DENVER BASIN GROUND WATER MODEL REPORT

Colorado Division of Water Resources

October 24, 1985

Technical Appendix C

CONTENTS

TABLES

ILLUSTRATIONS

DENVER BASIN GROUND WATER MODEL REPORT

Subsequent to the passage of Senate Bill 5 the State Engineer's office undertook the task of defining, within each of the Denver Basin aquifers, areas of nontributary ground water. Senate Bill 5 provides that a determination be made by the State Engineer if the ratio of rate of stream depletion (q) to the pumping rate (Q) of a proposed well is less than 0.1%. A determination of q/Q less than 0.1%, when the q used is that after 100 years of pumping, defines a nontributary well.

USGS 3-DIMENSIONAL GROUND WATER MODEL

The State Engineer's office chose the "Moduler Three-Dimensional Finite-Difference Ground Water Flow Model" (1) to compute the stream depletion (q) for various well locations to determine areas of nontributary ground water.

(General Equation)

Assuming constant temperature, viscosity and density the general equation for 3 dimensional ground water flow is:

$$\frac{\partial}{\partial x}\left[T(x,y,t)\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T(x,y,t)\frac{\partial h}{\partial y}\right] = S(x,y)\frac{\partial h}{\partial t} + W(x,y,t), \quad \text{Eq (1)}$$

where

x, y are space coordinates

T (x,y,t) is the transmissivity (L^2T^{-1})
h is the hydraulic head (L)
W (x,y,t) is the volumetric flux per unit volume (LT^{-1}) and can be used to represent pumping, flows to or from streams and vertical leakage between aquifers
S (x,y) is the specific yield of the porous material (dimensionless)
t is time

Equation (1) is the general ground water flow equation, which constitutes a mathematical model of ground water flow when boundary conditions and initial hydraulic heads are specified.

Equation Solution

An analytic solution to equation (1) is unobtainable in all but the very simplest of hydrologic systems. Therefore, approximation methods for solving systems of simultaneous linear algebraic difference equations are used. The accepted method of S.I.P. (Strongly Implicit Procedure) of Weinstein, Stone and Kwan (2) was utilized to solve for head distribution. Head distribution is then used to compute stream depletion (q) after 100 years of pumping.

The finite difference model utilizes rectilinear parallel piped cells whose dimensions are specified by the user. Finite difference equations are formulated at the beginning of each iteration. The system of simultaneous equations are then approximately solved for the head distribution and compared to the head distribution at the previous iteration. If the comparison results in a head difference greater than a user specified "head closure criteria", the finite difference equations are again formulated and solved. Iterations proceed until closure is met. The Denver Basin Models used head closure criteria of .001 or 0.0001 feet. The method of approximate solution for ground water models is generally accepted because even if an exact solution to the system of simultaneous ground water flow equation could be obtained, it would still only be an approximation of true field conditions.

Options used in the Ground Water Model

The USGS Finite Difference Ground Water Model has different options for simulating hydrologic conditions and specifying output. The reader is referred to "Input Data For and Results From the Ground Water Models" which specifically details input data and options selected for each aquifer modeled.

The optional packages selected for this study were the Well Package, River Package, and General Head Boundary Package. The River Package is used to simulate induced ground water flow from the stream systems to the aquifer as a result of pumping. The well package is used to specify pumping. The General Head Boundary Package simulates flow between aquifer layers in the vertical direction.

Output

The output from the USGS Ground Water Model includes the stream depletion rate (q). The value of (q) during the 100th year of pumping, as provided by the final time step budget of the model was used as the numerator of the ratio of q/Q.

DATA INPUT & VERIFICATION

Data files representing aquifer parameters were developed for the Laramie-Fox Hills, the Lower Arapahoe, the Upper Arapahoe, the Denver, the Lower Dawson, and the Upper Dawson Aquifers.

A rectilinear grid system with 120 rows and 84 columns was selected for input of aquifer parameters. Each grid in the system represents a square mile and the assumption was made that sections of land correspond to the grids in the system. The grid system is shown on Figure 6.

Values for the elevations of the aquifer bases, the elevations of aquifer tops and thicknesses of water yielding materials were read for the center of each section from Figures 1A, 1B, 1C, 1D, 1E, 2A, 2B, 2C, 3A, 3B, 3D, 3E, 3F, 4A, 4B, and 4C and entered directly into the computer from the keyboard. Similarly, values for hydraulic conductivity of water yielding materials were coded from maps adopted from Hydrologic Atlas HA-659. The values coded for conductivities are shown in Figures 7, 8, 9 and 10. In and near the aquifer outcrops, the elevations of the potentiometric surfaces were also coded. Hydrologic Atlases HA-647 and HA-646 were used for the potentiometric surfaces

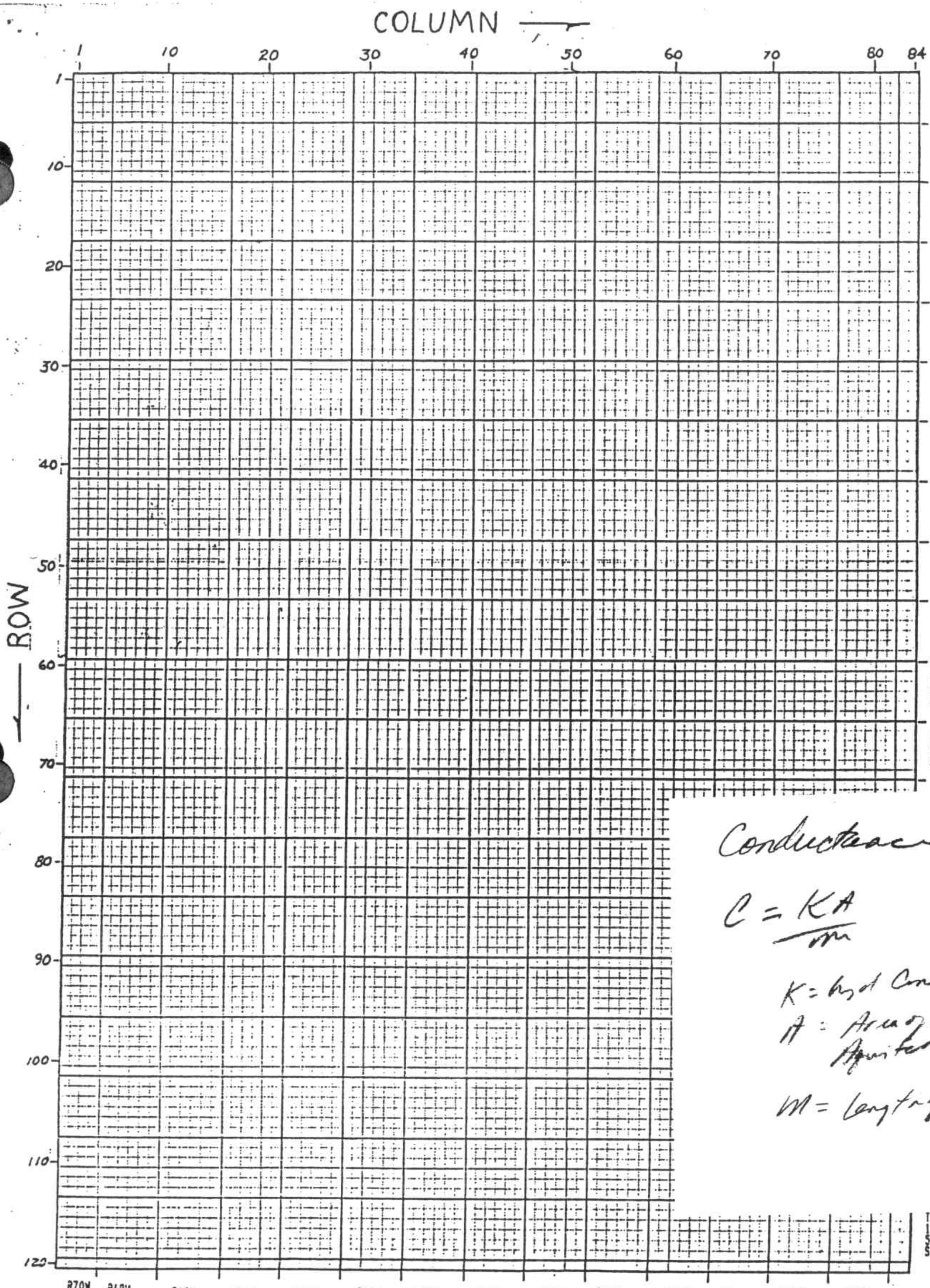

Figure 6. -- Map showing grid system used for data input.

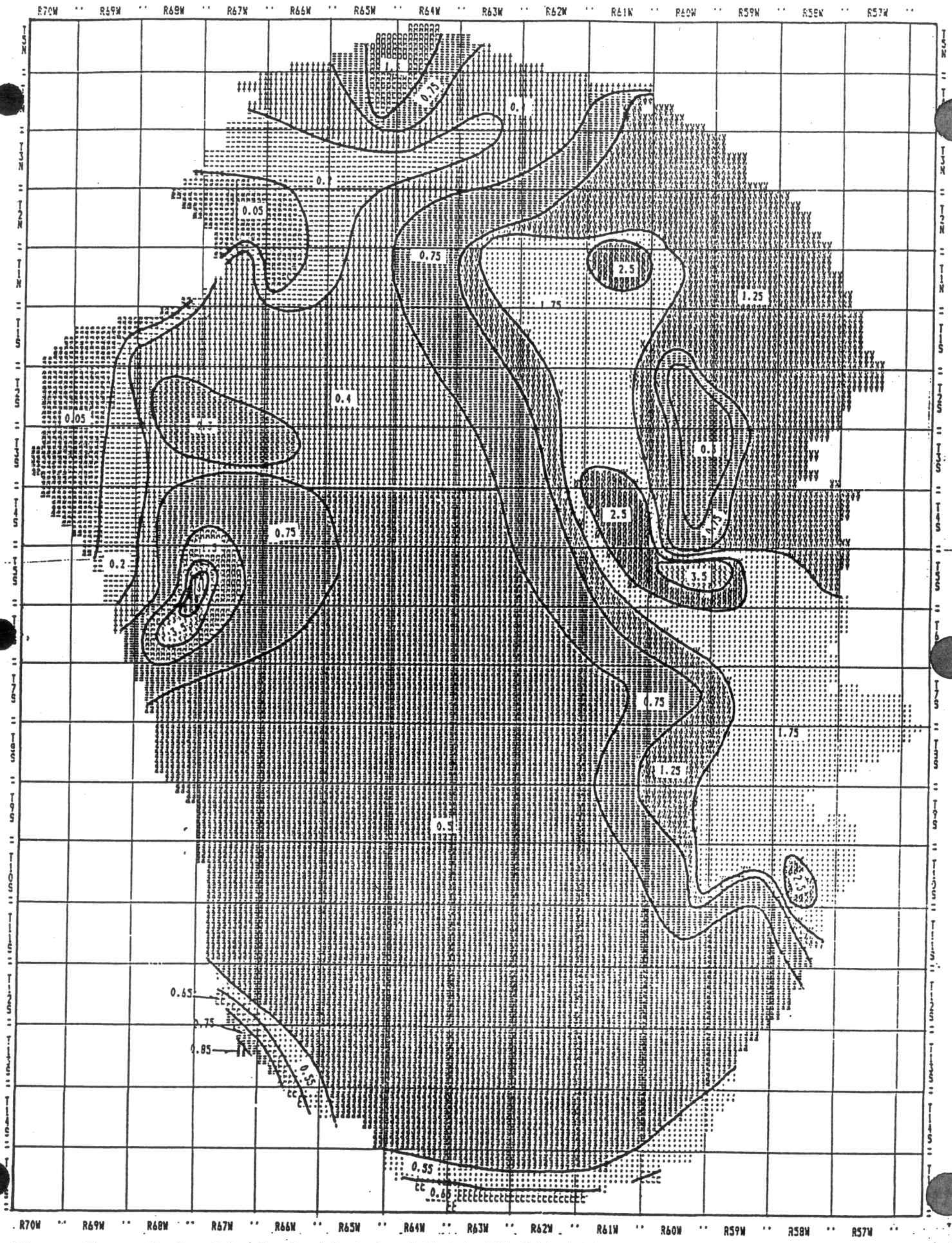

Figure 7. -- Hydraulic Conductivity of Water Yielding Materials for Laramie-Fox Hills aquifer (ft./day)

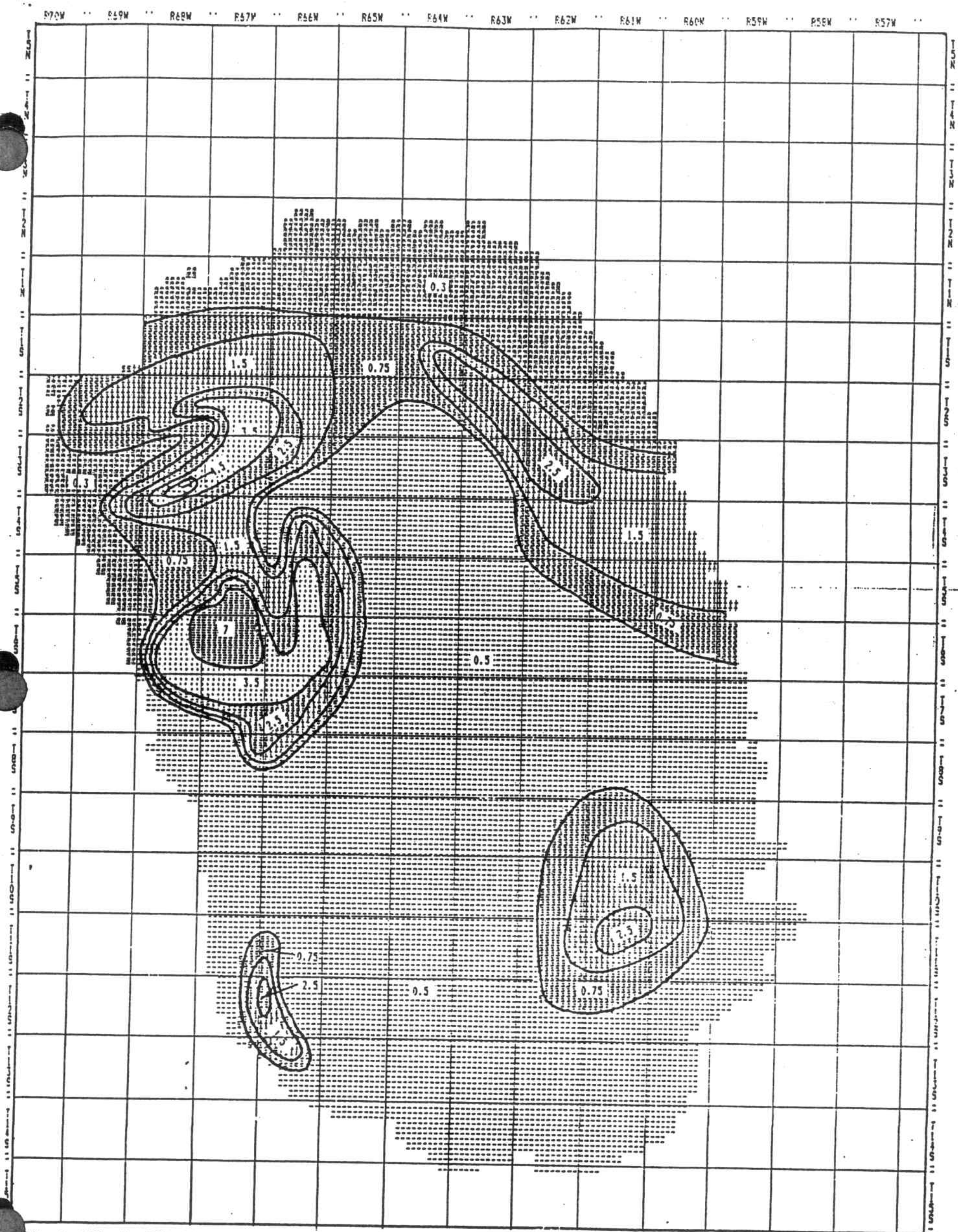

Figure 8. -- Hydraulic Conductivity of Water Yielding Materials for Arapahoe aquifer (ft./day)

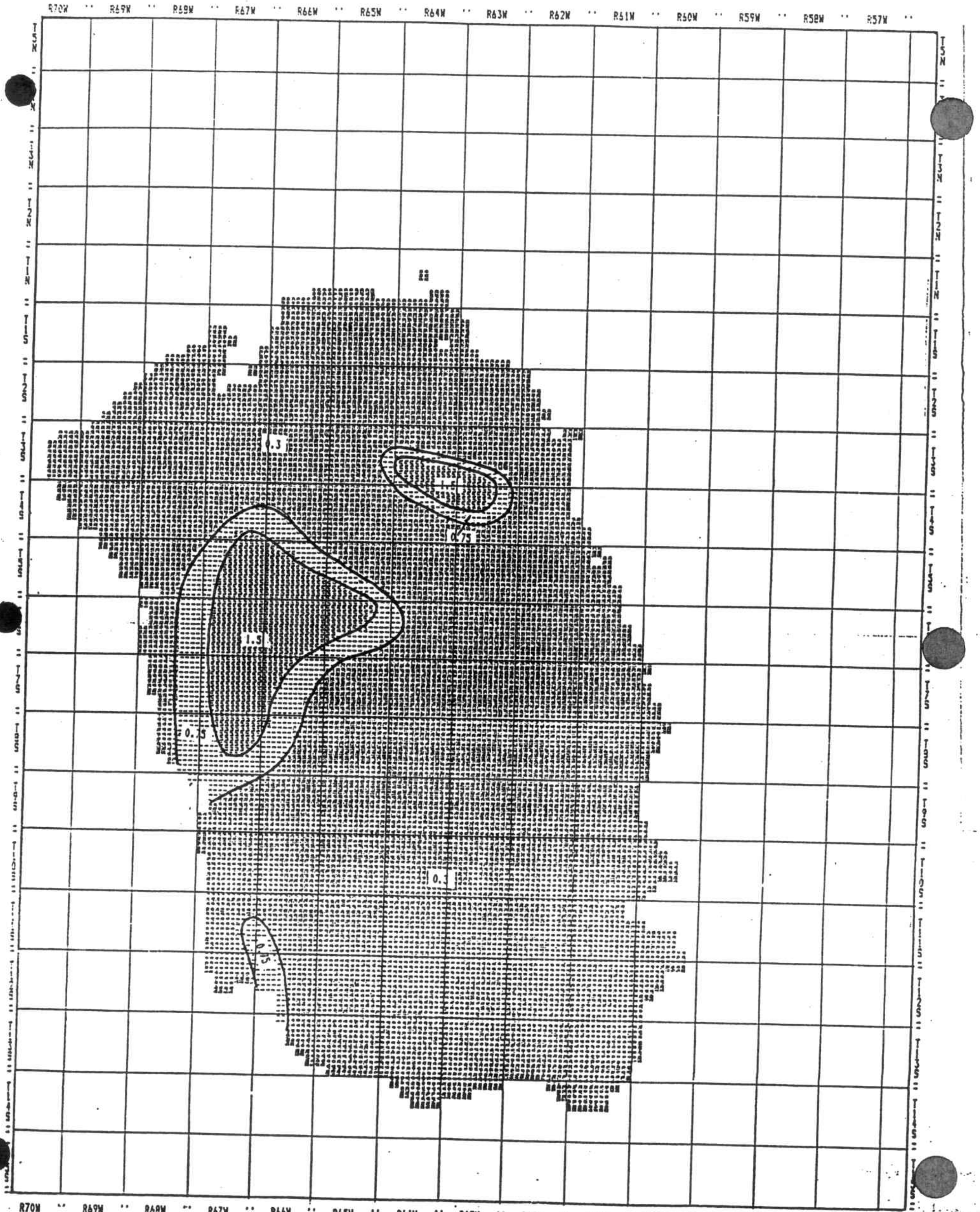

Figure 9. -- Hydraulic Conductivity of Water Yielding Materials for Denver aquifer (ft./day)

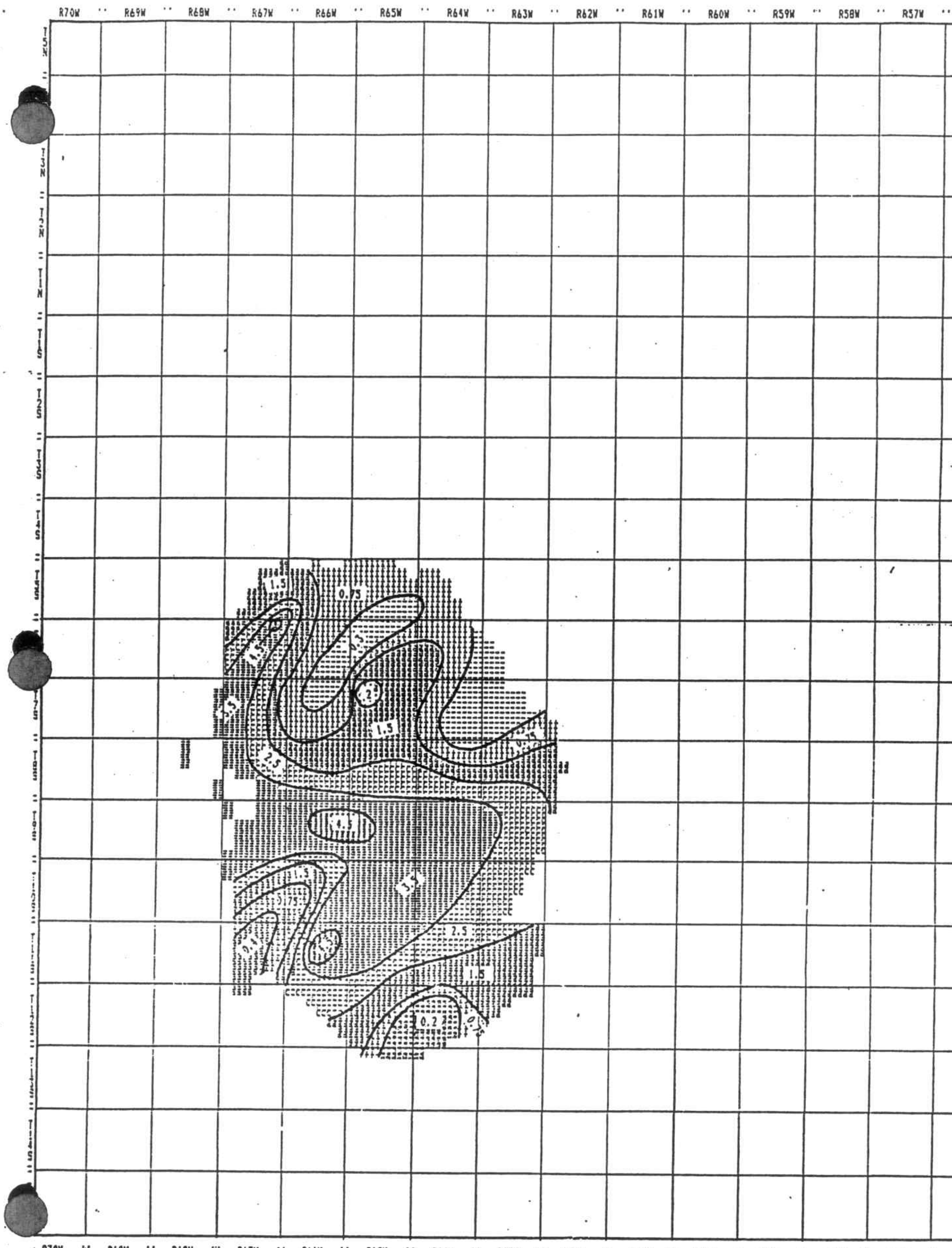

Figure 10. -- Hydraulic Conductivity of Water Yielding Materials for Dawson aquifer (ft./day)

of the Arapahoe and Denver aquifers respectively. The potentiometric surface of the Laramie-Fox Hills aquifer was taken from figure 38. Figure 11 was used for the potentiometric surface of the Dawson aquifers. Figure 11 was adopted from Hydrologic Atlas HA-643 and reflects refinements based on water levels measured by the Division of Water Resources personnel.

A set of values for a particular aquifer parameter can be referred to as a two-dimensional array. Each valve in a two-dimensional array has row and column index numbers which relate the value to the appropriate grid location in the grid system shown on Figure 6. Any section for which a value for an aquifer parameter was not coded or for which the aquifer was absent is represented in the array with a zero (0).

A computer program named "D2DF" was developed to facilitate data entry and to check the accuracies of data entry. The program required the user to specify a maximum and a minimum value to be accepted into an array. Any value entered from the keyboard less than the minimum value or greater than the maximum value was rejected. For example, if values for an aquifer surface should fall between 4000' and 6000' and the computer operator incorrectly entered 5000' as 500' or 50,000', the incorrect value would be rejected and a message to that effect would appear on the computer monitor. After arrays were coded, "D2DF" was used to generate contour maps of the arrays. Figure 12 represents an example of such a map. The lines of equal thickness were added to Figure 12 for clarity. The contour maps were checked against the parent maps for accuracy and any erroneous values in the arrays were corrected.

Using the computer, elevations of the base of each aquifer were compared to the elevations of the top of the aquifer to insure that the base of the aquifer was indeed lower than the top. The thicknesses of the water yielding materials were compared to the total aquifer thicknesses to insure that the thicknesses of the water yielding materials did not exceed total aquifer thicknesses. The last mentioned test caused the thickness of water yielding materials array for the Upper Dawson to be discarded as some of the sand intervals originally mapped were above the water table. Some discrepancies in the remaining aquifers were found requiring adjustments to values in the arrays and in some instances corrections to the parent map.

The average aquifer thickness, the average thickness of water yielding materials and the average thickness of water yielding materials to total aquifer thickness ratio were computed for each aquifer except the Upper Dawson. The results are shown in Table 1. An average thickness of water yielding materials to total aquifer thickness ratio of 0.5 was calculated for the Upper Dawson from a study of geophysical logs.

Table 1. Aquifer Statistics. -- Average aquifer thicknesses (m), average thicknesses of water yielding materials (n), and average thickness of water yielding materials to aquifer thickness ratios (n/m).

Aquifer	m (feet)	Area Averaged (Sq. Mi.)	n (feet)	Area Averaged (Sq. Mi.)	n/m	Area Averaged (Sq. Mi.)
Laramie-Fox Hills	286	5405	175	5560	.6478	5317
Lower Arapahoe	222	1194	91	1062	.4149	1060
Upper Arapahoe	408	3522	172	4053	.4738	3461
Denver	883	1373	207	3127	.3427	1371
Lower Dawson	235	696	97	654	.4426	654

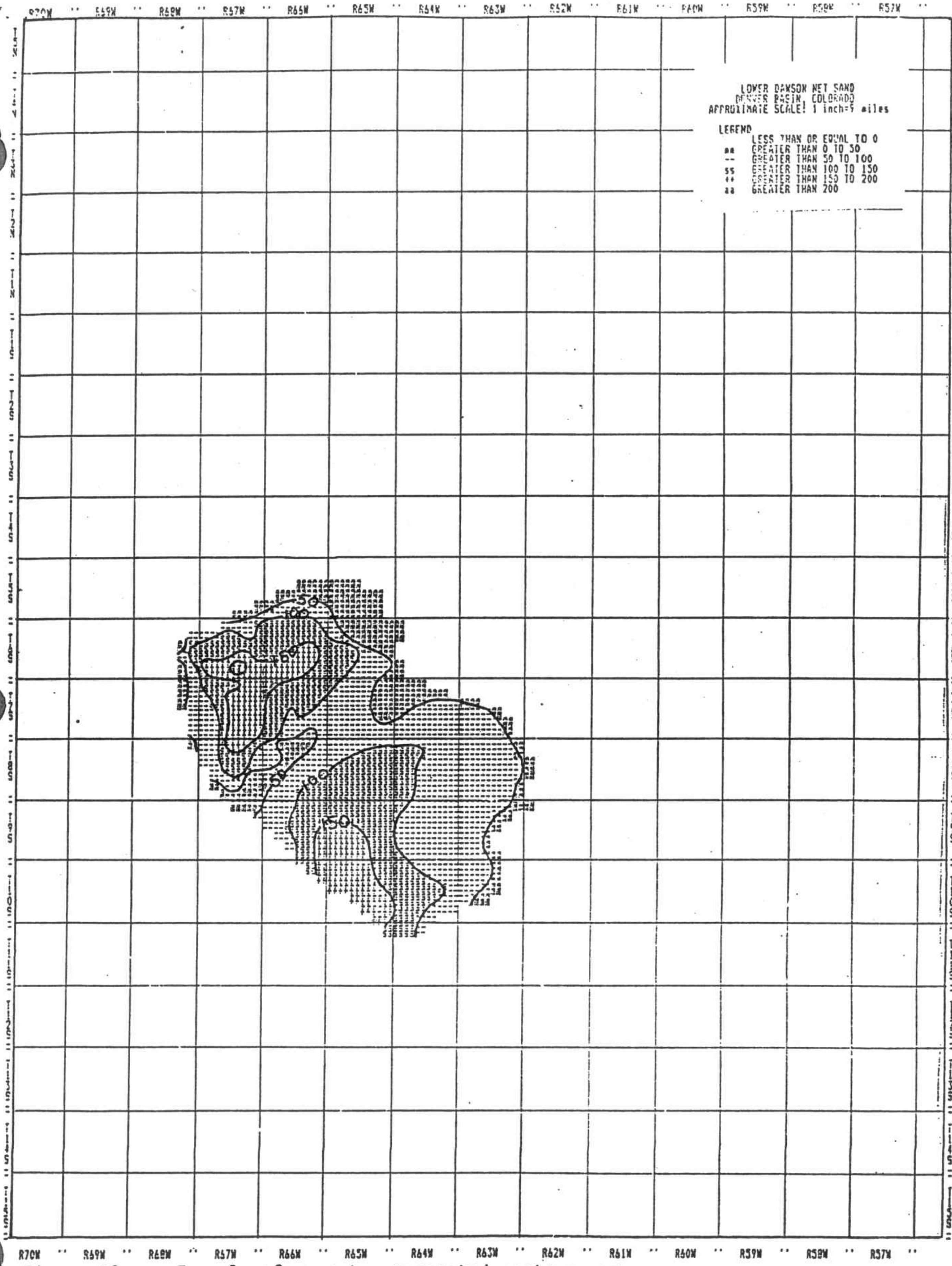

Figure 12. -- Example of computer generated contour map.

Before the data could be used in the models, additional adjustments were required as follows:

1. Where aquifers were confined the potentiometric surfaces were changed to correspond to the elevations of the top of the aquifers.

2. Where the potentiometric surface was below the base of the aquifer, all aquifer parameters for the cell were set at zero (0), to indicate that the cell was dry.

3. For the Laramie-Fox Hills, where the aquifer was confined and the top of the aquifer had not been coded, the top was set 286 feet (average thickness from Table 1) above the base of the aquifer. In Townships 1 South through 4 South, Ranges 70 West and 69 West, the thickness of water yielding materials was set at 200 feet and the bottom was set 309 feet below the top.

4. Where the thickness of water yielding materials had not been coded, the aquifer thickness was computed and multiplied by the appropriate ratio n/m from Table 1 to arrive at the value for the thickness of water yielding materials.

The above resulted in four finished arrays for each aquifer: the hydraulic conductivity of water yielding materials; the elevation of the base of the aquifer; the water table elevation and the thickness of water yielding materials. These finished arrays are referred to as master arrays.

THE HYDROLOGIC MODELS

Eighteen separate digital models were constructed to allow computation of the depletion to stream systems as a result of well pumping in the six aquifers. Aquifer data required for model input was derived from the master arrays. The bedrock aquifers were modeled within designated ground water basins as well as outside of designated ground water basins.

The areas included in the models are shown on Figures 13-18. Each area is referred to as a window. Where window boundaries disect the aquifers, no-flow boundaries were inserted in the models. To insure that computed stream depletion would not be affected by those no-flow boundaries, windows were made to overlap and window dimensions were generally designed with the no-flow boundaries away from the simulated pumping.

Each window was divided into a number of cells using a rectangular grid system. "Input Data For and Results From the Ground Water Models" shows the grid system, model input and results for each of the windows. Most cells are one mile on a side. Nodes at the center of each cell define data point locations as well as locations for computed water table changes. The differential equation describing two-dimensional flow is approximated with a finite difference equation and solved with the computer at each node to predict changes in water level and the resulting depletion to stream systems.

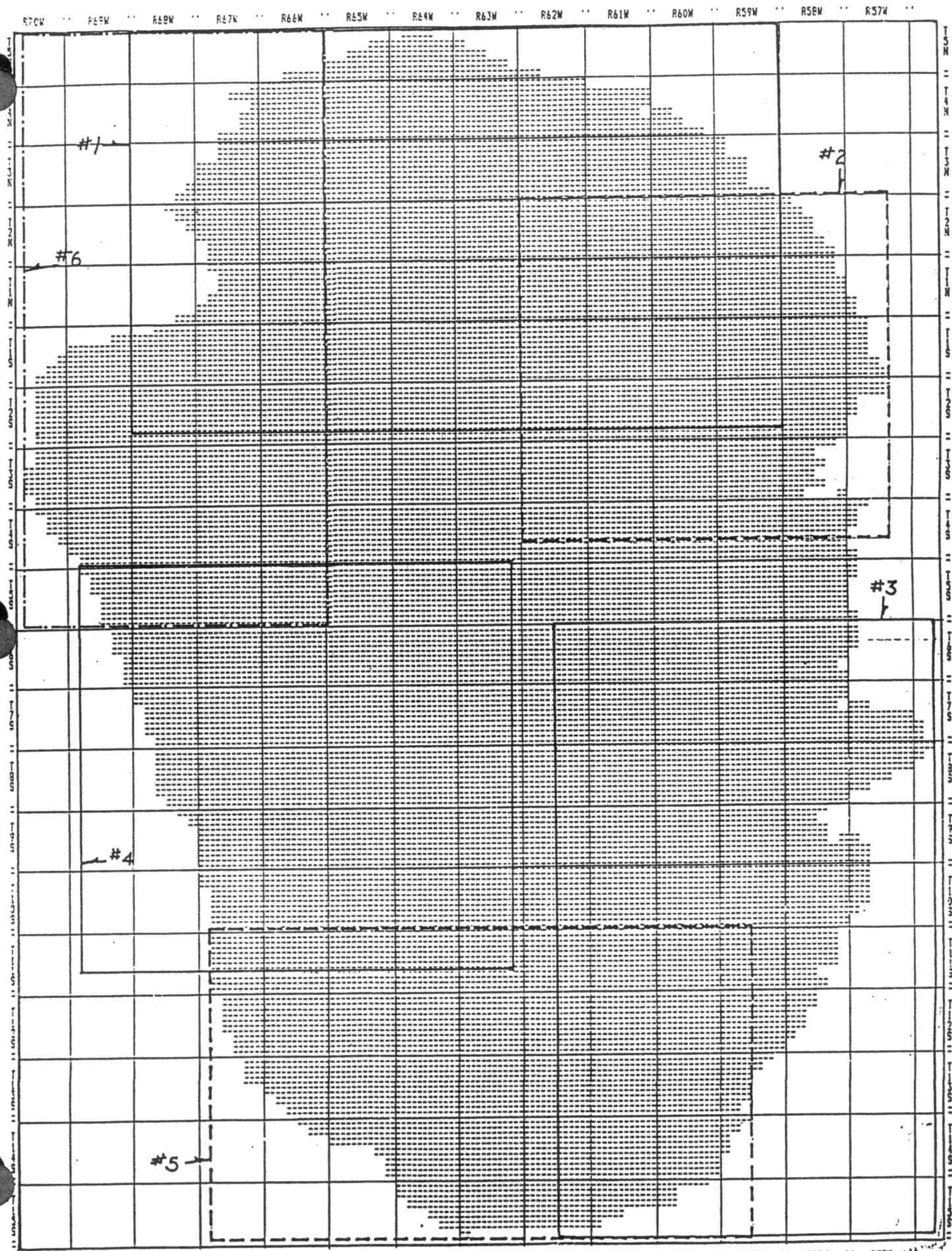

Figure 13. -- Window Location Map for the Laramie-Fox Hills Aquifer.

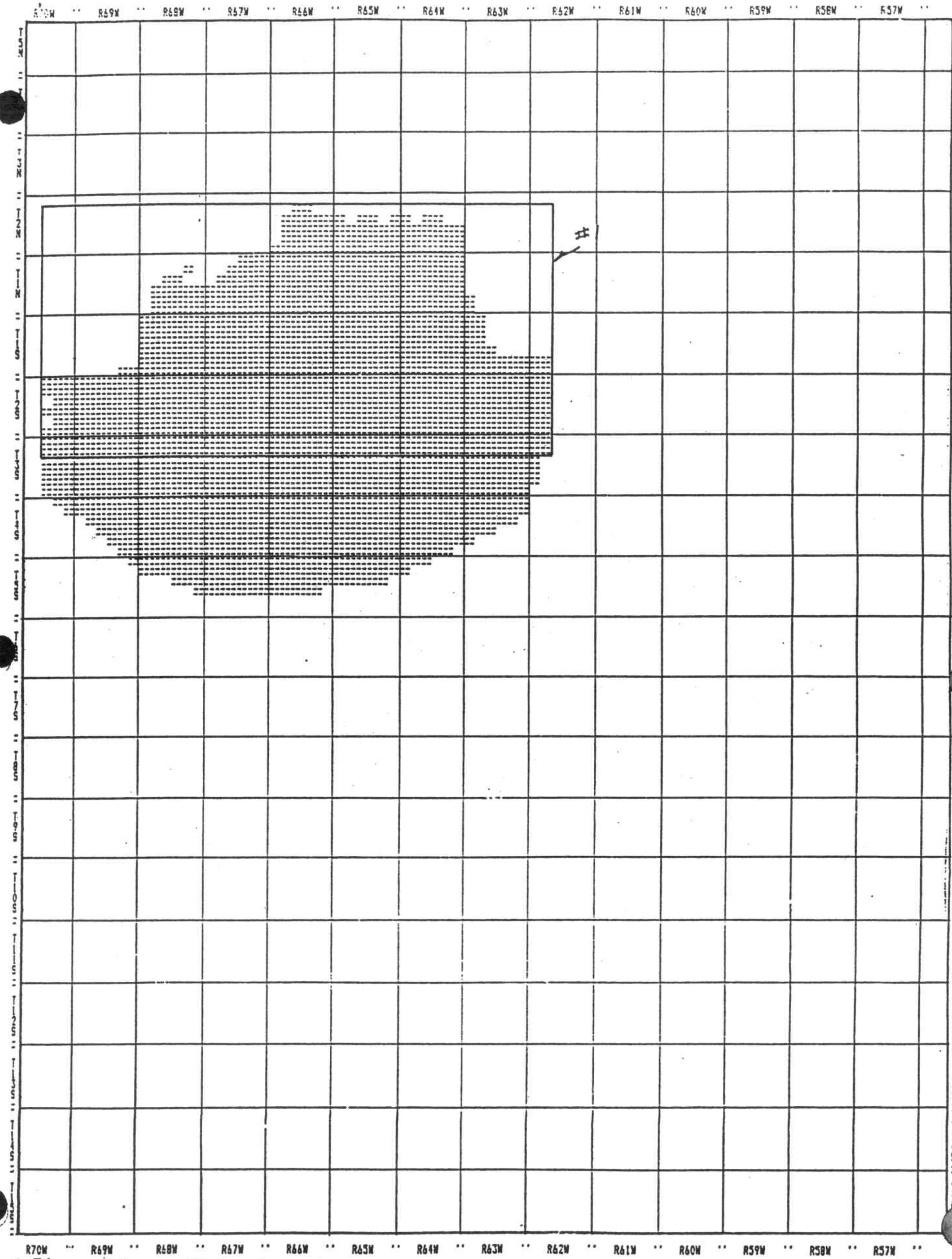

Figure 14. -- Window Location Map for the Lower Arapahoe Aquifer.

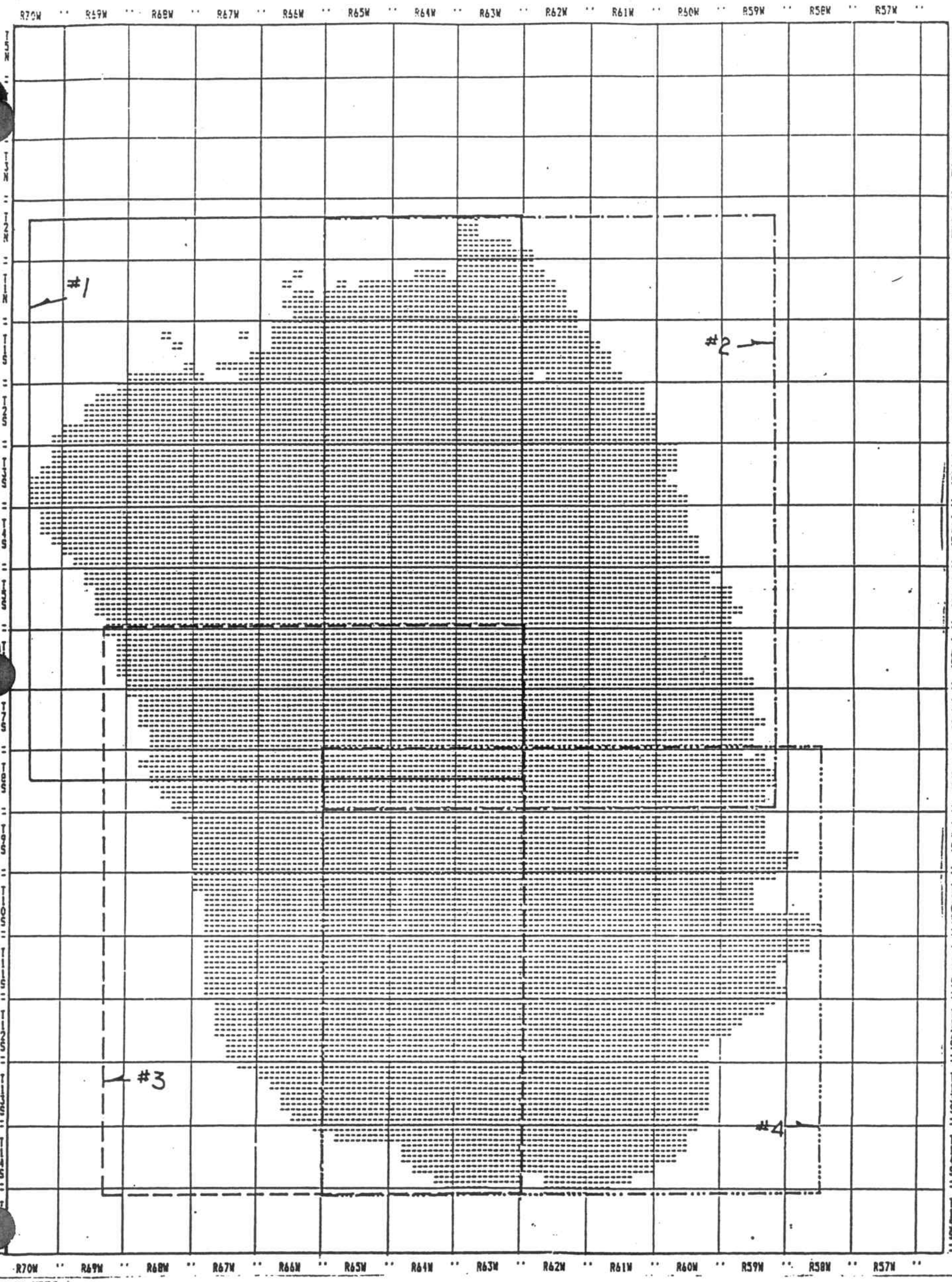

Figure 15. -- Window Location Map for the Upper Arapahoe Aquifer.

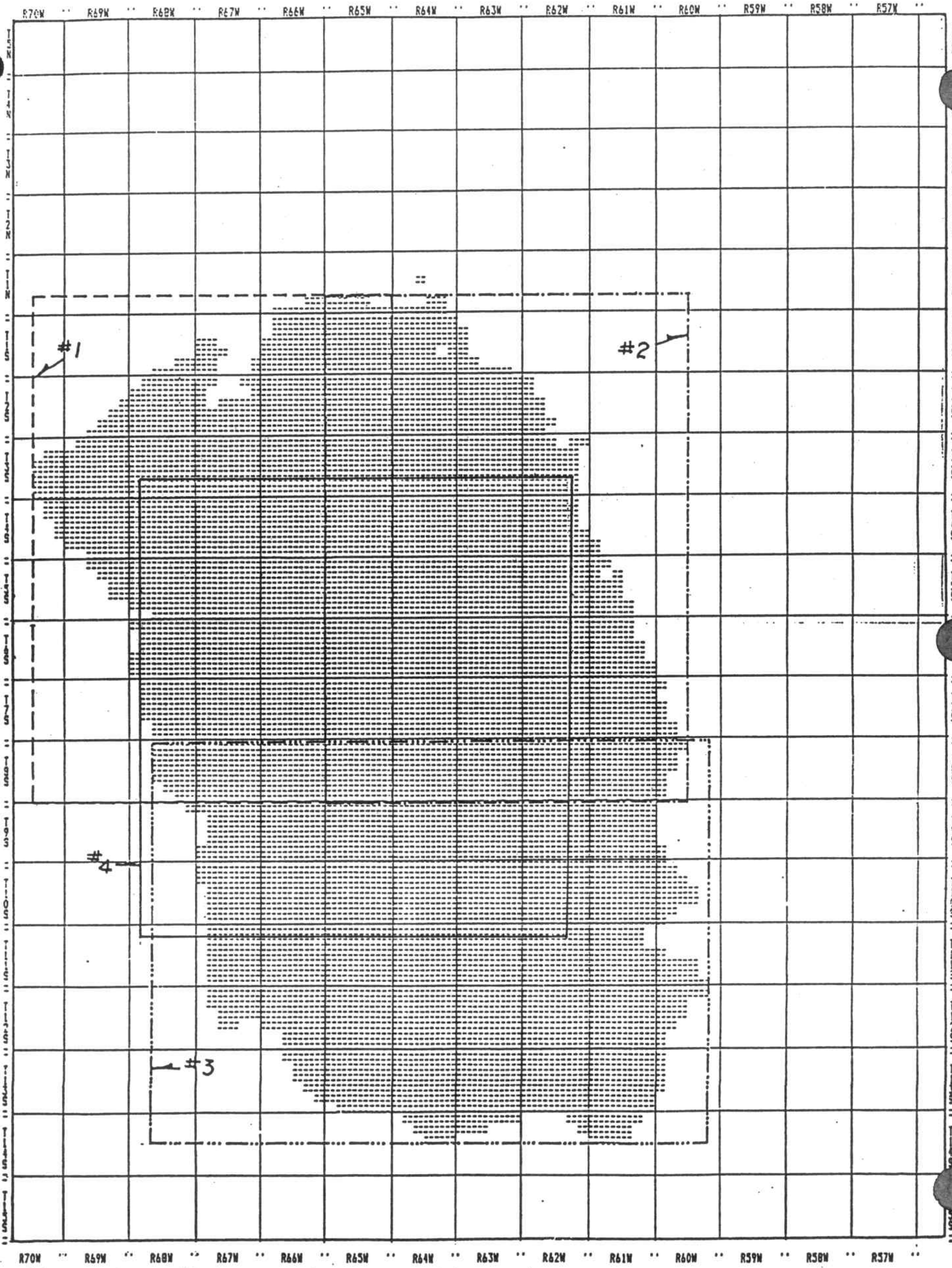

Figure 16. -- Window Location Map for the Denver Aquifer.

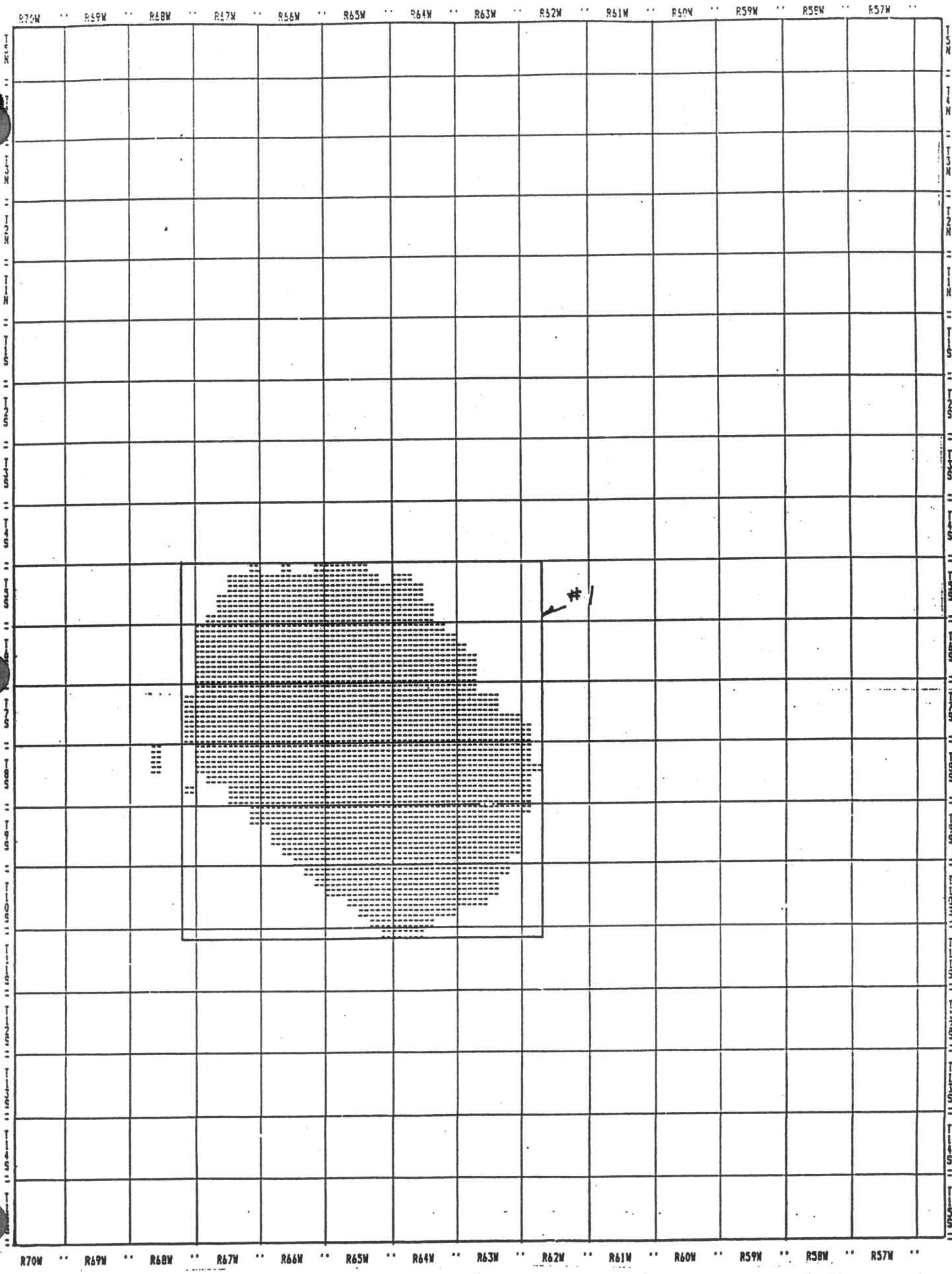

Figure 17. -- Window Location Map for the Lower Dawson Aquifer.

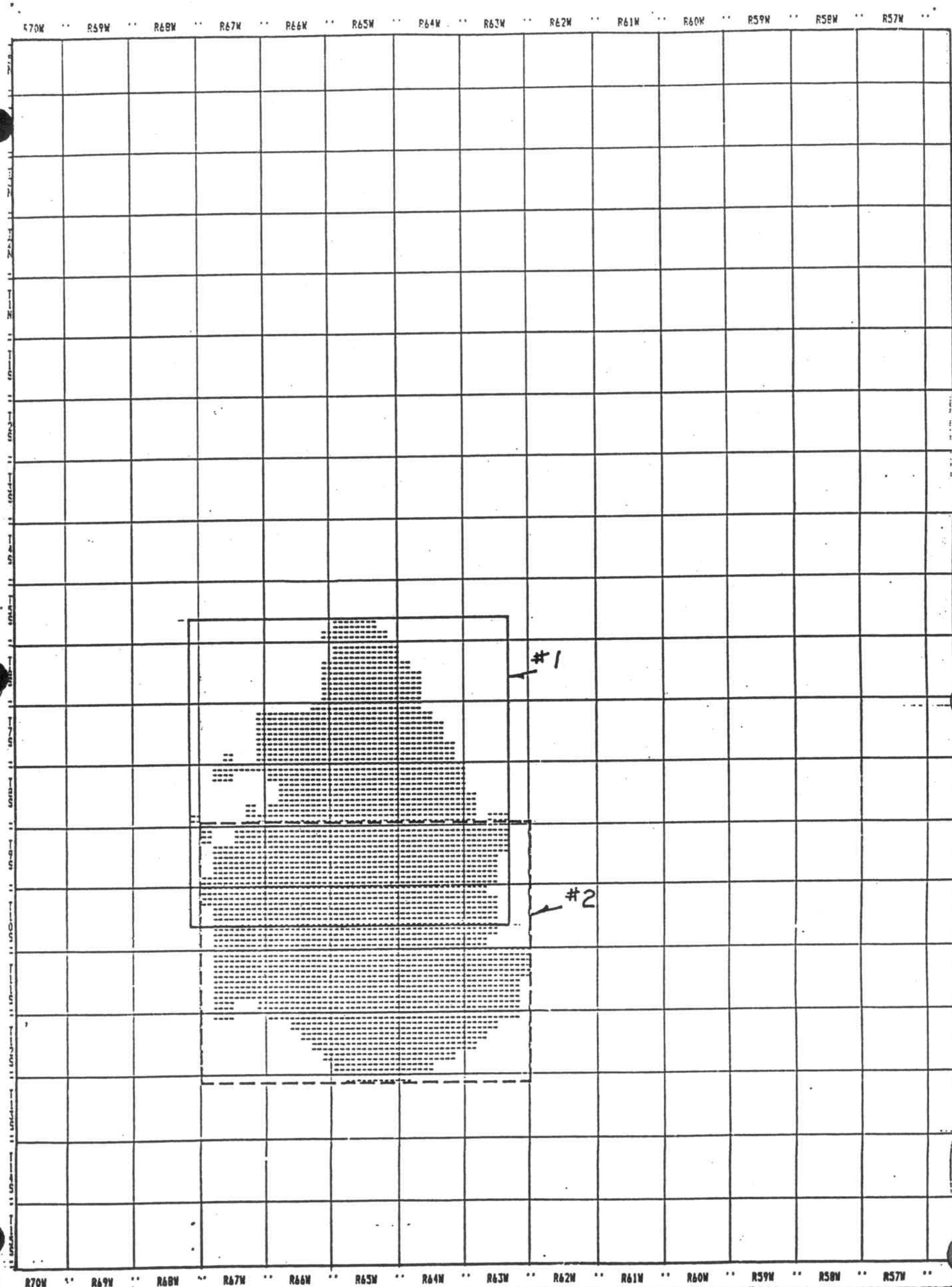

Figure 18. -- Window Location Map for the Upper Dawson Aquifer.

Data input for aquifer parameters

For each cell in the models, the elevation of the starting water level (head) and the elevation of the bottom of the aquifer must be specified. Cells therefore have vertical dimensions equivalent to the aquifer thicknesses. The heads for the start of a simulation were initialized at a uniform value. Using the appropriate data from the master arrays, the aquifer thickness of each cell was computed and the bottom was set at a distance below the starting head equivalent to the aquifer thickness.

The specific yields of the water yielding materials given in rule 7 were used to compute the average specific yields. At each model cell the specific yield for the water yielding materials was first multiplied by the thickness of the water yielding materials and then divided by the aquifer thickness. Similarly at each model cell, the hydraulic conductivity of the water yielding material was multiplied by the thickness of the water yielding material and then divided by the aquifer thickness to arrive at average hydraulic conductivity.

If model cells have vertical dimensions equivalent to the total aquifer thickness, the hydraulic conductivities and the specific yields input to the model must represent average values for the aquifer within each cell. To do otherwise would cause drawdowns and gradients to be computed which would not represent drawdowns and gradients in the real aquifer. The volume of a model cell multiplied by the modeled specific yield must equal the drainable water in storage in that cell.

Recharge and Discharge

Ground water moves vertically between aquifers. Senate Bill 5 provides all aquifers be reduced at least to water table conditions thereby limiting movement of ground water between aquifers to a downward direction.

When an aquifer is pumped the water level is lowered below the top of the aquifer and a fully saturated connection with the overlying aquifer would no longer exist. Movement of water from an overlying aquifer into an aquifer would be independent of water levels in the lower aquifer. Leakage from an overlying aquifer was not simulated in any of the models.

Pumping an aquifer would change the flow from that aquifer into an underlying aquifer in proportion to the change in water level. Changes in leakage from an aquifer into an underlying aquifer were simulated for all aquifers except the Laramie-Fox Hills and the Lower Arapahoe. The relatively impermeable clays and shales of the Pierre and Laramie formations effectively prevent the vertical movement of water. Changes in flow from an aquifer into an underlying aquifer were simulated using the General-Head Boundary Package described in McDonald Harbaugh (1) which utilizes the equation

$$Q = K (H_b - H_a) A/L$$

where

Q is the flow rate

K is the effective vertical hydraulic conductivity between aquifers

L is the length of the flow path

H_b is the head in the underlying aquifer

H_a is the head in the aquifer

A is the horizontal area of the cell

In the models, 3×10^{-5} ft./day was used for the value of K (Robson, (8)). The length of the flow path was computed at each cell as one-half the sum of the aquifer thickness and the underlying aquifer thickness. Changes in stream system depletion were computed for Lower Dawson window #1 with and without leakage into the Denver aquifer. Nearly identical results were obtained.

The bedrock aquifers receive recharge from precipitation falling directly on the aquifer outcrops. Pumping from the bedrock aquifers will not appreciably change this direct precipitation recharge and was not simulated in the models.

Bedrock aquifers of the Denver Basin discharge water to streams and/or their alluviums. Stream systems also recharge the bedrock aquifers. Any computed change in that discharge or recharge as a result of pumping is stream depletion. Changes in flow between bedrock aquifers and stream systems were simulated in the models using the River Package of the McDonald Harbaugh (1). The River Package utilizes the equation

$$Q = C\ (H_r - H_a)$$

where

Q is the flow rate

C is the river conductance

H_r is the head in the stream or its alluvium

H_a is the head in the aquifer

The river conductance in turn may be expressed as $C = KA/M$

where

K is the effective hydraulic conductivity between the stream or stream alluvium and the aquifer

A is the area of the stream system-aquifer contact

M is the length of the flow path

The area of the stream system-aquifer contact was estimated from Figure 5 and was input for each aquifer cell in contact with a stream system. The ratio K/M was input as a constant 0.00001 day^{-1}. K is dependent upon the stream alluvium and more dependent upon the ability of the bedrock aquifer to conduct water. The bedrock aquifers may not readily transmit water across the stream system-aquifer contact in a vertical direction particularly if the aquifer is interbedded with clay lenses. Where streams have eroded channels into the bedrock aquifers causing permeable materials in the aquifers to be at the stream system-aquifer contacts, water may be conducted much more readily in a horizontal direction across the contacts. Robson (1984) used a maximum recharge rate of 0.133 cfs/sq. mile of alluvial-bedrock interface in his bedrock aquifer model. If it is assumed that the maximum rate occurs when the water level in the aquifer is 40 feet or more below the water level in the alluvium, the ratio K/M would approximate 0.00001 day-1. A ground water flow of 0.31 cfs per square mile of alluvium was calculated from the Lower Dawson aquifer into the alluvium of Cherry Creek using Darcy's Law. Assuming the flow into Cherry Creek alluvium occurs as a result of about 40 feet of head difference, the ratio K/M would approximate 0.000024 day^{-1}. Use of K/M = 0.00001 in the models is therefore reasonable. A test model (window) was constructed in the Laramie-Fox Hills to determine the sensitivity of computed depletion to the ratio K/M. A portion of the test window is shown on Figure 19. Ratios of 0.001, 0.0001, and 0.00001 day^{-1} were tested. Decreasing the ratio from .001 to .00001 day^{-1} has the effect of moving the 0.1% stream depletion line about one mile closer to the stream.

Pumping

The intent of model simulations was to compute the depletion of a single well on stream systems. Model runs were made with a well yielding 0.1, 0.01 or 0.001 cfs. Most runs were made with a pumping rate of 0.01 cfs to avoid having model cells drying up during a simulation, particularly near aquifer outcrops where aquifer thicknesses and transmissivities were minimal.

Using the Laramie-Fox Hills Window Number 1, the stream depletion was computed for eight locations using well yields of 0.01, 0.1 and 0.2 cfs. The results of that sensitivity are shown in Table 2. Decreasing the pumping rate from 0.1 cfs to 0.01 cfs resulted in an average 4.4% increase in the computed depletions. Increasing the pumping rate from 0.1 cfs to .2 cfs resulted in an average 5.6% decrease in the computed depletions. Because the 0.1% depletion lines are reported to the nearest section line, the 0.1% depletion line did not move for the four miles tested. Model results are insensitive to pumping rate.

Discretization of Time

The length of time simulated was 100 years (36525 days) in all model runs. The 100 year simulation period (pumping period) was further divided into 20 time steps using a time step multiplier of 1.01. Each time step is 1% larger than the preceeding time step. The length of the first time step was about 4.5 years and about 5.5 years for the last time step.

For a model of an ideal aquifer different time discretization schemes were tried. Ten time steps with a multiplier of 1.5 was found to be inadequate. Using a time step multiplier of 1.01, 20 and 40 time steps were tried. Twenty

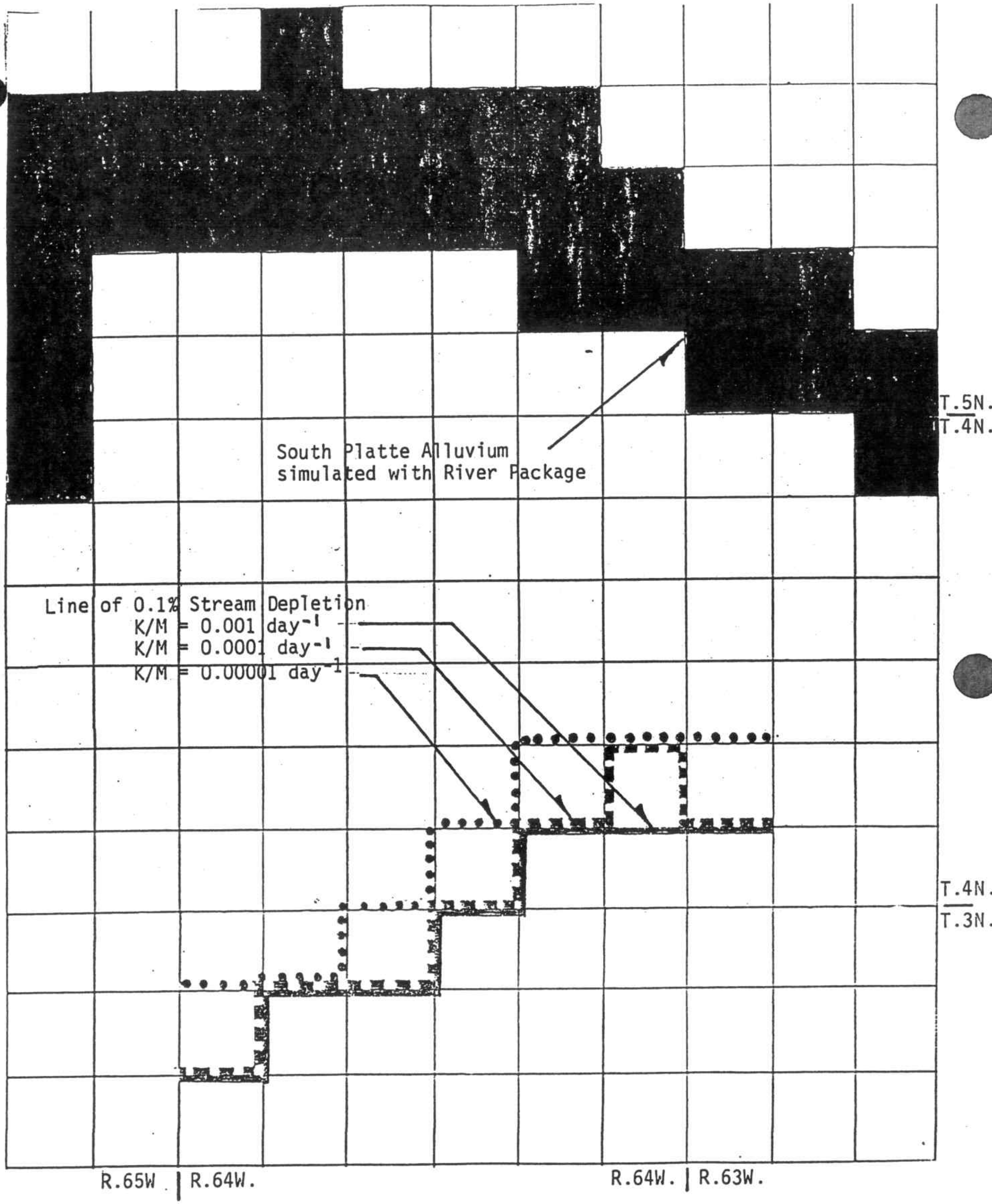

Figure 19. -- Map of Portion of Test Window Showing Sensitivity to the Ratio K/M

Table 2. -- Sensitivity to Pumping Rate for Laramie-Fox Hills Window #1

Cell		Q = .1 cfs	Q = .01 cfs		Q = .2 cfs	
Row	Col	q/Q - %	q/Q - %	Δ-%	q/Q - %	Δ-%
13	31	.02084	.02171	4.2	.01971	5.4
12	31	.1687	.1763	4.5	.1587	5.9
12	32	.05975	.06233	4.3	.05643	5.6
11	32	.4269	.4481	5.0	.3995	6.4
11	33	.1380	.1433	3.8	.1314	4.8
12	33	.02200	.02295	4.3	.02079	5.5
12	34	.05335	.05565	4.3	.05036	5.6
11	34	.2788	.2912	4.4	.2634	5.5
			Aver.	4.4		5.6

time steps was selected as increasing the number of time steps to 40 only slightly improved the results, moving the 0.1% depletion line about 0.2 miles closer to the stream for the ideal aquifer.

RESULTS

Over 3000 computer runs were made resulting in definition of the 0.1% stream depletion lines to the nearest section line. Depletions were computed for cells (sections) on either side of the 0.1% stream depletion lines. The study included areas inside and outside of designated ground water basins. The 0.1% depletion lines, in conjunction with designated ground water basin boundaries and aquifer boundaries, define areas of non-tributary ground water as shown on figures 1F, 1G, 2D, 3G, 3H, and 4D.

Reference:

(1) McDonald, M.G., Harbaugh, A.W., 1984, A Modular Three-Dimensional Finite-Difference Ground Water Flow Model: U.S. Geological Survey Open-File Report 83-875, 528 p.

(2) Weinstein, H.C., Stone, H.L., and Kwan, T.V., 1969, Interative procedure for solution of systems of parabolic and elliptic equations in three dimensions: Indus. Engineering Chemistry Fundamentals, V.8, No. 2, p. 281-287.

(3) Robson, S.G., Banta, E.R. (in press) 1986, Geology and Hydrologly of the Deep Bedrock Aquifers in Eastern Colorado, Water Resource Investigations, USGS, 6 plates.

(4) Robson, S.G., 1983, Hydraulic Characteristics of the Principal Bedrock Aquifers in the Denver Basin, Colorado, Hydrologic Investigations Atlas, HA-659, U.S. Geological Survey, 3 plates.

(5) Robson, S.G., Romero, J.C., 1981, Geologic Structure, Hydrology and Water Quality of the Denver Aquier in the Denver Basin, Colorado, U.S. Geological Survey Hydrologic Invest. Atlas, HA-646, 3 plates.

(6) Robson, S.G., Romero, J.C., 1981, Geologic Structure, Hydrology, and Water Quality of the Dawson Aquifer in the Denver Basin, Colorado, Hydrology, and Water Quality of the Dawson Aquifer in the Denver Basin, Colorado, Hydrologic Investigations Atlas, HA-643, 3 plates.

(7) Robson, S.G., Romero, J.C., Zawistoski, S., 1981, Geologic Structure, Hydrology, and Water Quality of the Arapahoe Aquifer in the Denver Basin, Colorado, Hydrologic Investigations, Atlas, HA-647, 2 plates.

(8) Robson, S.G., 1984, Bedrock Aquifers in the Denver Basin, Colorado -- A Quantitative Water Resources Appraisal., U.S. Geological Survey Oper. File Report 84-431, 111 p.