieve #100 | | 0.0276 | |
| Plasticity Index | -0.400 | -0.527 | |
| $R^2$ | 0.994 | 0.979 | 0.911 |
| $R^2$ adjusted | 0.985 | 0.898 | 0.874 |

### 7.2.4. A-6 soil

**Table 28. $k_1$ correlations for A-6 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | -310.073 | 10,283.1 | -619.055 |
| Max Dry Density | | -103.349 | |
| Optimum Moisture | | -300.289 | -267.314 |
| % passing 3/8” sieve | 76.988 | | |
| % passing sieve #4 | | 125.033 | |
| % passing sieve #10 | | | 102.802 |
| % passing sieve #50 | | -65.528 | -95.223 |
| Liquid Limit | -166.875 | | 105.575 |
| $R^2$ | 0.997 | 0.909 | 0.883 |
| $R^2$ adjusted | 0.992 | 0.849 | 0.806 |

### Table 29. $k_2$ correlations for A-6 soil group

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 3.413 | -11.124 | -6.135 |
| $\gamma_d$ | | | 0.0331 |
| In-situ Moisture | | 0.113 | |
| Optimum Moisture | | | 0.0783 |
| Max Dry Density | | 0.0596 | |
| % passing 3/8” sieve | -0.0245 | | |
| % passing sieve #10 | | 0.0240 | |
| % passing sieve #16 | | | -0.0102 |
| % passing sieve #40 | | | 0.0325 |
| % passing sieve #200 | | 0.0145 | |
| Liquid Limit | -0.0278 | | |
| $R^2$ | 0.971 | 0.848 | 0.842 |
| $R^2$ adjusted | 0.914 | 0.696 | 0.684 |

**Table 30. $k_3$ correlations for A-6 soil group**

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Intercept | 17.795 | 18.296 | 22.977 |
| $\gamma_d$ | | | -0.0646 |
| In-situ Moisture | -0.391 | -0.361 | |
| % passing 2” sieve | | | |
| % passing sieve #4 | -0.199 | -0.217 | -0.239 |
| % passing sieve #50 | | 0.0191 | 0.0808 |
| % passing sieve #200 | | | 0.118 |
| Liquid Limit | 0.413 | 0.396 | |
| Plasticity Index | -0.679 | -0.685 | -0.553 |
| $R^2$ | 0.939 | 0.949 | 0.867 |
| $R^2$ adjusted | 0.877 | 0.864 | 0.647 |

The following equations represent the top regression model for each soil type:

***Soil Type: A-1-b;*** $M_r$ model$\{R^2 = 0.905, R^2_{adj} = 0.903\}$

$k_1 = 1123.7 - 33.69\ P_{1\ in} + 72.01\ P_{\#10} - 42.29\ P_{\#200}$ $\{R^2 = 0.999$, Adj. $R^2 = 0.999\}$
(59)

$k_2 = 0.1149 + 0.0833\ P_{\#100} - 0.0611\ P_{\#200} - 0.134\ PI$ $\{R^2 = 0.999$, Adj. $R^2 = 0.999\}$
(60)

$k_3$ = 6.284 + 1.109 MC - 0.605 $P_{\#50}$ + 0.876 PI {$R^2$ = 0.999, Adj. $R^2$= 0.999}
(61)

***Soil Type: A-2-4;*** $M_r$ model {$R^2$ = 0.497, $R^2_{adj}$ = 0.488}

$k_1$= 3218.3–102.91$P_{2in}$+74.275 $P_{\#4}$+353.85 PI {$R^2$ = 0.999, Adj. $R^2$ = 0.995}
(62)

$k_2$ = 7.16 - 0.0753 $P_{2\ in}$ + 0.0199 $P_{\#16}$ - 0.155 PI {$R^2$ = 0.999, Adj. $R^2$ = 0.999}
(63)

$k_3$ = 1.402 + 0.0131 $P_{2\ in}$ - 0.168 $P_{\#100}$ {$R^2$ = 0.999, Adj. $R^2$ = 0.999} (64)

***Soil Type: A-4***; $M_r$ model {$R^2$ = 0.608, $R^2_{adj}$ = 0.595}

$k_1$ = 13475 - 91.386 $P_{1/2\ in}$ – 408.65 PI {$R^2$ = 0.995, Adj. $R^2$ = 0.986}
(65)

$k_2$ = 2.622 - 0.00816 $P_{1/2\ in}$ - 0.215 PI {$R^2$ = 0.985, Adj. $R^2$ = 0.954}
(66)

$k_3$ = -3.974 - 0.0866 $\gamma_d$ + 0.133 $P_{3/8\ in}$ - 0.401 PI {$R^2$ = 0.962, Adj. $R^2$ = 0.924} (67)

***Soil Type: A-6;*** $M_r$ model {$R^2$ = 0.730, $R^2_{adj}$ = 0.721}

$k_1$ = -310.07 + 76.988 $P_{3/8\ in}$ – 166.875 LL {$R^2$ = 0.997, Adj. $R^2$ = 0.992}
(68)

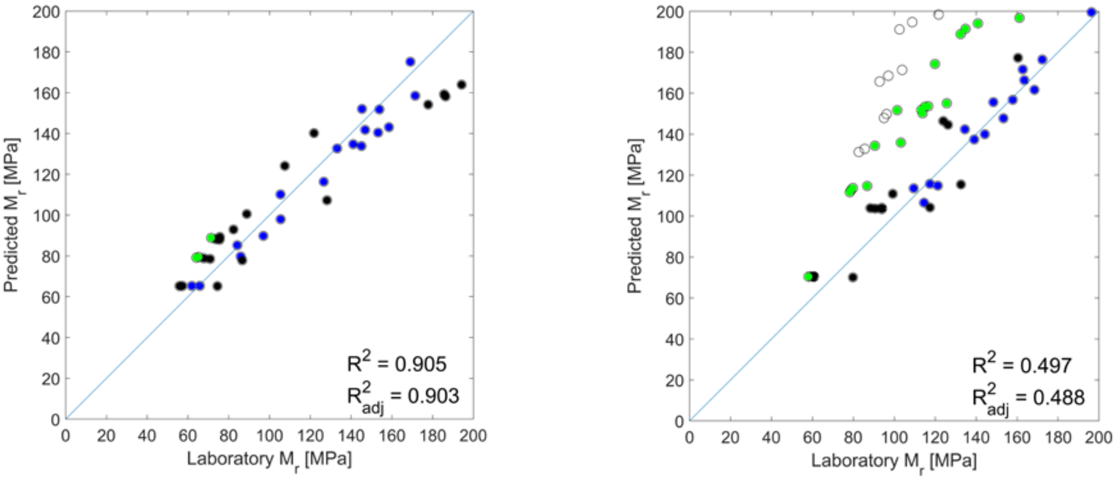
$$k_2 = 3.413 - 0.0245\ P_{3/8\ in} - 0.0278\ LL\ \{R^2 = 0.972,\ Adj.\ R^2 = 0.915\} \quad (69)$$

$$k_3 = 17.795 - 0.391\ MC - 0.199\ P_{\#4} + 0.413\ LL - 0.679\ PI\ \{R^2 = 0.939,\ Adj.\ R^2 = 0.877\} \quad (70)$$

The plots of the $M_r$ predicted using equations 59 through 70 and the laboratory measured $M_r$ are presented in Figure 16. The diagonal line indicates that the predicted value was equal to the laboratory value. The percentage of the cases that had predicted values within ±10%, ±20%, and ±50% for each soil type are presented in Table 31. The predicted values that were within ±10% of the laboratory value are shown in blue-filled circles, those within ±20% of the laboratory value are shown in black-filled circles, and those within ±50% of the laboratory values ae shown in green-filled circles. All other cases are plotted in hollow circles. The A-2-4 soil has a relatively low $R^2$ adjusted value for its correlation and therefore a more extensive testing program was conducted to study this specific soil type. In addition, given that the database of resilient modulus tests lacked the information on the percent of silt and clay and all tests were not systematically conducted for the specific soil types of interest, the use of these regression models requires caution. Specifically, the A-6 soil had the least percentage of predictions within the specified ranges and the use of the regression models is not suggested without additional evaluation.

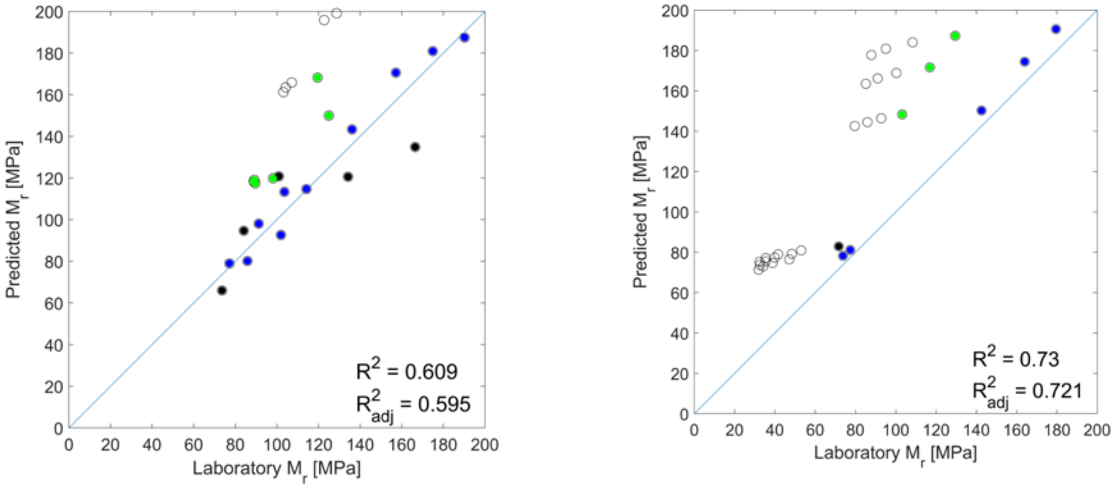
**Table 31. Specimens with predicted Mr within certain percentages of the laboratory values**

| | A-1-b soil | A-2-4 soil | A-4 soil | A-6 soil |
|---|---|---|---|---|
| ±10% | 41% | 21% | 37% | 17% |
| ±20% | 86% | 41% | 57% | 20% |
| ±50% | 100% | 67% | 83% | 30% |



(a) A-1-b soil

(b) A-2-4 soil



(c) A-4 soil

(d) A-6 soil

**Figure 16. Predicted versus laboratory $M_r$ values for all soil types in established database**

## 8. A-2-4 soil testing and resilient modulus prediction model

After a detailed review of the relevant literature and conducting a preliminary regression analysis, we determined that two important independent variables were not available in the existing Colorado database. These two property variables (the percent silt and clay content) are not reported in a sieve analysis test and can be found only through a hydrometer test. Therefore, because they are used in several existing linear regressions to predict $M_r$ (e.g., Titi et al., 2006; Malla and Joshi, 2007), it was determined that a more comprehensive database that included the hydrometer test results was needed. As shown in Table 17 , AASHTO soil type A-2-4 was the most prevalent in the existing database and was selected as the target soil type for a more detailed and systematic testing and regression analysis. While the remainder of the study presented here focuses on A-2-4 soil type, the same methodology and correlation analysis can be applied to other soil types of interest.

## 8.1 Laboratory testing program for A-2-4 soils

The resilient modulus values were determined from the repeated load triaxial test following the AASHTO T307 procedure. Figure 17 shows the apparatus used for conducting the tests on specimens, which were 101.6 mm (4 inches) in diameter and were compacted in five lifts with the moisture content as close as possible to the optimum moisture content. While maintaining a constant confining stress on the specimen, the repeated axial load was applied to the top of the specimen. A haversine-shaped loading waveform was applied for one second, comprised of a 0.1 second load duration and a 0.9 second rest period. The recoverable axial strain, $\varepsilon_r$ (equation 71) then was measured and used to calculate the resilient modulus as a function of the applied bulk and octahedral shear stresses.

$$M_r = \frac{\sigma_d}{\varepsilon_r} \tag{71}$$

Table 32 provides the list of sites from which the 30 A-2-4 soil specimens were collected for systematic testing. In this study, reconstituted test specimens were used for resilient modulus testing. The reconstituted samples have been compacted to approximately 95% of Maximum Dry Density and near optimum moisture content. With the addition of the hydrometer test, the following new independent variables were added: percent silt content ($\%Silt$, particles 0.074-0.002 mm) and percent clay content ($\%Clay$, particles smaller than 0.002 mm). Additional composite variables also were constructed and explored as follows: moisture content multiplied by the plasticity index ($MC.PI$), the plasticity index multiplied by the percent passing sieve #200 ($PI.P_{\#200}$), the optimum moisture multiplied by the maximum dry density ($OM.MDD$), the dry unit weight multiplied by the maximum dry density ($\gamma_d.MDD$), and the ratio of the percent passing sieves #200 and #40 ($P_{40}^{200} = P_{\#200}/P_{\#40}$).

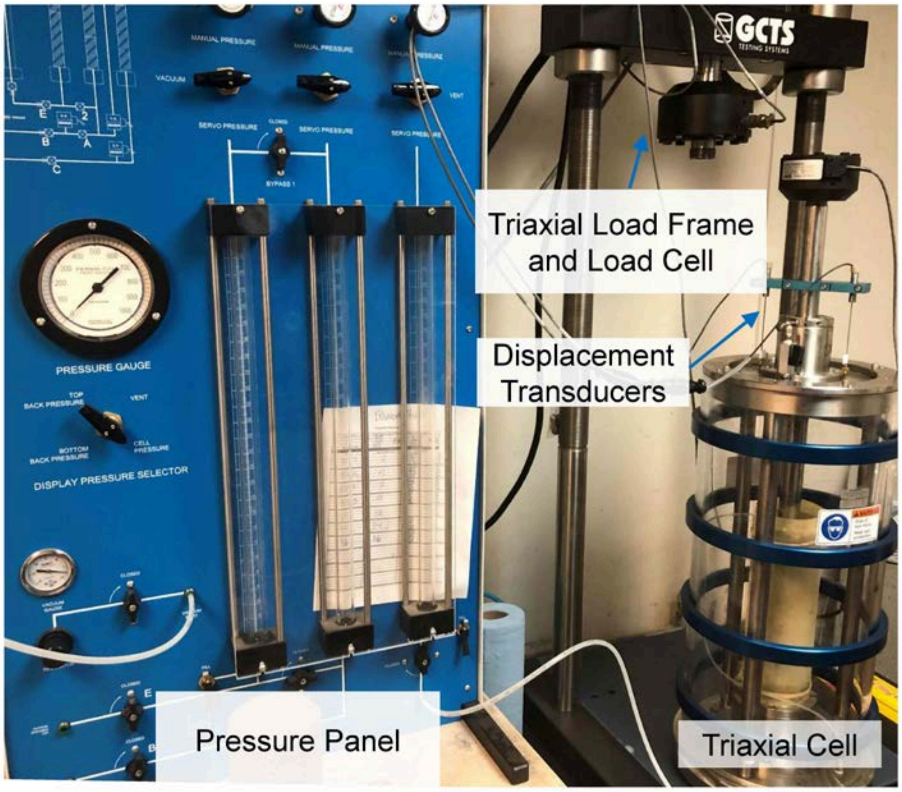


**Figure 17. GCTS testing system at the Ground Engineering Consultants Laboratory**

**Table 32. Site Locations and the associated sample numbers**

| Site Location | Sample Number |
|---|---|
| I-25 South (The Gap) | 6210 |
| NE Denver 17th Ave Pkwy & Fairfax | 6211 |
| 39th Ave @ Greenway, Denver | 6212 |
| Stapleton Filing 57 at Dallas St and 61st Ave, Denver | 6213 |
| E-470 | 6759-6590 |
| National Western Equestrian Center | 6760-6554 |
| Richter Dental | 6761-6548 |
| Denver Art Museum | 6762-6536 |
| National Western Equestrian Center | 6763-6553 |
| The Pulse on Brighton | 6764-6487 |
| North Water Reclamation Facility | 6765-6458 |
| North Water Reclamation Facility | 6766-6454 |
| Gateway Hotel | 6767-6418 |
| E69th Ave and Race St | 6768-6387 |
| PAR 1235 | 6770-6328 |
| CO Blvd and 72nd Ave | 6771-5942 |
| 3525 Clay St | 6772-6617 |
| CSU Animal | 6773-6638 |
| Buckhorn Exchange | 6774-6647 |
| Transform Fitness | 6775-6655 |
| CoDOT Spngfield-Fowler | 6776-6521 |
| CoDOT Spngfield-Fowler- sample 2 | 6776-6521 @ +2 |
| Security PFOS, Colorado Springs | 7086 |
| Gun Club Rd water line | 7087 |
| 10601 Fulton St Denver | 7088 |
| Promenade @ Castle Rock | 7089 |
| Sand Creek | 7221-6936 |
| 4104 Winoa | 7222-6922 |
| DEN5 Parking Lot | 7223-6943 |
| Kit Carson | 7224-6947 |

Appendix A includes the test results conducted by the Ground Engineering Consultants on the 30 A-2-4 soil specimens.

## 8.2 Regression analysis methodologies

### 8.2.1. Regression based on k-parameters

From the literature review, we determined that the most common method for predicting the resilient modulus for a soil sample was multiple linear regression to first predict the $k_{1-3}$ coefficients, which then were used to calculate the $M_r$ value using the constitutive model presented in Equation 9. Every possible combination of the independent variables was considered with up to six variables per combination. The general form of the multiple linear regression equation is as follows:

$$k_1, k_2, k_3 = \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \varepsilon \tag{72}$$

where $\beta_1, \beta_2, \ldots, \beta_n$ are the regression coefficients, $x_1, x_2, \ldots, x_n$ are the independent variables (basic soil properties), and $\varepsilon$ represents the error of the model. To assess the goodness-of-fit for each model, $R^2$ and RMSE were used as performance indices, along with the adjusted $R^2$. One of the issues with the use of $R^2$ for selection of the best model is that this metric does not protect against overfitting. The addition of another independent variable results in an increase in the $R^2$ value. To overcome this issue, the adjusted $R^2$ value was used (Equation 73). The adjusted $R^2$ penalizes the addition of non-contributing independent variables, meaning that the addition of a non-significant variable increases the ordinary $R^2$ but decreases the adjusted $R^2$. A higher adjusted $R^2$ is a clear measure of the addition of significant independent variables.

$$R^2_{adj} = 1 - \frac{SS_{residuals} / (n-K)}{SS_{total} / (n-1)} \tag{73}$$

where $K$ represents the number of variables in the model. Another common concern when performing linear regression is the multi-collinearity between independent variables. This effect is detrimental when pairs of independent variables are highly correlated with each other because including both variables in the model does not add any new information. Using the adjusted $R^2$ is one way to overcome this issue, but the multicollinearity problem also can be evaluated through the variance inflation factor (VIF) as an additional metric. The VIF is the diagonal of the correlation matrix for all the independent variables included in a regression, and a value below 10 is generally considered acceptable. The VIF was constrained to less than 10 for the $k_{1-3}$ regressions but the combination of all three regressions used to predict $M_r$ often had VIF values that were higher. However, the $k_{1-3}$ coefficients were correlated with one another because all three were calculated from the same $M_r$ test, so multi-collinearity was less of a concern in this situation than other linear regressions. Since the main goal of the regression was to reliably predict the $k_{1-3}$ coefficients and as a result the $M_r$, the high VIF issue was not considered as important as long as the model prediction was acceptable. The VIF would have been an important parameter to evaluate if the goal of the regression was to elucidate the underlying mechanism under which the soil properties would affect the $k_{1-3}$ coefficients.

### 8.2.2. Stepwise regression for model development

An alternative to the multiple linear regression presented in section 8.2.1 is to directly predict the $M_r$ value without the individual correlations for the $k_{1-3}$ coefficients. Elimination of the middle step, predicting the $k_{1-3}$ coefficients to be used in the constitutive model, was attempted to reduce the compounding of errors and to improve the prediction quality for the $M_r$ values. This alternative approach was taken to explore whether the quality of the regression could be increased. The general form of this regression is as follows:

$$\log\left(\frac{M_r}{P_a}\right) = \log(k_1) + k_2 \log\left(\frac{\sigma_b}{P_a}\right) + k_3 \log\left(\frac{\tau_{oct}}{P_a} + 1\right) \tag{74}$$

$$\log\left(\frac{M_r}{P_a}\right) = \left(a_1x_1 + \ldots + a_nx_n\right) + \left(b_1x_1 + \ldots + b_nx_n\right)\log\left(\frac{\sigma_b}{P_a}\right) + \left(c_1x_1 + \ldots + c_nx_n\right)\log\left(\frac{\tau_{oct}}{P_a} + 1\right) \tag{75}$$

The logarithmic version of the constitutive model was used so that the $k_2$ and $k_3$ coefficients were linear rather than exponential. Once the best regression model was identified, the individual $k_{1-3}$ coefficient models could be determined as follows:

$$\log(k_1) = a_1x_1 + \ldots + a_nx_n \tag{76}$$

$$k_2 = b_1x_1 + \ldots + b_nx_n \tag{77}$$

$$k_3 = c_1x_1 + \ldots + c_nx_n \tag{78}$$

These new $k$ coefficients could be predicted for each soil specimen and used in either the linear or logarithmic version of the constitutive model to predict $M_r$. The stepwise regression technique was used to determine the optimum combination of independent variables instead of testing every possible combination, which reduced the computational cost by an order of magnitude. Using this method, a linear regression was developed by first computing the p-value associated with adding each independent variable to the regression. A threshold p-value was specified for adding variables; and when the minimum p-value calculated was less than the threshold, the variable was added to the model. The procedure thus continued in this same way, and at each step the p-values were calculated for adding new variables and removing the existing variables in the model. Depending on the calculated p-values, the program added or removed variables until it reached the maximum number of steps specified or until all the p-values were less than the threshold value. For this regression analysis, the entrance threshold of 0.05 and exit threshold of 0.10 were selected. Figure 18 shows the general approach for the stepwise regression.

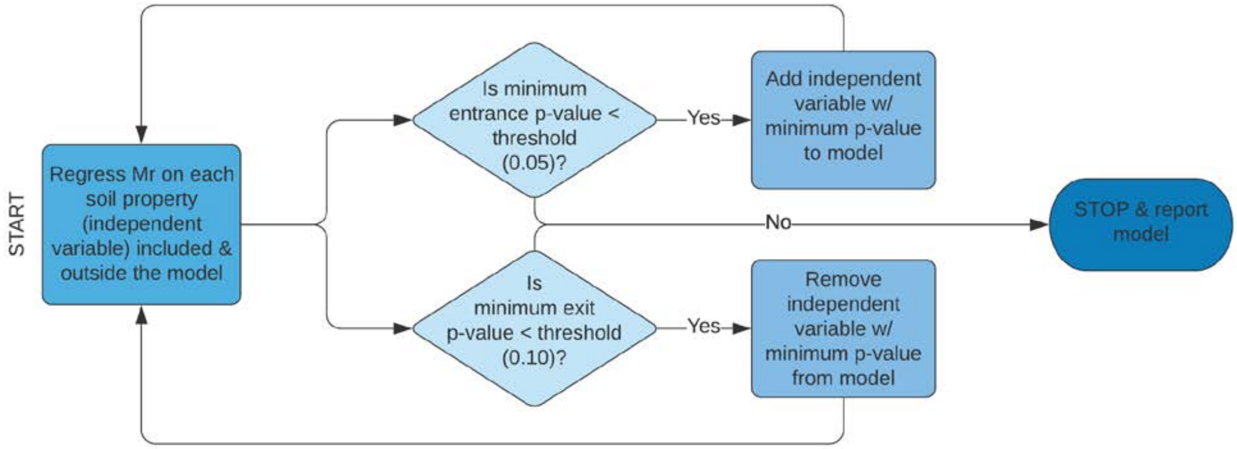


**Figure 18. Flowchart Illustrating the Process and Conditions for Stepwise Regression**

## 8.3 Results of regression analysis

### 8.3.1. Prediction models based on k-regression

Thirty specimens of A-2-4 soil type were used in the linear regression to predict the $k$ coefficients. Of these samples, 10 were non-plastic and did not have a value for $LL$; thus, only 20 observations were used in the regression. The results of the regressions to predict the $k$ coefficients are shown in Figure 19 and equations 79 through 81.

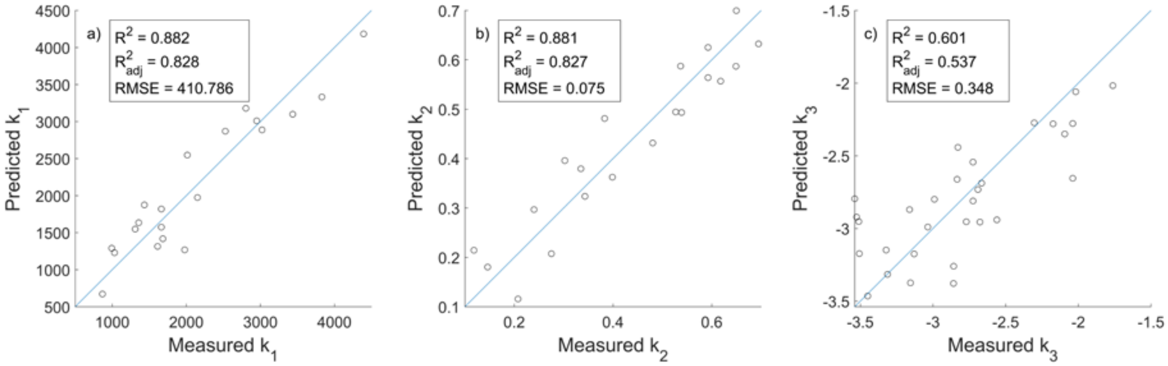


**Figure 19. Plots of Predicted versus Measured Values for (a) $k_1$, (b) $k_2$, and (c) $k_3$**

$$\log(k_1) = 67011.03 - 153.91P_{\#4} - 35.72P_{\#50} + 287.16P_{\#200} - 179.615\%silt$$
$$-90.76LL - 53544.95\gamma_d.MDD \tag{79}$$

$$k_2 = -7.07 + 0.0344\gamma_d + 0.0338P_{3/8in} + 0.0179P_{\#50} - 0.0480\%silt - 0.0734\%clay + 0.0256LL \tag{80}$$

$$k_3 = -1.137 - 0.009P_{\#40} - 0.061\%silt - 0.0148MC.PI + 0.00415PI.P_{\#200} \tag{81}$$

After the $k_1, k_2$, and $k_3$ coefficients were predicted for each specimen, the constitutive model (Equation 9) was used, along with the $\sigma_b$ and $\tau_{oct}$ values for specific tests to predict the $M_r$ value. Since each test had 15 cycles with unique combinations of $\sigma_b$ and $\tau_{oct}$, there were 15 times more observations for the $M_r$ prediction compared to $k_1, k_2$, and $k_3$ coefficients.

Figure 20 shows the measured and predicted $M_r$ values using this multiple linear regression method. The green dots represent the predictions within +/- 50% of the actual values, black dots represent the cases with +/- 20% of the actual values, and blue dots represent the cases with +/- 10% of the actual values. This model had an $R^2$ adjusted value of 0.71, but seemed to be somewhat biased to overpredicting the true value.

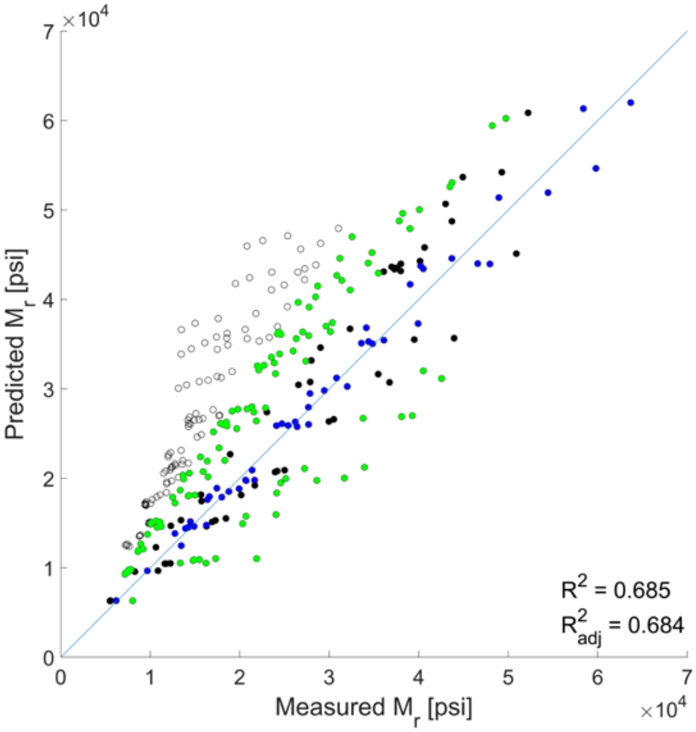


**Figure 20. Comparison of Predicted versus Measured Values for $M_r$ using k-regressions**

### 8.3.2. Prediction models based on the stepwise regression

The same dataset was used for the stepwise regression. After the best model was identified while adopting equation 74, the model was separated into parts that corresponded to the $\log(k_1)$, $k_2$ and $k_3$ using the method shown in equations 75 through 78. The results of the stepwise regression for the $k_{1-3}$ coefficients are presented in equations 82 through 84.

$$\ln(k_1) = 28.678 + 0.0726MC - 0.208P_{1in} - 0.00858P_{\#40} + 0.0494\%clay - 0.0095MC*PI \\ +0.0034PI*P_{\#200} - 0.0012OM*MDD + 0.911P_{40}^{200} \quad (82)$$

$$k_2 = 0.00968P_{\#16} - 0.00036\%clay*P_{\#16} \quad (83)$$

$$k_3 = -0.016P_{\#4} - 0.022P_{\#100} - 0.018\%silt \quad (84)$$

Figure 21 shows that the new stepwise method was less biased than the k-based model, as evidenced by the fact that all predictions fall within +/- 50% of the true values and a greater proportion of the points fall within the +/- 20% of the true values, compared to the step-wise regression.

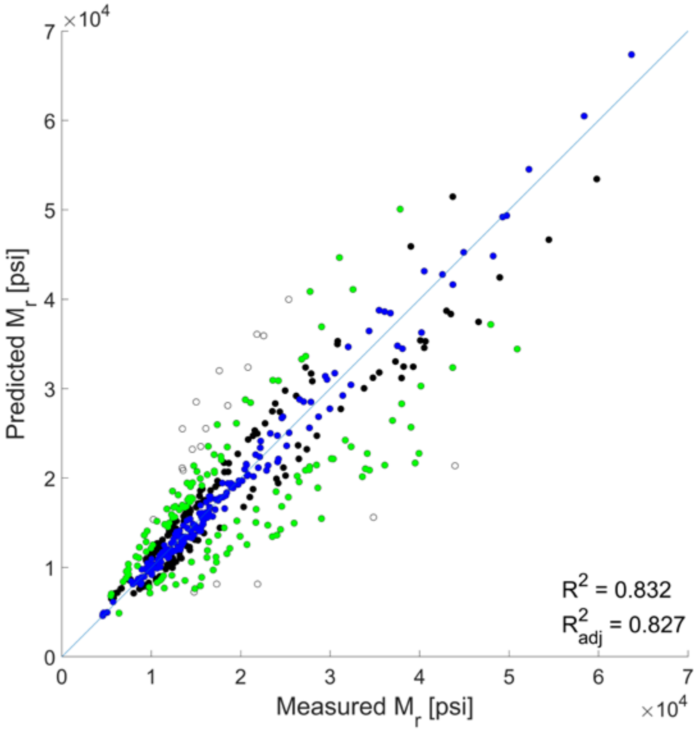


**Figure 21. Predicted versus Measured $M_r$ Values using the Stepwise Regression**

### 8.3.3. Comparison between the k-based and stepwise regression models

Both the k-based regression and the stepwise regression predicted the $M_r$ values with a high degree of accuracy. The k-based model, however, consistently overpredicted the true $M_r$ value compared to the stepwise model. By comparing Figures 20 and 21, it can be seen that the proportion of points where the predicted value was within +/- 20% of the measured value was higher for the stepwise regression (65.7%) than the k-based regression (32.0%). The prediction interval was also a very useful metric for evaluating the prediction models.

## 9. Correlations between resilient modulus and R-value for A-2-4 soil

The resilient modulus and R-value tests are inherently different. While both tests determine measures of the soil's resistance to deformation under load, the resilient modulus test is a dynamic test in nature and is conducted over 15 cycles with different load combinations. The resilient modulus is calculated based on the recoverable strain due to the load application cycles. The R-value test, however, requires at least three static tests that expresses the material's resistance to deformation as a function of the ratio of the transmitted lateral pressure to the applied vertical pressure. Due to the static nature of the R-value testing, is it hard to capture the dynamic properties of soil under repeated (traffic) loads. CDOT's earlier study concluded that equations based on the soil's R-value cannot predict resilient modulus with a high degree of accuracy (Chang et al., 1994).

In this study, using the collected test data for the thirty A-2-4 soil specimens, R-value was used an independent variable to predict the resilient modulus of the soil. To obtain a unique value of the resilient modulus, the resilient modulus for sequences of 7,8, 9, 12,13, and 14 were averaged to obtain a constant and unique value. This practice was suggested by the Ground Engineering Consultants and presented in the reports in Appendix A.

Four regression models were developed in this study based on the combination of available index properties, as follows:

**Model 1:** $$\mathrm{Mr(psi)} = -1180400 + 14511.\mathrm{P}_{1in} - 1673.\mathrm{P}_{\#4} - 39902.\mathrm{R}^{0.2} \quad (\mathrm{R}^2=0.715) \tag{85}$$

**Model 2:** $$\mathrm{Mr(psi)} = 84815 - 16964.\log(\mathrm{R}) \quad (\mathrm{R}^2=0.607) \tag{86}$$

**Model 3:** ($\mathrm{R}^2$=0.942)

For R≥50: $$\begin{aligned}\text{Mr (psi)} &= -2.9244\times10^{5}+589.81\times P_{\#100}-1976.6\times\%\text{Silt}+\\ &\quad 2274.2\times \text{PI}+2417.1\times \text{MDD}\end{aligned} \tag{87}$$

For R<50: $$\text{Mr (psi)} = 7945.6+368.66\times P_{\#100}-255.83\times P_{\#16}-648.61\times \text{OM} \tag{88}$$

**Model 4:** ($R^2$=0.532)

$$\begin{aligned}k_1 &= 7999.7-54.72\times P_{\#4}+307.37\times \text{percentClay}-12.41\times R-394.67\times \text{OM}\\ &\quad -4.05\times\%\text{Clay}\times R+4.66\times \text{OM}\times R\end{aligned} \tag{89}$$

$$k_2 = 0.635-0.0057\times P_{\#10}+0.007\times R \tag{90}$$

$$k_3 = -1.78-0.065\times\%\text{Silt} \tag{91}$$

The first model (equation 85) uses both the R-value and the particle gradation information while the second model (equation 86) uses only the R-value information. Both models predict the average value of *Mr* for each specimen with a reasonable degree of accuracy as demonstrated by $R^2$ values for the regression ranging from 0.6 to 0.7. The third model uses the R-value information as a basis to identify the proper equation for determination of *Mr* value. In third model, when the R-value is greater than or equal to 50, gradation information along with the density and plasticity index are used for the prediction and when the R-value is less than 50, gradation information along with the optimum moisture content are used for the prediction. This model achieved the highest $R^2$ value. To use this model (equations 87 and 88), the hydrometer testing information is necessary. Finally, the fourth model allows us to predict the $k_{1\text{-}3}$ coefficients for each sample using the R-value test and other basic soil properties. The *Mr* constitutive model presented in equation 9 is then used to calculate the *Mr* value based on the specific octahedral and confining stresses. Although this feature makes model 4 the most versatile, model 3 can be considered as the most accurate given its high $R^2$ value. Figure 22 shows the predicted versus average value of resilient modulus for A-2-4 soil specimens using the developed models. The diagonal line represents the case where the predicted value was equal to the laboratory value. Model 3, shown in blue circles, had the highest $R^2$ value with closest points to the diagonal line.

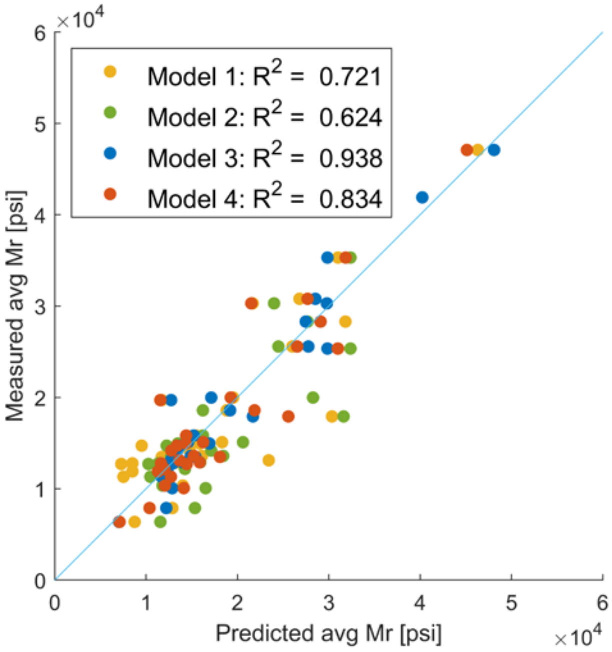


**Figure 22. Predicted versus measured Mr values by the four proposed models**

## 10. Conclusions and Recommendations

### 10.1 Conclusions

Resilient modulus is an important mechanical property of soil and $M_r$ testing therefore is an important part of pavement analysis and design as it can properly describe the stress-dependent elastic modulus of soil materials under traffic loading. In addition to the resilient modulus in pavement analysis and design, the R-value is commonly used to measure the strength of the subgrade, subbase, and base course materials used in pavements. Both tests are expensive and time consuming, however, and establishing accurate correlations between the test results and the soil's physical properties can save a considerable amount of time and money in the testing and analysis process. This research project aimed at developing correlations for estimating the resilient modulus and R-values of available soil types in established databases for Colorado soils from basic physical soil properties. Using the correlation value between field and laboratory testing would save a considerable amount of time and money in testing and analyzing the material properties. The need to leave the construction field trailer to perform additional R-value or resilient modulus testing to verify the quality of construction materials would be considerably reduced if not totally eliminated.

In the study, an extensive database of systematically conducted resilient modulus and R-value tests along with the basic soil properties for Colorado soils was established. We performed multiple linear regression analysis to develop prediction models for estimating the R-value and the resilient modulus based on the basic properties of Colorado soils. The database includes over 2,500 R-value data points and the associated soil basic properties as well as over 200 resilient modulus tests and associated soil basic properties.

In addition, a laboratory testing program for A-2-4 soil type was developed and then conducted to establish a systematic database of test results for 30 specimens of A-2-4 soil. The program included tests to evaluate basic soil properties, R-value, and repeated load triaxial. High quality resilient modulus testing was ensured by evaluating the multiple correlation coefficient, $R^2$, equal to 0.90 or higher from the regression analysis of the generalized constitutive model. By performing two tests on soil specimens with lower $R^2$ values, a database of high-quality test results for A-2-4 specimens was developed and then used for the regression analysis.

Two groups of prediction models were developed for the R-values of AASHTO soil types for the following cases: (a) with the knowledge of the moisture content for the soil at the associated exudation pressure of interest; and (b) no prior knowledge of the relation between the soil's moisture content and the exudation pressure.

The adjusted $R^2$ values obtained for the prediction models for the $k_{1\text{-}3}$ coefficients, using the generalized resilient modulus model that was developed through NCHRP Project 1-28A, ranged from 0.488 to 0.903. It was determined that in the existing databases of resilient modulus and R-value tests, two important independent variables being the percent silt and clay content, determined from hydrometer testing, were not reported. Because these two parameters are used in many regression equations, it was determined that a more comprehensive database that included the hydrometer test results was needed. Given the limited number of tests budgeted in the research study, the preference of the study panel for evaluation of the behavior of A-2-4 soil, and its prevalence in Colorado, a comprehensive study was conducted to develop reliable correlations for prediction of the resilient modulus values for A-2-4 soil type.

The main goal was to develop correlations among the resilient modulus, R-value, and basic soil properties through a systematic testing and comprehensive regression analysis of A-2-4 soil. Two methodologies for prediction of the resilient modulus values were utilized: 1) the $k$-based method and 2) the stepwise regression method. A detailed laboratory-testing program on thirty A-2-4 soil specimens collected from sites in Colorado was developed, which included the basic soil properties tests and repeated loading triaxial tests to determine the resilient modulus. The resilient modulus models developed by the stepwise regression method showed higher accuracy compared to the models developed by the $k$-based method.

## 10.2 Recommendations and Implementations

### 10.2.1. Summary of Developed Regression Models for R-value

Based on the results of this research study, several regression models were developed to be used for estimation of the R-value of several soil types based on the basic soil properties. It should be noted that the developed equations are only available for AASHTO soil types that were available in the CDOT database.

Table 33 provides a summary of the developed regressions for A-1-a, A-1-b, A-2-4, A-2-6, A-2-7, A-4, A-6, and A-7-6 soil types when the knowledge of the moisture content for the soil at the exudation pressure of interest is available. For example, the final R-value is reported for the exudation pressure of 300 psi and the prediction models presented in Table 33 require the knowledge of the moisture content at the exudation pressure of 300 psi. This information can be obtained from an existing R-value test report from a representative soil specimen. For cases where no R-value test information is available, regression models presented in Table 34 can be used for the estimation of the R-value. Please note that because of the dependency of the R-value on the moisture content, the regression models in Table 33 provide a more accurate estimation of the R-value than those presented in Table 34, as evidenced by the higher percentage of prediction cases within the specified error level of 20%.

**Table 33. Summary of developed models for estimation of R-value when moisture content information is available**

| | Prediction Model | % of cases with ±20% error in estimation |
|---|---|---|
| A-1-a | R-value = -293.9 + 134 SpG + 4.45 Abs – 1.16 PI – 1.08 $P_{\#200}$ +0.0147 EP + 3.93 MC – 9.85 | 98 |
| A-1-b | R-value = -12.711 + 0.772 MDD – 2.232 LL +2.99PL – 0.633 $P_{\#40}$ -0.570 $P_{\#200}$ + 0.0231 EP – 8.893 MCDRPI | 90 |
| A-2-4 | R-value = 150.64 – 28.82 SpG – 3.83 OM + 0.730 LL - 1.233 PI + 0.170 $P_{\#10}$ – 0.999 $P_{\#200}$ + 0.0422 EP –7.055 $MC_{diff}$ | 73 |
| A-2-6 | R-value = 179.143 + 4.167 Abs + 1.527 MDD – 0.469 LL + 2.379 PL + 0.582 $P_{\#40}$ – 1.846 $P_{\#200}$ + 0.0616 EP – 6.199 $MC_{diff}$ | 64 |
| A-2-7 | R-value = -35.954 + 1.152 PL + 0.969 $P_{\#10}$ – 1.663 $P_{\#40}$ + 1.118 $P_{\#200}$ + 0.0411 EP – 2.859 $MC_{diff}$ | 57 |
| A-4 | R-value = -427.651 – 156.304 SpG + 4.472 MDD – 1.967 LL + 9.381 PL + 3.316 $P_{\#10}$ – 1.986 $P_{\#200}$ + 0.0281 EP – 23.262 MCDRPI | 80 |
| A-6 | R-value = 29.865 – 3.508 Abs + 5.013 OM – 0.951 PI +0.0430 EP – 4.452 MC | 48 |
| A-7-6 | R-value = 0.920 + 5.379 Abs – 0.557 LL + 0.473 $P_{\#4}$ + 0.0137 EP – 4.265 $MC_{diff}$ + 0.897 MCDRPI | 46 |

The independent variables used in table 33 are as follows: specific gravity (SpG), absorption (Abs), maximum dry density (MDD), optimum moisture content (OM), moisture content (MC) corresponding to the exudation pressure of interest, liquid limit (LL), plastic limit (PL), plasticity index (PI), percent passing of #4 sieve ($P_{\#4}$), percent passing of #10 sieve ($P_{\#10}$), percent passing of #40 sieve ($P_{\#40}$), percent passing of #200 sieve ($P_{\#200}$), difference in moisture content ($MC_{diff}$ = MC-OM), moisture content differential ratio multiplied by plasticity index (MCDRPI = ($MC_{diff}$)/OM×PI), and exudation pressure (EP). For the determination of the final R-value, the EP needs to be considered as 300 psi and the moisture content (MC) associated with the 300 psi of exudation pressure needs to be used. Such value can be obtained from an existing R-value test on a representative soil. If unavailable, the regression models summarized in table 34 can be used.

## Table 34. Summary of developed models for estimation of R-value without the knowledge of moisture content for the soil

| | Prediction Model without moisture information | % of cases with ±20% error in estimation |
|---|---|---|
| A-1-a | $R-value=-1016.8676-21.1913*Abs+7.1937*MDD$ $+37.5803*OM-5.1423*LL+0.738*PI$ $-1.935*P_{\#10}+3.035*P_{\#40}$ | 83 |
| A-1-b | $R-value=193.785-39.483*SpG-1.917*PL+0.0396*EP$ | 88 |
| A-2-4 | $R-value=133.698-22.206*SpG-0.749*LL$ $-0.958*P_{\#200}+0.0627*EP$ | 64 |
| A-2-6 | $R-value=-183.827+1.590*MDD+3.797*OM+0.739*LL$ $-2.404*PI+0.185*P_{\#4}-1.368*P_{\#200}+0.0775*EP$ | 57 |
| A-2-7* | $R-value=-47.628+1.654*LL-1.899*PI$ $+0.428*P_{\#10}+0.0507*EP$ | 31 |
| A-4 | $R-value=-692.704+6.402*Abs+3.448*MDD+5.631*PL$ $+2.834*P_{\#4}+0.521*P_{\#40}-1.732*P_{\#200}+0.0839*EP$ | 71 |
| A-6* | $R-value=7.234-6.588*Abs+3.794*OM-1.917*PL$ $-2.658*PI+0.946*P_{\#4}-0.741*P_{\#40}+0.0731*EP$ | 45 |
| A-7-6* | $R-value=-318.529+107.764*SpG-4.612*LL+3.967*PL$ $+0.975*P_{\#10}-0.565*P_{\#200}+0.0423*EP$ | 42 |

*As demonstrated by the % of the prediction cases with the error within ±20% of the laboratory measured R-value, the use of these regression models for soils A-2-7, A-6, and A-7-6 requires caution.

The independent variables used in table 34 are as follows: specific gravity (SpG), absorption (Abs), maximum dry density (MDD), optimum moisture content (OM), moisture content (MC) corresponding to the Exudation pressure of interest, liquid limit (LL), plastic limit (PL), plasticity index (PI), percent passing of #4 sieve ($P_{\#4}$), percent passing of #10 sieve ($P_{\#10}$), percent passing of #40 sieve ($P_{\#40}$), percent passing of #200 sieve ($P_{\#200}$) and exudation pressure (EP). For the determination of the final R-value, the EP needs to be considered as 300 psi.

### 10.2.2. Example for R-value Estimation

For the following A-2-4 soil specimen with the basic soil properties listed in table 35, the R-value can be estimated using the equations presented in Tables 33-34.

Table 35. Soil properties for A-2-4 soil

| Soil Property | Value |
|---|---|
| Specific Gravity (SpG) | 2.49 |
| Optimum Moisture Content (OM) | 9.05% |
| Liquid Limit (LL) | 25 |
| Plastic Limit (PL) | 17 |
| Plasticity Index (PI) | 8 |
| % passing of Sieve No. 10 ($P_{\#10}$) | 54 |
| % passing of Sieve No. 200 ($P_{\#200}$) | 15 |
| Exudation Pressure (EP) for final R-value determination | 300 psi |
| Moisture Content (MC) at EP=300 psi (obtained from an existing R-value reports for this site) | 7.896% |
| MCdiff=MC-OM | -1.154 |
| MCDRPI = ($MC_{diff}$)/OM×PI | -1.020 |

In this example, the two regression models presented in tables 33 and 34 for the A-2-4 soil type can be used to provide an estimate for the R-value of this soil.

Using the regression model in table 33, R-value can be calculated as follows:

R-value = 150.64 – 28.82 SpG – 3.83 OM + 0.73 LL - 1.233 PI + 0.170 $P_{\#10}$ – 0.999 $P_{\#200}$ + 0.0422 EP –7.055 $MC_{diff}$ = 150.64 – 28.82*(2.49)- 3.83*(9.05) + 0.730*(25)- 1.233*(8) + 0.170*(54)– 0.999*(15)+ 0.0422*(300)–7.055*(-1.154)= 67.6

The above value is calculated for this A-2-4 soil specimen with the knowledge of the MC at the EP of 300 psi which was obtained from an existing R-value test for the same soil type. When such information is not available, the regression model presented in table 34 for soil type A-2-4 can be used.

Using the model presented in table 34, R-value can be calculated as follows:

R-value = 133.698 - 22.206 SpG -0.749 LL -0.958 $P_{\#200}$ +0.0627 EP = 133.698 - 22.206*(2.49) - 0.749*(25) -0.958*(15) +0.0627*(300) = 64.1

Similar procedure can be followed for other soil types of interest using the regression models presented in tables 33 and 34.

### 10.2.3. Summary of Developed Regression Models for Resilient Modulus

Table 36 provides a summary of the developed regression models for the estimation of the resilient modulus of soil types A-1-b and A-4, based on the analysis of Ground Engineering Database, and the resilient modulus of A-2-4 soil, based on the analysis of systematically conducted experiments in this project.

**Table 36. Summary of developed models for estimation of Mr value**

| Soil Type | Proposed model | % of cases with ±20% error in estimation |
|---|---|---|
| All | $\frac{M_r}{P_a} = k_1 \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a}+1\right)^{k_3}$ ; $P_a$=101.4 kPa or 14.7 psi | NA |
| A-1-b* | $k_1$ = 1123.7 - 33.69 $P_{1\text{ in}}$ + 72.01 $P_{\#10}$ - 42.29 $P_{\#200}$<br>$k_2$ = 0.1149 + 0.0833 $P_{\#100}$ - 0.0611 $P_{\#200}$ - 0.134 PI<br>$k_3$ = 6.284 + 1.109 MC - 0.605 $P_{\#50}$ + 0.876 PI | 86 |
| A-4* | $k_1$ = 13475 - 91.386 $P_{1/2\text{ in}}$ – 408.65 PI<br>$k_2$ = 2.622 - 0.00816 $P_{1/2\text{ in}}$ - 0.215 PI<br>$k_3$ = -3.974 - 0.0866 $\gamma_d$ + 0.133 $P_{3/8\text{ in}}$ - 0.401 PI | 57 |
| A-2-4 | $\ln(k_1) = 28.678 + 0.0726MC - 0.208P_{1in} - 0.00858P_{\#40}$<br>$+0.0494\%clay - 0.0095MC*PI + 0.0034PI*P_{\#200}$<br>$-0.0012OM*MDD + 0.911P_{40}^{200}$<br>$k_2 = 0.00968P_{\#16} - 0.00036\%clay*P_{\#16}$<br>$k_3 = -0.016P_{\#4} - 0.022P_{\#100} - 0.018\%silt$ | 66 |

*The use of the regression model for soils A-1-b and A-4 requires caution as two important properties of clay and silt content were not available in the established database and also the database did not include systematically conducted experiments for these soil types.

In table 36, the following independent variables were used: percent silt ($\%Silt$, particles 0.074-0.002 mm) and percent clay content ($\%Clay$, particles smaller than 0.002 mm), maximum dry density (MDD) (pcf), dry unit weight ($\gamma_d$) (pcf), optimum moisture content (OM), in-situ moisture content (MC), liquid limit (LL), plastic limit (PL), plasticity index (PI), percent passing of 1 inch sieve ($P_{1\ in}$), percent passing of 1/2 inch sieve ($P_{1/2\ in}$), percent passing of 3/8 inch sieve ($P_{3/8\ in}$), percent passing of #4 sieve ($P_{\#4}$), percent passing of #40 sieve ($P_{\#40}$), percent passing of #50 sieve ($P_{\#50}$), percent passing of #100 sieve ($P_{\#100}$), percent passing of #200 sieve ($P_{\#200}$), moisture content multiplied by the plasticity index ($MC.PI$), the plasticity index multiplied by the percent passing sieve #200 ($PI.P_{\#200}$), the optimum moisture multiplied by the maximum dry density ($OM.MDD$), and the ratio of the percent passing sieves #200 and #40 ($P_{40}^{200} = P_{\#200}/P_{\#40}$).

### 10.2.4. Examples for Mr Determination

Using the resilient modulus model, the resilient modulus value will be a function of the bulk stress and octahedral stress. Assuming that the confining stress, $\sigma_c$, of 6 psi and deviator stress, $\sigma_d$ of 2 psi are applied to the soil, the stress components will be as follows:

$$\frac{M_r}{P_a} = k_1 \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{k_3}$$ ; $P_a$=101.4 kPa=14.7 psi

Bulk Stress ($\theta$)= Sum of principal stresses= $3\sigma_c + \sigma_d = 3(6) + 2 = 20\,psi$

Octahedral Stress ($\tau_{oct}$) = $= \sqrt{2}/3(\sigma_d) = (1.414/3)(2) = 0.943\,psi$

The following examples are provided for the soil types with developed regressions in table 36.

**(a) A-1-b soil type**

Table 37 lists the values of the independent variables needed for the estimation of the $k_{1-3}$ constants for the A-1-soil.

**Table 37. Soil properties for A-1-b soil**

| **Soil Property** | **Value** |
|---|---|
| Moisture Content (MC) | 9.6% |
| Plastic Limit (PL) | 17 |
| Liquid Limit (LL) | 21 |
| Plasticity Index (PI) | 4 |
| % passing of Sieve 1 inch ($P_{1in}$) | 98 |
| % passing of Sieve No. 10 ($P_{\#10}$) | 55 |
| % passing of Sieve No. 50 ($P_{\#50}$) | 36 |
| % passing of Sieve No. 100 ($P_{\#100}$) | 25 |
| % passing of Sieve No. 200 ($P_{\#200}$) | 22.3 |

According to table 36,

$k_1$ = 1123.7 - 33.69 $P_{1\ in}$ + 72.01 $P_{\#10}$ - 42.29 $P_{\#200}$=1123.7 - 33.69*(98) + 72.01*(55) - 42.29*(22.3)=839.6
$k_2$ = 0.1149 + 0.0833 $P_{\#100}$ - 0.0611 $P_{\#200}$ - 0.134 PI = 0.1149 + 0.0833*(25) - 0.0611*(22.3) - 0.134*(4) =0.2988

$k_3$ = 6.284 + 1.109 MC - 0.605 $P_{\#50}$ + 0.876 PI = 6.284 + 1.109*(9.6) - 0.605*(36) + 0.876*(4) = -1.3456

$$M_r(psi) = k_1.P_a.\left(\frac{\theta}{P_a}\right)^{k_2}\left(\frac{\tau_{oct}}{P_a}+1\right)^{k_3} = 839.6(14.7)\left(\frac{20}{14.7}\right)^{0.2988}\left(\frac{0.943}{14.7}+1\right)^{-1.3456} = 12445\,psi$$

### (b) A-4 soil type

Table 38 lists the values of the independent variables needed for the estimation of the $k_{1-3}$ constants for the A-4 soil for the defined stress conditions.

**Table 38. Soil properties for an A-4 soil**

| **Soil Property** | **Value** |
|---|---|
| Moisture Content (MC) | 13.2% |
| Dry unit weight ( $\gamma_d$ ) | 121.2 pcf |
| Plastic Limit (PL) | 21 |
| Liquid Limit (LL) | 27 |
| Plasticity Index (PI) | 6 |
| % passing of Sieve 1/2 inch ($P_{1/2in}$) | 92 |
| % passing of Sieve 3/8 inch ($P_{3/8in}$) | 85 |

According to table 36, for A-4 soil type,

$k_1$ = 13475 - 91.386 $P_{1/2\text{ in}}$ – 408.65 PI =13475 - 91.386*(92) – 408.65*(6)=2615.6
$k_2$ = 2.622 - 0.00816 $P_{1/2\text{ in}}$ - 0.215 PI =2.622 - 0.00816*(92) - 0.215*(6) =0.5812

$k_3$ = -3.974 - 0.0866 $\gamma_d$ + 0.133 $P_{3/8\text{ in}}$ - 0.401 PI = -3.974 - 0.0866*(121.2)+ 0.133*(85)-0.401*(6)=-5.571

$$M_r(psi) = k_1.P_a.\left(\frac{\theta}{P_a}\right)^{k_2}\left(\frac{\tau_{oct}}{P_a}+1\right)^{k_3} = (2615.6)(14.7)\left(\frac{20}{14.7}\right)^{0.5812}\left(\frac{0.943}{14.7}+1\right)^{-5.571} = 32521 psi$$

### (b) A-2-4 soil type

Table 39 lists the values of the independent variables needed for the estimation of the $k_{1\text{-}3}$ constants for the A-2-4 soil for the defined stress conditions.

**Table 39. Soil properties for an A-2-4 soil**

| **Soil Property** | **Value** |
|---|---|
| Maximum dry density (MDD) | 133.9 pcf |
| Optimum moisture content (OM) | 7.2 |
| Moisture content (MC) | 7.1 |
| Liquid limit (LL) | 32 |
| Plastic limit (PL) | 22 |
| Plasticity index (PI) | 10 |
| Percent passing of 1 inch sieve ($P_{1\text{ in}}$) | 100 |
| Percent passing of #4 sieve ($P_{\#4}$) | 88 |
| Percent passing of #16 sieve ($P_{\#16}$) | 67 |
| Percent passing of #40 sieve ($P_{\#40}$) | 50 |
| Percent passing of #100 sieve ($P_{\#100}$) | 37 |
| Percent passing of #200 sieve ($P_{\#200}$) | 29.7 |
| $P_{40}^{200} = P_{\#200} / P_{\#40}$ | 0.594 |
| % clay | 16 |
| % silt | 14 |

According to table 36, for A-2-4 soil type,

$$\begin{aligned}&\ln(k_1) = 28.678 + 0.0726MC - 0.208P_{1in} - 0.00858P_{\#40} + 0.0494\%clay - 0.0095MC*PI + 0.0034PI*P_{\#200}\\ &-0.0012OM*MDD + 0.911P_{40}^{200} = 28.678 + 0.0726(7.1) - 0.208(100) - 0.00858(50) + 0.0494(16) -\\ &0.0095(7.1)(10) + 0.0034(10)(29.7) - 0.0012(7.2)(133.9) + 0.911(0.594) = 8.4744\end{aligned}$$

$$k_1 = e^{8.4744} = 4790.5$$

$$k_2 = 0.00968P_{\#16} - 0.00036\%clay*P_{\#16} = 0.00968(67) - 0.00036(16)(67) = 0.2626$$

$$k_3 = -0.016P_{\#4} - 0.022P_{\#100} - 0.018\%silt = -0.016(88) - 0.022(37) - 0.018(14) = -2.474$$

$$M_r(psi) = k_1.P_a.\left(\frac{\theta}{P_a}\right)^{k_2}\left(\frac{\tau_{oct}}{P_a}+1\right)^{k_3} = (4790.5)(14.7)\left(\frac{20}{14.7}\right)^{0.2626}\left(\frac{0.943}{14.7}+1\right)^{-2.474} = 65464\,psi$$

## 10.3 Additional recommendations

In this study, in addition to the development of models for prediction of resilient modulus and R-value from basic soil properties, direct correlations between the resilient modulus and R-value were explored for A-2-4 soil type and four models were developed, as presented in section 9. Most of the models require information about soil basic properties beyond the R-value and considering the higher accuracy of resilient modulus models summarized in Table 33 for the A-2-4 soil type, we suggest using equations 82 through 84 for determination of the resilient modulus based on stress conditions for A-2-4 soils.

This study focused on systematic testing of A-2-4 soil type and for additional AASHTO soil types of interest, we recommend testing at least 30 samples using the same testing program to obtain the same basic soil properties for all studied soil specimens. We suggest performing the hydrometer testing for all applicable soil of interest since the percent silt and clay content in soil were identified as important independent variables from our regression analysis and the literature.

Please note that the existing CDOT database for R-value lacked the exudation pressure and moisture content information for soil specimens that were tested for R-values, and the reported R-value was the final value corresponding to 300 psi exudation pressure. The R-value is strongly affected by the change in the moisture content (exudation pressure), especially for cohesive soils. An increase in the moisture content generally reduces the R-value for the cohesive soils. Since the exudation pressure has such a large effect on the R-value, we suggest recording the exudation pressures and the associated moisture contents for each of the R-value tests in the database.

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