RUNS OF PRECIPITATION SERIES

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Jose Llamas and M. M. Siddiqui

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HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
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ABSTRACT

Three quantitative measures are introduced for the concepts of "surplus" and "deficit" in hydrologic series. These are: run-length, run-sum and run-intensity. Positive and negative runs of a series are defined in terms of a fixed value, say c, of the variable under consideration, namely precipitation. The distribution function, moments, and other statistical properties of the three variables, run-length, run-sum, and run-intensity, are obtained analytically under the following alternative assumptions on the sequence of annual precipitations:

1. It is independent and normally distributed.
2. It is independent and gamma distributed.

For monthly precipitations, z_t, the series was first standardized by the transformation

$$x_t = \frac{z_t - \mu_\tau}{\sigma_\tau} ,$$

where t is of the form $t = 12(n-1) + \tau$, $\tau = 1, \ldots, 12$, $n = 1,2, \ldots$, and where μ_τ and σ_τ are mean and standard deviation of the series corresponding to the month τ. Calling "$x_t \leq c$" as state "0" and "$x_t > c$" as state "1," the series is then analyzed as a two-state Markov chain with stationary transition probabilities.

Annual precipitation from 27 stations in Colorado, and monthly precipitation from 219 stations in the Western United States are analyzed.

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Jose Llamas* and M. M. Siddiqui**

Chapter I

INTRODUCTION

1.1 Subject of this study. The major objective of this study is to carry out the mathematical analysis of some parameters by which the concept of runs of a precipitation series may be defined with reference to the series itself. One of the main problems in water resource projects is to predict accurately the amount of water available during a period of operation and to determine whether or not it will be sufficient. The total amount of water necessary in a given period of time, whether for one particular project or for a number of projects in one region, can be considered as the water demand of that region. Of course, the demand changes from region to region or from country to country. For instance, in arid areas the water demand must be necessarily less than in the humid regions because of different water availability. The same situation is encountered in the agricultural countries as compared to the industrial ones. If in one period of time the supply of water is smaller or greater than the demand, then this period can be considered as dry or wet, respectively.

The concepts of dry or wet periods ought to be taken only in a relative sense so that they depend on a certain level, c. This value, c, can be a constant or a variable according to the characteristics of the water demand. In the case of agricultural projects on a constant surface of land and for the same kind of annual crops, the consumptive use of water is usually constant every year. In the case of urban development, the future requirement of water is related to the growth of the population and to the expected industrial expansion.

1.2 Background of the problem. The problem of runs of a precipitation series has been initiated by Downer, Siddiqui and Yevjevich [1], and Yevjevich [2]. In these two papers, the authors define a dry or a wet run or a negative or positive run as the period in which the total amount of precipitation is less or greater than a certain constant, c. This constant may correspond to the concept of water demands previously defined. Three main factors may be used in order to characterize a particular negative or positive run: run-length, run-sum, and run-intensity. The run-length of a wet (positive) or a dry (negative) run is the number of terms in a complete positive or negative run, respectively. This is also the duration of a positive or a negative run. This quantity is particularly important in water resource problems because the knowledge of the expected duration of drought or rainfall provides the engineer with the necessary design information.

The run-sum (or the magnitude of a run) is defined as the sum of deviations from a level (water demand) of precipitation over the run-length. These deviations are negative or positive when the run is dry or wet, respectively. In some water resource problems the run-sum is the most important factor. The total capacity of water that must be stored and then supplied depends on the expected run-sum of the future dry negative run. The run-sum of positive or negative runs is directly related to the sizing of reservoir capacities, design and operation of hydroelectric structures, projects of water pollution, sizing of pumps, problems of erosion and sedimentation, and so on.

The third factor characterizing the runs is the run-intensity, which is defined throughout this study as the average intensity or the ratio of run-sum to run-length. This quantity of run-intensity may be used as an index for the classification of regions with respect to precipitation patterns. In this study, the probability distributions of these three quantities will be obtained taking into account several possible cases of the original variable, which is the amount of precipitation in a unit of time.

First, since the unit of time for the precipitation measurement is one year, three different situations are then considered:

(a) One single process of annual precipitation.

(b) Two processes of annual precipitation that are mutually independent.

(c) Two processes of annual precipitation that are dependent.

The term "process" is used in the narrow sense of "stochastic process." It is assumed that any functional dependence on time, such as trend or periodicity, has been removed from any process under consideration. The total amount of annual precipitation is considered as the original random variable.

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With respect to the original process, two alternative assumptions are made:

(a) The annual precipitations are independent identically distributed normal random variables.

(b) The annual precipitations are independent identically distributed gamma-type random variables.

The hypothesis of independence in annual precipitation is supported by Markovic [3], and physically speaking, seems to be realistic because only some factors of small effects (carry-over of water in river basins, evaporation, etc.) may affect the amount of precipitation of the following year. This hypothesis may be easily verified (autocorrelation test, run test, etc.) before analyzing data for positive and negative runs.

The hypothesis of normality of the annual precipitation is, in fact, one of truncated normality (no negative precipitation) at origin. In the regions where the probability of zero annual precipitation is high (arid or arctic regions), neither the hypothesis of normality nor the hypothesis of gamma distribution are applicable.

From the analysis of the samples from 1141 stations in the western United States, Markovic [3] found that, on the average, the annual precipitations are positively skewed, and the gamma distribution hypothesis is more realistic than the hypothesis of normality.

Second, the three main variables, run-length, run-sum, and run-intensity are analyzed also with monthly precipitation as the random variables. In this case the hypothesis of independence is tested and stated at the beginning of analysis, and no hypothesis of distribution of monthly precipitation is made.

The critical level (water demand) considered in this study is the mean value of the process. In the case of annual precipitation, the second order stationarity of the process is assumed. Therefore, the critical level is assumed to be invariant in time. In the case of monthly precipitation, the stationarity is obtained by standardization of the process.

In order to simplify the algebraic operations, the annual precipitation series is standardized, and in both cases (annual and monthly precipitation) the critical level is assumed to be zero.

Chapter II

A SEQUENCE OF INDEPENDENT VARIABLES

2.1 Introduction. In this chapter, a single process of annual precipitation is analyzed in order to obtain the statistical properties of the three main variables characterizing the positive and negative runs: run-length, run-sum, and run-intensity. This type of single record analysis is necessary before one can study several series and obtain correlation properties of one station with another or of one region with another.

2.2 Formulation of the problem. The problem was formulated by Downer, Siddiqui and Yevjevich [1]. For the sake of ready reference, however, it seems desirable to summarize the essentials of that paper. Some of their results are reported here in a strengthened form, and some new results are also included.

Let X_n, $n = 1,2,\ldots$, be independent identically distributed random variables with a common distribution, F, which is assumed to be continuous. In the application to be followed after the derivation of theoretical results, X_n is the total precipitation at a given station during the n-th year. However, it can also represent the sum of precipitations over several stations in a given region. Also the unit of time may be shorter or longer than a year.

A level, c, in the range of values of X_n is chosen such that $0 < F(c) < 1$, and the n-th year is classified as a surplus year if $X_n > c$ and, in that case, refer to $X_n - c$ as the surplus. Similarly, the n-th year is called a deficit year if $X_n \le c$, in which case, $c - X_n$ is called the deficit. Thus defined, these surpluses and deficits are all positive random variables.

A consecutive sequence of k surplus years preceded and succeeded by a deficit year is called a positive run-length k, the sum of surpluses $\Sigma(X_n-c)$ over such a run is the positive run-sum, and this run-sum divided by the run-length is called the positive run-intensity. Similar definitions hold for negative runs.

For $j = 1,2,\ldots$, let N_{1j} denote the length of the j-th negative run-length and N_{2j} the length of the following positive run-length. If the initial observation, X_1, is greater than c, the initial positive run is disregarded. Suppose that the j-th negative run starts with X_{i+1}. Set

$$S_{1j} = \sum_{k=1}^{N_{1j}} (c-X_{i+k}) , \qquad I_{1j} = \frac{S_{1j}}{N_{1j}} ,$$
$$S_{2j} = \sum_{k=1}^{N_{2j}} (X_{i+N_{1j}+k} - c), \qquad I_{2j} = \frac{S_{2j}}{N_{2j}} \tag{2.1}$$

Then S_{1j}, S_{2j} are the negative and positive run-sums, respectively, for the j-th run, and I_{1j}, I_{2j} are the corresponding run-intensities. The properties of N_{ij}, S_{ij}, I_{ij}, $i = 1,2$, $j = 1,2,\ldots$ are studied in the further text.

2.3 The independence of $\{(N_{1j}, S_{1j})\}$ and $\{(N_{2j}, S_{2j})\}$. For convenience the following notations are introduced:

$$p = F(c) = P(X_n \le c) , \quad q = 1 - p ;$$
$$F_1(x) = \frac{F(c) - F(c-x)}{F(c)}, \text{ if } x \ge 0 ,$$
$$= 0 \qquad x < 0 ;$$
$$F_2(x) = \frac{F(x+c) - F(c)}{1 - F(c)} , \text{ if } x \ge 0,$$
$$= 0 \quad , \text{ if } x < 0 . \tag{2.2}$$

Let X^*_{1n}, $n = 1,2,\ldots$, be a sequence of independent random variables each with the distribution F_1 and X^*_{2n}, $n = 1,2,\ldots$, another sequence of independent random variables, independent of the sequence X^*_{1n}, with the distribution F_2. Then

$$P(X^*_{1n} \le x) = P(c - X_n \le x | X_n \le c) = F_1(x) ,$$
$$P(X^*_{2n} \le x) = P(X_n - c \le x | X_n > c) = F_2(x) ,$$
$$P(\sum_{j=1}^{m} X^*_{1j} \le x) = P(\sum_{j=1}^{m} (c-X_j) \le x | X_j \le c, j = 1,..m) = F_1^{\circledast m}(x),$$
$$P(\sum_{j=1}^{m} X^*_{2j} \le x) = P(\sum_{j=1}^{m} (X_j - c) \le x | X_j > c, j=1,\ldots,m) = F_2^{\circledast m}(x) \tag{2.3}$$

where, for any distribution function H and $m = 1,2,\ldots$, $H^{\circledast m}$ denotes the m-fold convolution of H with itself.

First consider the distribution of N_{1j}. If $X_1 \le c$, then $P(N_{11} = k | X_1 \le c) = P(X_i \le c, i = 1,\ldots,k, X_{i+k} > c | X_1 \le c) = qp^{k-1}$, $k = 1,2,\ldots$.

If $X_1 > c$, then

$$P(N_{11} = k | X_1 > c) = \sum_{j=1}^{\infty} P(X_i > c, i = 1,\ldots,j, X_{j+i} \le c, i = 1,\ldots,k, X_{j+k+1} > c | X_1 > c) = p^k \sum_{j=1}^{\infty} q^i = qp^{k-1},$$
$k = 1,2,\ldots$.

Hence, the unconditional distribution of N_{11} is

$P(N_{11} = k) = P(N_{11} = k|X_1 \leq c)\ P(X_1 \leq c) + P(N_{11}$

$= k|X_1 > c)\ P(X_1 > c) = qp^{k-1},\ k = 1,2,\ldots\ .$ m(2.4)

Similarly,

$$P(N_{11} = k_1, N_{21} = k_2) = p^{k_1} q^{k_2};\ P(N_{21} = k_2)pq^{k_2-1},$$

so that

$$P(N_{11} = k_1, N_{21} = k_2) = P(N_{11} = k_1)\ P(N_{21} = k_2)\ ,$$

and N_{11} and N_{21} are independent. This argument can be extended to show that N_{11}, N_{21}, N_{12}, N_{22}, ... are mutually independent, $\{N_{1j}\}$ are identically distributed, and $\{N_{2j}\}$ are identically distributed.

Now, look at the joint distribution of (N_{11},S_{11}). From (2.3) it follows that

$$P(S_{11} \leq x|N_{11} = k) = F_1^{\circledast k}(x)\ , \tag{2.5}$$

hence

$$P(N_{11} = k,\ S_{11} \leq x) = qp^{k-1}F_1^{\circledast k}(x)\ , \tag{2.6}$$

Similar expressions hold for (N_{21},S_{21}). Finally,

$$F_s(x) = P(S_{11} < x) = \sum_{k=1}^{\infty} qp^{k-1}F_1^{\circledast k}(x)\ . \tag{2.7}$$

Again, one can show that the sequence of vectors (N_{1j},S_{1j}) is mutually independent and identically distributed with (2.6). This sequence is also independent of (N_{2j},S_{2j}), which themselves are mutually independent and identically distributed. Since the treatment of one vector sequence is exactly parallel to the other, only one is considered. (In fact $X_n \leq c$ is equivalent to $-X_n > -c$ so that a negative run for X_n at level, c, is equivalent to a positive run for $-S_n$ at level, -c). We choose to concentrate on the negative run (N_{1j},S_{1j}). We drop the subscript, j, and write it as (N_1,S_1) unless the whole sequence is considered.

2.4 The distribution of S_1 in some special cases. From (2.7), the distribution function, F_s of S_1, is directly related to F_1 rather than to F. Since $0 < p < 1$, $p^n \to 0$, terms after some k = n may be negligible. For example, if p = 1/2, $p^7 < 0.01$ and the series may be truncated at the sixth term with the error of approximation less than one percent uniformly for all x. Actually, since $F^{\circledast k}(x) \leq 1$, then

$$F_s(x) = \sum_{k=1}^{n} qp^{k-1}\ F_1^{\circledast k}(x) \leq \sum_{k=n+1}^{\infty} qp^{k-1} = p^n \tag{2.8}$$

For example, let $F_1(x) = F(x,\lambda,r)$ with the density function

$$f(x,\lambda,r) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}\ ,\ x > 0\ ,$$

and $r \geq 0$. If r = 1, then

$$f_s(x) = F_s'(x) = q\lambda e^{-\lambda x}[1+p\lambda x + \frac{(p\lambda x)^2}{2!} + \ldots]$$

$$= q\lambda e^{-q\lambda x}\ ,\ x > 0\ , \tag{2.9}$$

an exponential distribution. For arbitrary r > 0,

$$F_s(x) = q[F(x,\lambda,r)+p\ F(x,\lambda,2r) + p^2F(x,\lambda,3r) + \ldots]$$

$$= q \sum_{k=1}^{n} p^{k-1}\ F(x,\lambda,kr) + R_n(x)\ ,$$

where

$$R_n(x) = q[p^nF(x,\lambda,(n+1)r) + p^{n+1}\ F(x,\lambda,(n+2)r) + \ldots]$$

$$\leq qp^n\ F(x,\lambda(n+1)r)[1+p+p^2+\ldots] = p^n\ F(x,\lambda,(n+1)r). \tag{2.10}$$

This follows because, for x > 0 ,

$$F(x,\lambda,kr) < F(x,\lambda,(k-1)r)\ .$$

Usually, n = 2 or 3 may give a satisfactory approximation.

2.5 Moment generating functions. If y is any random variable with the distribution function, J(y), and E is the mathematical expectation operator, then

$$M_y(\theta) = Ee^{\theta y} = \int_{-\infty}^{\infty} e^{\theta y}\ dJ(y)$$

is called the moment generating function of Y or of the distribution, J. θ is taken to be a complex number and $M_y(\theta)$ exists at least for Re $\theta \leq 0$. If for some r = 1,2,..., the r-th moment of Y exists, then it is given by

$$\mu_r'(y) = EY^r = M_y^{(r)}(0)\ ,$$

where $M^{(r)}(0)$ is the r-th derivative of M evaluated at θ = 0.

If Y_1, Y_2, ... ,Y_n are independent and $Y = Y_1+\ldots+Y_n$, then

$$M_y(\theta) = M_{y_1}(\theta)\ M_{y_2}(\theta)\ \ldots\ M_{y_n}(\theta)\ .$$

The function, $K_y(\theta) = \ln M_y(\theta)$, is called the cumulant generating function of Y. The r-th cumulant of Y exists if the r-th moment of Y exists and is given by

$$k_r(y) = \text{r-th cumulant of } Y = K_y^{(r)}(0) .$$

Clearly, for $Y = Y_1+\dots+Y_n$, where $Y_1,\dots,Y_n$ are independent

$$K_y(\theta) = K_{y_1}(\theta) + \dots + K_{y_n}(\theta)$$

$$k_r(y) = k_r(y_1) + \dots + k_r(y_n) .$$

Recall that $\{X_{1n}\}$ have the common distribution function, F_1. Set

$$M_1(\nu) = Ee^{\nu X_{11}^*} = \int_{-\infty}^{\infty} e^{\nu x}\, dF_1(x) = \frac{e^{c\nu}}{p}\int_{-\infty}^{c} e^{-\nu x}\, dF(x) ,$$

$$K_1(\nu) = \ln M_1(\nu) .$$

From Downer, Siddiqui and Yevjevich [1],

$$M_1(u,\nu) = Ee^{uN_1+\nu S_1} = \frac{q\exp[u - K_1(\nu)]}{1-p\exp[u-K_1(\nu)]} ,$$

$$K_1(u,\nu) = \ln M_1(u,\nu) .$$

Also

$$EN_1 = \frac{1}{q} , \text{ var } N_1 = \frac{p}{q^2}$$

$$ES_1 = \frac{EX_{11}^*}{q} , \text{ var } S_1 = \frac{q \text{ var}X_{11}^* + p(EX_{11}^*)^2}{q^2}$$

$$\text{Cov}(N_1,S_1) = \frac{p}{q^2} = EX_{11}^*$$

$$\rho(N_1,S_1) = \frac{pEX_{11}^*}{\sqrt{pq \text{ var } X_{11}^* + p^2(EX_{11}^*)^2}} \qquad (2.12)$$

where var X is the variance of X, and cov (X,Y) is the covariance, and $\rho(X,Y)$ is the correlation between X and Y.

The authors just mentioned did not give the moment generating function of $I_1 = \frac{S_1}{N_1}$ but, in a similar argument,

$$M_{I_1}(\theta) = Ee^{\theta I_1} = Ee^{\theta\frac{S_1}{N_1}} = \sum_{n=1}^{\infty} qp^{n-1}\{M_1(\tfrac{\theta}{n})\}^n . \qquad (2.13)$$

The evaluation of the moments of I_1 involves sums of the form

$$A_r(z) = \sum_{n=}^{\infty} \frac{z^{n-1}}{n^r} , \quad r = 1, 2, \dots \quad 0 < z < 1 .$$

Now,

$$A_1(z) = 1 + \frac{z}{2} + \frac{z}{3} + \dots = -\frac{1}{z}\ln(1-z) .$$

Integrating both sides from 0 to p gives

$$-\int^{p}\frac{1}{z}\ln(1-z) = p + \frac{p^2}{2^2} + \frac{p^3}{3^2} + \dots + \frac{p^n}{n^2} + \dots = \frac{1}{p}A_2(p),$$

and so on.

Finally,

$$EI_1 = EX_{11}^*, \text{ var } I_1 = \frac{q}{p}\text{ var } X_{11}^* \ln(\tfrac{1}{q}). \qquad (2.14)$$

There is little point in giving the algebraic form of higher moments as they can be numerically calculated in a specific situation.

Chapter III

NORMAL AND GAMMA DISTRIBUTED VARIABLES

3.1 Normal sequence. Suppose that the original sequence $\{X_n\}$ is an independent normally distributed sequence with $EX_n = 0$, var $X_n = 1$. (If $EX_n = \mu$ and $X_n = \sigma^2$, then consider the standardized sequence $(X_n - \mu)/\sigma$.) Downer, Siddiqui and Yevjevich [1] have studied this situation exhaustively for (N_{2j}, S_{2j}). Their results are equally applicable to (N_{1j}, S_{1j}). Now consider the case of $c = EX_n = 0$ to illustrate a method of approximating to the distribution of S_{11} and I_{11}. Thus

$ES_1 = 1.59577$, var $S_1 = 2.0$, $ES_1^3 = 18.8615$, $ES_1^4 = 93.0225$

where an approximation to the value of π is used. Also

$EI_1 = 0.797885$, $EI_1^2 = 0.888496$, $EI_1^3 = 1.263186$,

$EI_1^4 = 2.064387$ $EN_1 = 2$, var $N_1 = 2$, $\mathrm{cov}(N_1, S_1) =$

$= 1.59577$, $\rho(N_1, S_1) = 0.798$.

The coefficients of skewness of S_1 is

$$C(S_1) = 1.82 \ .$$

Since this is positive, a gamma distribution is chosen to approximate to $f_s(x)$. Siddiqui [4] gives the method for this type of approximation.

Let

$$f(x;g,h) = \frac{e^{-x/2g}\, x^{k/2-1}}{(2g)^{h/2}\Gamma(h/2)} \quad \text{for } x > 0$$

$$= 0 \qquad \text{otherwise,} \tag{3.1}$$

be the probability density function of a gamma variate, to which the probability density function of S_1 is approximated.

In (3.1), g is a scale factor and h is the effective number of degrees of freedom.

The approximation to the probability density of S_1 can be improved as follows:

$$f(x) = f(x;g,h) \sum_{m=o}^{\infty} \frac{m!\Gamma(h/2)}{\Gamma(m+h/2)} \frac{d_m}{(2g)^m} L_m^{(\frac{h}{2}-1)}(\frac{x}{2g}) \tag{3.2}$$

where $L_m^{(c)}(y)$ is the Laguerre polynomial of degree, m.

$$L_m^{(c)}(y) = \sum_{j=0}^{m} \binom{m+c}{m-j} \frac{(-y)^j}{j!} \tag{3.3}$$

and

$$d_m = \sum_{j=o}^{m} \binom{m-1+h/2}{m-j} (-1)^j \ (2g)^{m-j} \ \frac{\gamma_j}{J!} \tag{3.4}$$

where

$$\gamma_j = E(S_1)^j \ .$$

The parameters g and h can be computed by the method of moments, i.e., equating the first two moments of the probability density function in Eq. (3.1) with the moments, ES_1 and $E(S_1)^2$, already found. The first two moments of the distribution in Eq. (3.1) are gh and $g^2h(h+2)$. Thus, setting $gh = ES_1$ and $g^2h(h+2) = ES_1^2$,

$$g = 0.626657, \quad h = 2.546482 \ .$$

Then

$$f(x;g,h) = 0.816398 \ e^{-0.797885x} \ x^{0.273241}$$

In this kind of approximation, only the first few polynomials are really important. As a general rule, the order, m, of the last polynomial considered must be such that:

(a) No appreciable oscillations appear in the probability density function.

(b) The coefficient of x^m must be small in comparison with the coefficients of the terms of lower order.

With those considerations, the probability density function of S_1 is truncated at $m = 4$.

Table 1 shows the different computations. In this table, $A_m = \frac{m!\Gamma(h/2)}{\Gamma(m+h/2)}$.

Finally, the probability density function of S_1 (and S_2) is

$$f_s(x) = 0.816398e^{-0.797885x}x^{0.273241} \ (0.790207 +$$

$$+ 0.514732x - 0.265132x^2 + 0.042132x^3 - 0.001922x^4).$$

3.2 Approximated probability density functions of I_1 and I_2. As before, the probability density function of I_1 (and I_2) will be approximated by a function of a gamma-variate.

In this case,

$$g = \frac{\mathrm{Var}\ I_1}{2E\ I_1} = 0.157840$$

TABLE 1

IMPROVEMENT OF PROBABILITY DENSITY FUNCTION OF $S_i (i = 1,2)$

m	$\frac{d_m}{(2g)^m}$	A_m	$L_m^{(0.273241)} (\frac{x}{2g})$
0	1	1	1
1	0	-	-
2	0	-	-
3	-0.017781	0.633311	$1.579002-2.968478x+1.041905x^2-0.084658x^3$
4	-0.192013	0.592816	$1.686864-4.228341x+2.226157x^2-0.361764x^3+ +0.016887x^4$

$$h = \frac{2(EI_1)^2}{\text{Var } I_1} \approx 5.055031 .$$

The parameters d_m and A_m and the functions L_m, for m = 0 to 4, are given in Table 2.

Finally, the probability density function of I_1 and I_2 is

$$f_I(x)=13.650570e^{-3.167764x} x^{1.527516}(0.595968+2.070133x-$$

$$-2.848625x^2+1.356767x^3-0.198404x^4) . \quad (3.5)$$

Figures 1 and 2 show the probability density functions of S_1 (or S_2) and I_1 (or I_2).

3.3 Gamma distributed sequence. Let a random variable, X, have the distribution function, F(x), with the probability density function

$$f(x) = \frac{x^{r-1} e^{-x}}{\Gamma(r)} , \quad \text{if } x > 0$$
$$= 0, \quad \text{if } x \leq 0 ,$$

where r > 0. One can introduce a scale factor λ, but it will simply involve multiplying the k-th moment by λ^k. Since EX = r, var X = r, we consider the moments of, the standardized variable

$$X_1 = \frac{X - r}{\sqrt{r}} ,$$

and the sequence X_n, n = 1,2,..., which are identically distributed.

If X^*_{11} and X^*_{12} denote the truncated random variables with c = 0, then

$$EX^{*k}_{11} = r^{-k/2} EX^k_{11} ,$$

where X_{11} is the variable, X, truncated at EX = r. Similarly, $EX^{*k}_{21} = r^{-k/2} EX^k_{22}$.

Let F_1 and F_2 be the distribution functions obtained from Eq. (2.2). Then

TABLE 2

IMPROVEMENT OF PROBABILITY DENSITY FUNCTION OF $I_i (i = 1,2)$

m	$\frac{d_m}{(2g)^m}$	A_m	$L_m^{(1.527516)} (\frac{x}{2g})$
0	1	1	1
1	0	-	1
2	0	-	-
3	0.035602	0.148638	$6.727776-25.295999x+22.716197x^2-5.297942x^3$
4	-0.439633	0.107562	$9.296972-46.608005x+62.782081x^2-29.284471x^3+ +4.195661x^4$

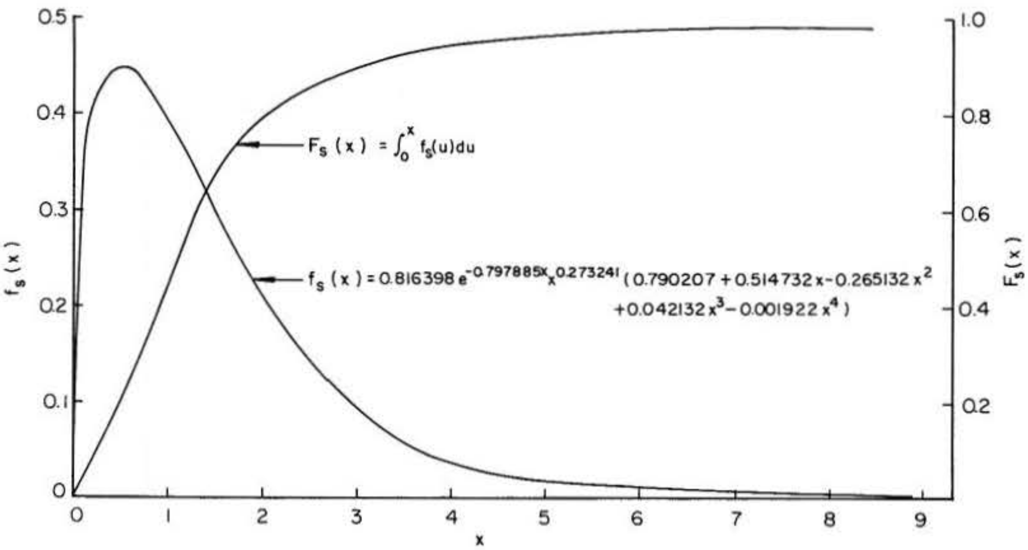

Figure 1 Distribution function and probability density function of S_1 and S_2

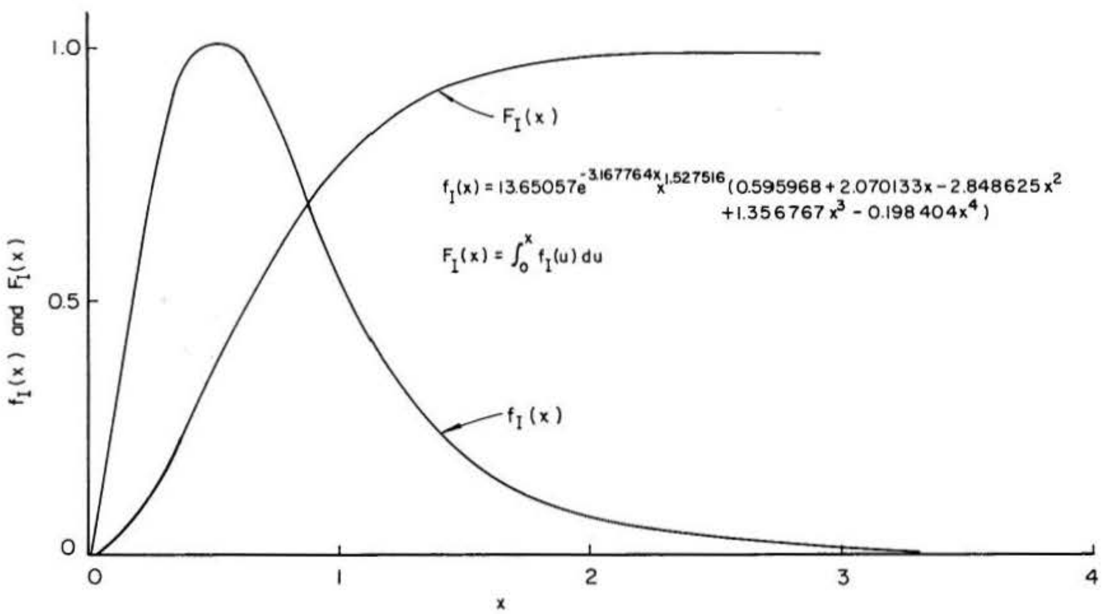

Figure 2 Distribution function and probability density function of I_1 and I_2

$$dF_1(x) = \frac{(r-x)^{r-1}\, e^{-(r-s)}}{\Gamma(r)\, P(r,r)}\, dx\ , \quad 0 \le x \le r$$

where

$$P(a,x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1}\, e^{-t}\, dt \quad a > 0,\ x > 0\ ,$$

is the incomplete gamma function.

Then

$$\Gamma(r)\, P(r,r)\, EX_{11}^k = \int_0^r x^k (r-x)^{r-1}\, e^{-(r-x)}\, dx$$

$$= \int_0^r y^{r-1}\, (r-y)^k e^{-y}\, dy$$

$$= \sum_{j=0}^{k} (-1)^j\, r^{k-j}\, \Gamma(r+j)\, P(r+j,r)\ .$$

Hence,

$$E(X_{11}^*)^k = \frac{EX_{11}^k}{r^{k/2}} = \frac{\sum_{j=0}^{k} (-1)^j r^{k-j} \Gamma(r+j) P(r+j,r)}{r^{k/2}\, \Gamma(r)\, P(r,r)}\ . \qquad (3.6)$$

In a similar fashion

$$E(X_{21}^*)^k = \frac{EX_{21}^{\ k}}{r^{k/2}} = \frac{\sum_{j=0}^{k} (-1)^j \binom{k}{j}\, r^j\, \Gamma(r+k-j)}{r^{k/2}\, \Gamma(r)\, P(r,r)} -$$

$$- (-1)^k\, E(X_{11}^*)^k \qquad (3.7)$$

In the distribution of N_{11} and N_{21},

$$p = F(r) = P(r,r), \; q = 1-p \; .$$

We define S_1 and S_2 in terms of the "normalized" variables X^*_{1n}, X^*_{2n} and then calculate their moments.

The following table shows the values of the first four moments of X^*_{11} and X^*_{21} for several values of r.

The values of the Incomplete Gamma Function have been taken from K. Pearson [5].

TABLE 3

MOMENTS OF X^*_{i1} (i=1,2) FOR SEVERAL VALUES OF r

r	EX^*_{11}	$E(X^*_{11})^2$	$E(X^*_{11})^3$	$E(X^*_{11})^4$	EX^*_{21}	$E(X^*_{21})^2$	$E(X^*_{21})^3$	$E(X^*_{21})^4$
1	0.58198	0.41802	0.32788	0.27027	0.58198	1.16396	3.49184	13.96753
2	0.64412	0.54456	0.51807	0.53441	0.64412	1.13809	2.89771	9.56150
4	0.69031	0.65486	0.72614	0.87779	0.69031	1.11027	2.49127	7.06530
6	0.71055	0.71012	0.84056	1.10099	0.71055	1.09454	2.31406	6.11766

The following tables show the first four moments of S_i and I_i for several values of r.

TABLE 4

MOMENTS OF S_i(i=1,2) FOR SEVERAL VALUES OF r

r	ES_1	$E(S_1)^2$	$E(S_1)^3$	$E(S_1)^4$	ES_2	$E(S_2)^2$	$E(S_2)^3$	$E(S_2)^4$
1	1.58198	4.30027	17.20120	91.63671	0.92068	2.46503	9.89973	53.01123
2	1.58768	4.33840	17.35705	92.41097	1.08383	2.86815	11.18544	57.70780
4	1.59252	4.38432	17.63093	94.21243	1.21849	3.24693	12.64762	65.03209
6	1.59359	4.40703	17.77899	95.26155	1.28230	3.44160	13.46745	69.57820

TABLE 5

MOMENTS OF I_i(i=1,2) FOR SEVERAL VALUES OF r

r	ES_1	$E(S_1)^2$	$E(S_1)^3$	$E(S_1)^4$	ES_2	$E(S_2)^2$	$E(S_2)^3$	$E(S_2)^4$
1	0.58198	0.38486	0.27422	0.20031	0.58198	0.98911	2.63477	9.77068
2	0.64412	0.49475	0.42168	0.37902	0.64412	0.96617	2.15717	6.50659
4	0.69031	0.59059	0.57961	0.60859	0.69031	0.94718	1.85078	4.74713
6	0.71055	0.63827	0.66659	0.75612	0.71055	0.93751	1.72266	4.09933

Comparing the moments of S_i and I_i, i = 1,2, obtained in this way with the same moments as for the normal, it follows that the moments corresponding to the gamma distribution of the original random variable, X_1, converge to the moments corresponding to the normal distribution of X_1. This convergence is almost independent of r for the moments of S_1, but for the other random variables, S_2, I_1 and I_2, both assumptions are similar for large values of r only as shown in Figs. 3, 4 and 5.

3.4 Example: Fort Collins, Station No. 5.3005.

Years of records: N = 69

Mean: $\mu = 14.62$

Standard deviation: $\sigma = 4.00$

Equating the mean and variance, it follows

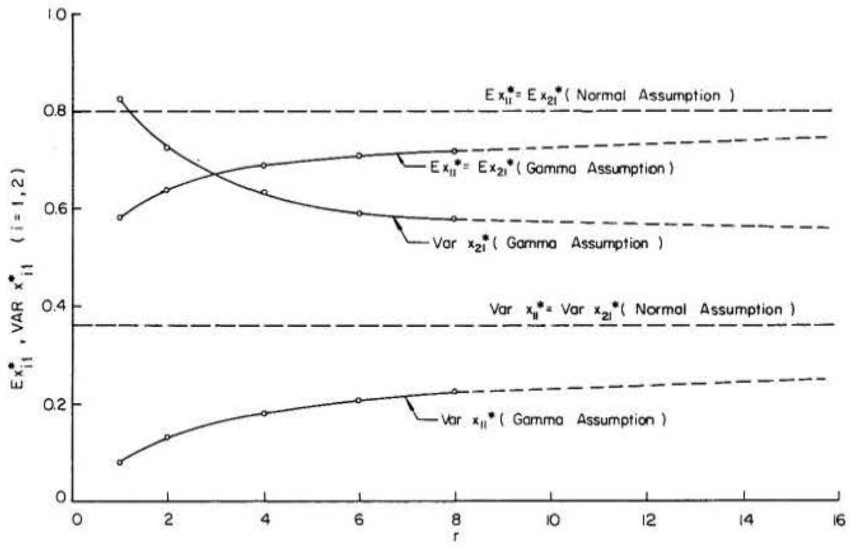

Figure 3 Expected values and variances of X_{11}^{*} and X_{21}^{*} for normal and gamma assumptions

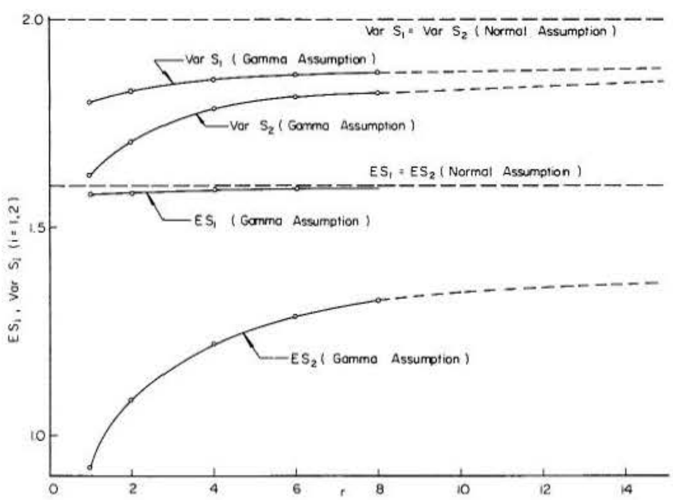

Figure 4 Expected values and variances of S_1 and S_2 for normal and gamma assumptions

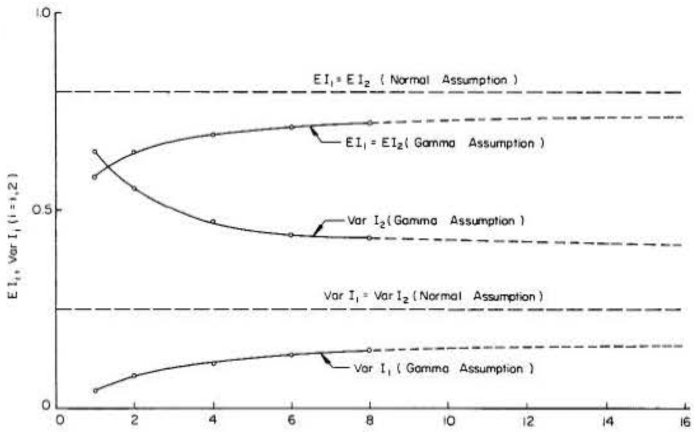

Figure 5 Expected values and variances of I_1 and I_2 for normal and gamma assumptions

TABLE 6

EXPECTED VALUES AND VARIANCES OF X_i and N_i (i=1,2) FOR DIFFERENT HYPOTHESES OF $\{X_n\}$

Hypothesis of $\{X_n\}$	EX^*_{11}	EX^*_{21}	$VarX^*_{11}$	$VarX^*_{21}$	EN_1	EN_2	$VarN_1$	$VarN_2$
Normal	0.79789	0.79789	0.36338	0.36338	2.00000	2.00000	2.00000	2.00000
Gamma	0.740	0.740	0.245	0.565	2.166	1.857	2.527	1.760
From the data	0.670	0.910	0.243	0.502	1.850	1.524	0.928	0.725

TABLE 7

EXPECTED VALUES AND VARIANCES OF S_i AND I_i(i=1,2) FOR DIFFERENT HYPOTHESES OF $\{X_n\}$

Hypothesis of $\{X_n\}$	ES_1	ES_2	$VarS_1$	$VarS_2$	EI_1	EI_2	$VarI_1$	$VarI_2$
Normal	1.59577	1.59577	2.00000	2.00000	0.79789	0.79789	0.25188	0.25188
Gamma	1.595	1.360	1.880	1.845	0.73800	0.73800	0.160	0.420
From the data	1.239	1.386	0.816	1.027	0.673	0.957	0.123	0.586

$$\mu = \frac{r}{\lambda} = 14.62$$

$$\sigma^2 = \frac{r}{\lambda^2} = 16 \ .$$

Then

$\lambda = 0.9138$ $\qquad r = 13.360$

The preceding example shows that the first moment of all random variables, obtained from the data, agrees quite well with the first moment of the theoretical hypothesis (better if the comparison is done with gamma hypothesis). For the random variables, N_i,S_i, and I_i(i=1,2), the disagreement between the higher moments in both cases, provided by the fact that the sample size and consequently the number of runs, is very small in this example; therefore, the estimation is obviously subject to large sampling fluctuations.

Chapter IV

TWO MUTUALLY INDEPENDENT PROCESSES

4.1 Introduction. In previous chapters, the parameters defining the negative and positive runs of annual precipitation were studied considering one single sequence of original random variables: the total amount of annual precipitation at one station. The concept of runs defined in this way can be generalized to several points in space simultaneously in order to study the behavior of those phenomena in the joint dimensions of time and space. This situation is often encountered in hydrology. For example, if a river is passing through two regions with similar or different meteorological conditions, the expected runs in a downstream storage project depend on the combined pattern of precipitation in both regions. In this case, two different sequences will be required in order to define the process. The same problem can arise in a large watershed in regard to the particular model of precipitation on its main tributaries.

4.2 Formulation of the problem. Consider a sequence of a two-dimensional process, (X_n, Y_n), $n = 1, 2, \ldots$, where these vectors are mutually independent and have a common distribution function, $F(x,y)$. Given two levels, c_1 and c_2, such that $0 < F(c_1,c_2) < 1$, we have four possible events:

$$A_n = \{X_n \le c_1,\ y_n \le c_2\} \qquad B_n = \{X_n \le c_1,\ y_n > c_2\}$$

$$c_n = \{X_n > c_1,\ y_n \le c_2\} \qquad D_n = \{X_n > c_1,\ y_n > c_2\}$$

Of these four, A_n and D_n are of interest to us. The n-th year will be called deficit for both sequences if A_n occurs and surplus if D_n occurs. A sequence of k consecutive A's followed and preceded by any other event is a negative run of length, k. A sequence of k consecutive D's followed and preceded by any other event is a positive run of length, k. (For the initial run the requirement of "preceded by" is dropped.) The situation is depicted in Fig. 6.

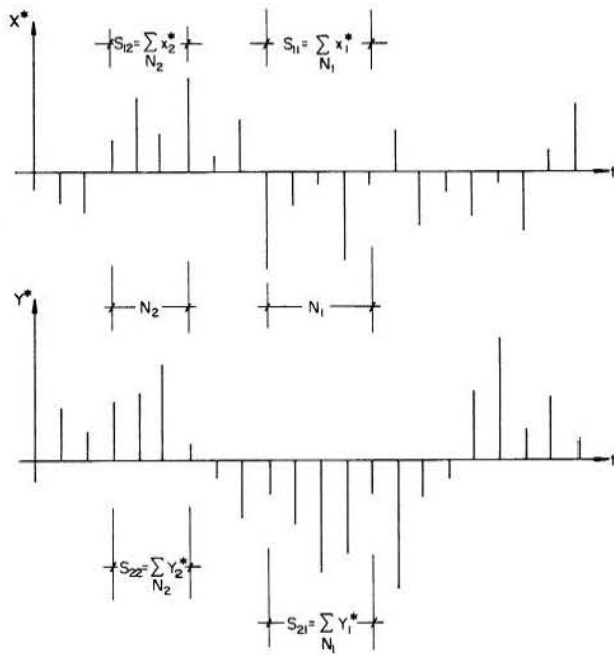

Figure 6 Graphical representation of the random variables S_{11}, S_{12}, S_{22}, N_1 and N_2

$$P(A_n) = F(c_1,c_2) = p, \text{ say}, P(B_n \cup C_n \cup D_n) = P(A_n^{\,c}) = 1-p = q \ .$$

Thus the distribution of N_{11} is still given by the formula

$$P(N_{11} = k) = qp^{k-1}, \quad k = 1,2,\ldots \ .$$

The difference is that now there is no guarantee that a negative run will be immediately followed by a positive run. In fact, it is quite possible that a negative run is followed by a few B and C type events, which in turn, are followed by another negative run. Also here $q \ne P(D_n)$. Since the discussion of the positive runs is parallel to that of negative runs, we omit their mention entirely. We now use the symbols S_{11} for $\Sigma(c-X_n)$ and S_{21} for $\Sigma(c-Y_n)$, where the summation is over a (common) negative run.

When $\{X_n\}$ is independent of $\{Y_n\}$, we have

$$F(c_1,c_2) = F(c_1)G(c_2) \ ,$$

where $F(x) = P(X_n \le x)$, $G(y) = P(Y_n \le y)$. For example, if both X_n and Y_n are standard normal and $c_1 = c_2 = 0$, then $F(0,0) = 1/4$. However, we are at liberty to choose F and G differently, for instance, F to be normal and G to be a gamma distribution.

Now, let

$$F_{12}(x,y) = P(c_1 - X_1 \le x,\ c_2 - y_1 \le y \mid X_1 \le c_1,\ Y_1 \le c_2)$$

$$= P(c_1 - X_1 \le x \mid X_1 \le c_1)\ P(c_2 - Y_1 \le y \mid Y_1 \le c_2), \qquad (4.1)$$

so that the random variables, $\{X_{1n}\}$ and $\{Y_{1n}\}$, can be defined independently by the truncation of F and G, respectively. The entire discussion of Chapter II carries through for S_{1j} and S_{2j} except for their covariance properties. We have

$$ES_{11}\,S_{21} = E[E(S_{11}S_{21}|N_1)] = \Sigma N_1^2\, qp^{N_1-1}\, EX_{11}^{*}EY_{11}^{*}$$

$$= EN_1^2\, EX_{11}^{*}\, EY_{11}^{*} \,,$$

so that

$$\operatorname{cov}(S_{11},S_{21}) = \operatorname{var} N_1\, EX_{11}^{*}\, EY_{11}^{*} = \frac{p}{q^2}\, EX_{11}^{*}\, EY_{11}^{*}. \quad (4.2)$$

4.3 Comment on the dependent case. From the equation (4.1), it is apparent that no general discussion can be carried very far if $\{X_n\}$ and $\{Y_n\}$ are not independent, i.e., when $F(x,y) \neq F(x)G(y)$. The essential difficulty is in finding the joint distribution of the truncated random variables (X_{11}^{*},Y_{11}^{*}). Even the marginal distribution of X_{11}^{*} (or of Y_1^{*}) depends on the joint condition $\{X_1 \leq c_1,\ Y_1 \leq c_2\}$.

Chapter V

MONTHLY PRECIPITATION SERIES

5.1 Introduction. In the three pervious chapters the annual precipitation as the basic random variable leading to an objective definition of runs was considered. The series in this form were independent within and strictly stationary.

However, in some cases it is preferable to reduce the length of this original random variable in order to create another process in which the length of observations will be longer. In practical terms this new process offers more advantages, in particular, in the study of those phenomena renewable in a short period of time. For example, the drought, defined as the negative run over the mean of annual precipitation, does not mean anything to a farmer so long as the precipitation is concentrated in the right period.

In this chapter, the monthly precipitation is the basic random variable. The time series formed by the total precipitation during a month are not stationary because of the seasonal variations. Each series must be considered as a sample of 12 different populations, and some transformations should be necessary in order to bring about stationarity.

5.2 Formulation of the problem. We consider a sequence of monthly precipitation. Let P_t, $t = \tau+12(n-1)$ be the total amount of precipitation in the τ-th month of the n-th year. Here, $\tau = 1, 2, \ldots, 12$ and $n = 1, 2, \ldots$. Fix τ and set

$$X_n = \frac{P_t - \mu_\tau}{\sigma_\tau} \quad , \; t = \tau + 12(n-1) \; ,$$

where μ_τ is the mean value and σ_τ is the standard deviation for the month τ. X_n, $n = 1,2,\ldots$, then corresponds to the standardized values of P_t for the same month of successive years. Clearly, this may be assumed to be either an independent sequence or a mildly dependent stationary sequence. What we assume concerning the dependence is the following. Let

$$x_n = 1, \quad \text{if } X_n > 0 \; ,$$
$$= 0, \quad \text{if } X_n \leq 0 \; .$$

We assume that the sequence of x_n forms a two state Markov process with stationary transition probabilities. That is,

$$P(x_n|x_{n-1},x_{n-2},\ldots,x_1) = P(x_n|x_{n-1}) = P(x_2|x_1). \quad (5.1)$$

Let

$$P(x_2 = 0|x_1 = 0) = 1 - \alpha, \; P(x_2 = 1|x_1 = 0) = \alpha$$

$$P(x_2 = 0|x_1 = 1) = \beta \quad P(x_2 = 1|x_1 = 1) = 1 - \beta$$

The transition matrix of the model is

$$P = \begin{matrix} & \to 0 & 1 \\ \end{matrix} \begin{bmatrix} 0 & 1-\alpha & \alpha \\ 1 & \beta & 1-\beta \end{bmatrix} ,$$

with equilibrium probabilities

$$\lim_{n\to\infty} P(x_n = 0) = \pi_o = \frac{\beta}{\alpha+\beta} \; , \; \lim_{n\to\infty} P(x_n = 1) = \pi_1 = \frac{\alpha}{\alpha+\beta} \; .$$

We further assume that the initial probability distribution is given by

$$P(x_1 = 0) = \pi_o, \quad P(x_1 = 1) = \pi_1$$

so that the chain is stationary. That is,

$$P(x_n = 0) = \pi_o, \; P(x_n = 1) = \pi_1, \text{ for all } n = 1,2,\ldots \; .$$

Now let N_{1j} be the j-th run of 0's and N_{2j} the following run of 1's. We have

$$P(N_{11} = k|x_1 = 0) = P(x_1 = 0,\ldots,x_k = 0, \; x_{k+1} = 1|x_1=0)$$

$$= (1-\alpha)^{k-1}\alpha \; ,$$

$$P(N_{11} = k|x_1 = 1) = \sum_{j=1}^{\infty} P(x_i=1, \; i=1,\ldots,j, \; x_{j+i} = 0 \; ,$$

$$i = 1,\ldots,k,$$

$$x_{j+k+1} = 1|x_1 = 1) = \sum_{j=1}^{\infty} \beta^{j-1}(1-\beta)(1-\alpha)^{k-1}\alpha$$

$$= (1-\alpha)^{k-1}\alpha \; .$$

Hence, the unconditional probability

$$P(N_{11} = k) = \alpha(1-\alpha)^{k-1} \; , \quad k = 1,2,\ldots,$$

which is the same as Eq. (2.4) with $p = 1 - \alpha$.

Similarly,

$$P(N_{21} = k) = \beta(1-\beta)^{k-1} \; , \; k = 1,2,\ldots \; .$$

Let

$$Z_n = \sum_{j=1}^{n} x_i, \; Y_n = n - Z_n \; .$$

Then Z_n is the number of surplus months out of n, and

Y_n is the number of deficit months. We know that [6] Y_n is asymptotically ($n\to\infty$) normally distributed with

$$EY_n \sim n \frac{\beta}{\alpha+\beta}, \; \text{var } Y_n \sim n \frac{\alpha\beta(2-\alpha-\beta)}{(\alpha+\beta)^3}$$

(Since $Y_n + Z_n = n$, var $(Y_n + Z_n) = 0$ for all n.)

5.3 Properties of runs. Defining S_1, I_1, S_2, I_2 as before, we note that for $c = 0$, the model outlined above is equivalent to the independent sequence model except that $q = \alpha$, $p = 1 - \alpha$ for (N_1,S_1,I_1) and the same (p,q) will not apply for (N_2,S_2,I_2) unless $\beta = 1 - \alpha$, which is the independent case. Thus, when discussing N_1,S_1,I_1, i.e., negative run-length, run-sum and run-intensity, we set $p = 1 - \alpha$, $q = \alpha$, in the formulas (2.11, 2.12 and 2.13).

5.4 Example.

Station 4.7740 San Diego WB APT

The probability density function of monthly precipitation for this station is given in Fig. 7. We obtain the following values for the parameters:

$\alpha = 0.290$

$\beta = 0.683$

$ES_1 = 1.596$;	$E(S_1)^2 = 4.6066$
$ES_2 = 1.609$;	$E(S_2)^2 = 5.5628$
$EI_1 = 0.4629$;	$E(I_1)^2 = 0.2508$
$EI_2 = 1.0941$;	$E(I_2)^2 = 2.3969$

From the data:

$\hat{E}S_1 = 1.5953$;	$\hat{E}(S_1)^2 = 4.2246$
$\hat{E}S_2 = 1.5987$;	$\hat{E}(S_2)^2 = 5.8756$
$\hat{E}I_1 = 0.4629$;	$\hat{E}(I_1)^2 = 0.2416$
$\hat{E}I_2 = 1.1023$;	$\hat{E}(I_2)^2 = 2.2147$

5.5 Explanation of appendices. In Appendix I the following tables are provided:

1. Table of incomplete gamma function P(a,x) for a = 1(1)14, and x = 1,2,4,6,10.
2. Data used in example of Chapter III.
3. Locations of precipitation stations in Colorado.
4. Table giving numerical values of means and variances of variables related to runs for the annual precipitation series at stations in Colorado.

Appendix II provides numerical values of parameters discussed in Chapter V, such as EX_1^*, α, β, π_o, π_1, EN_1, for monthly precipitation series at stations in the Western United States. Areal distribution of these stations is also provided.

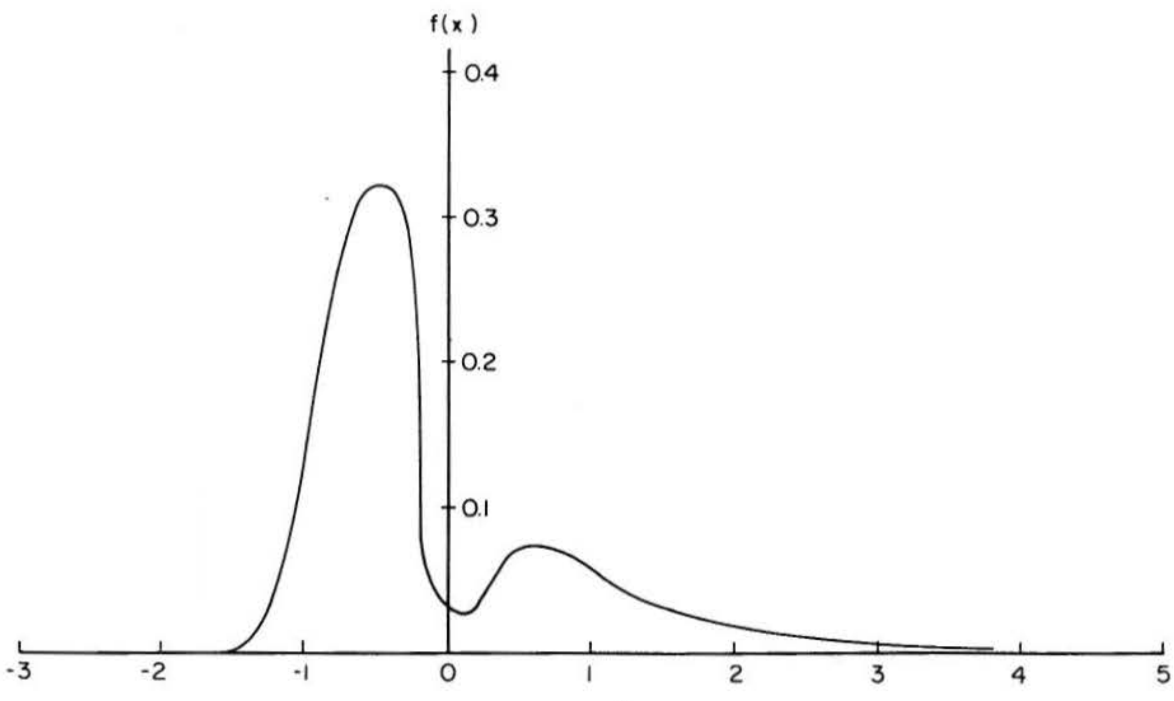

Figure 7 Probability density function of monthly precipitation. Station No. 4.7740. San Diego W.B. APT

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APPENDIX I

TABLE OF INCOMPLETE GAMMA FUNCTION IN THE FORM P(a,x)

USED IN CHAPTER III (FROM K. PEARSON [5]

a \ x	1	2	4	6	10
1	0.63212	0.86498	0.98168	0.99752	0.99995
2	0.26424	0.59430	0.90745	0.98257	0.99990
3	0.08030	0.32362	0.76441	0.93761	0.99959
4	0.01899	0.14317	0.56653	0.84880	0.99668
5	0.00366	0.05295	0.37099	0.71374	0.98663
6	0.00060	0.01686	0.21456	0.55412	0.96183
7	0.00009	0.00483	0.11026	0.39348	0.91243
8	0.00002	0.00139	0.05067	0.25579	0.83558
9	0.00001	0.00027	0.02135	0.15252	0.73226
10	0.00000	0.00005	0.00813	0.08367	0.60625
11	0.00000	0.00001	0.00274	0.04139	0.47678
12	0.00000	0.00000	0.00094	0.02017	0.35680
13	0.00000	0.00000	0.00028	0.00883	0.24992
14	0.00000	0.00000	0.00008	0.00356	0.16383

DATA USED IN EXAMPLE OF CHAPTER III

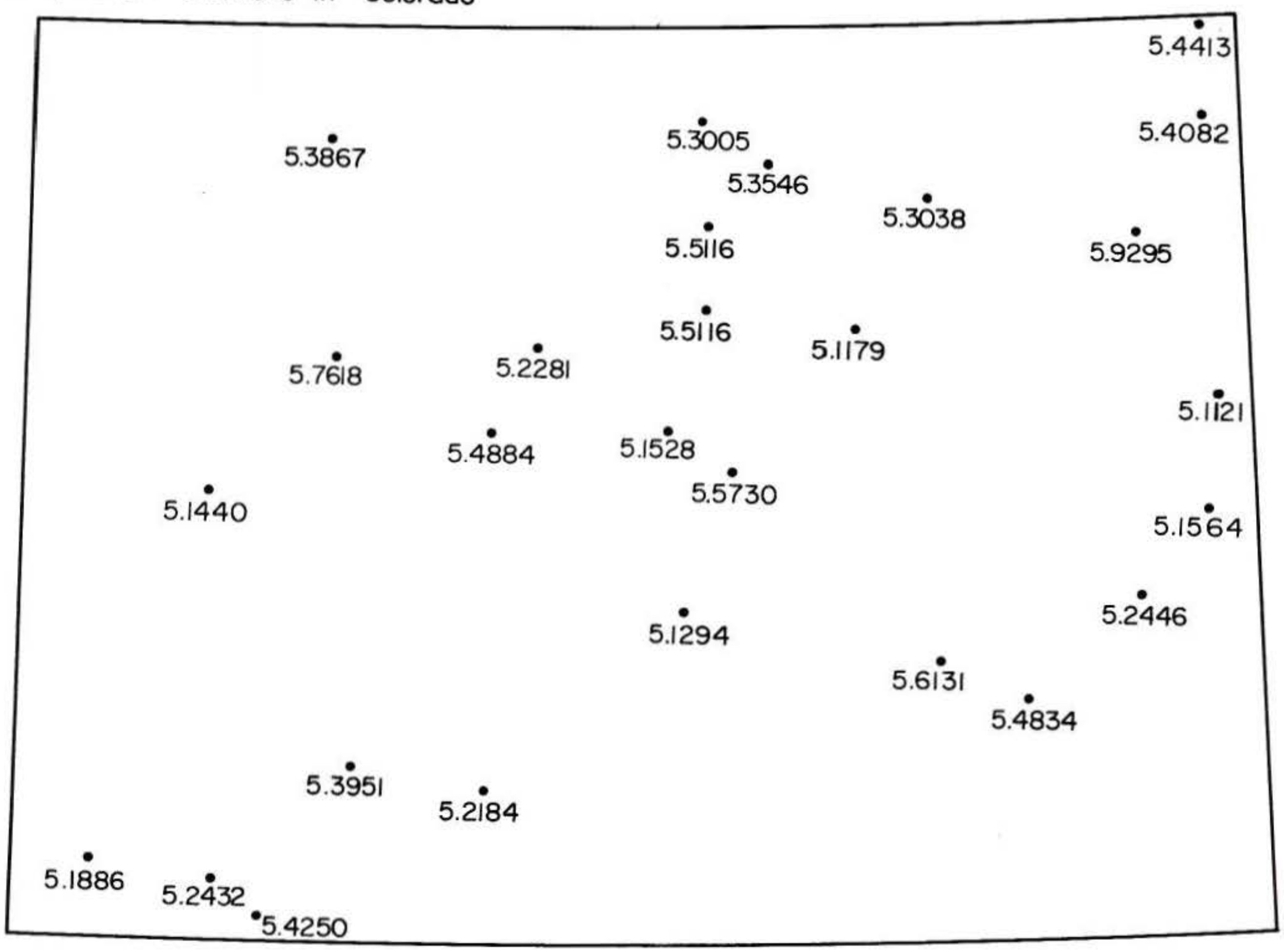
FORT COLLINS, COLO. STATION NO. 5.3005

Year	P	S	Year	P	S
1891	17.50	.71	1926	13.57	- .26
1892	13.58	- .26	1927	15.77	.28
1893	5.65	-2.24	1928	13.54	- .27
1894	12.35	- .56	1929	14.08	- .13
1895	18.07	.86	1930	15.17	.13
1896	15.76	.28	1931	9.88	-1.18
1897	15.24	.15	1932	12.80	- .45
1898	11.03	- .89	1933	15.65	.25
1899	16.19	.39	1934	8.87	-1.43
1900	19.21	1.14	1935	15.95	.33
1901	21.17	1.63	1936	11.81	- .70
1902	18.43	.95	1937	12.93	- .42
1903	11.63	- .74	1938	19.72	1.27
1904	13.13	- .37	1939	7.85	-1.69
1905	19.85	1.30	1940	13.94	- .17
1906	19.88	1.31	1941	17.81	.79
1907	11.64	- .74	1942	21.19	1.63
1908	17.22	.64	1943	12.27	- .58
1909	16.24	.40	1944	13.53	- .27
1910	12.92	- .42	1945	15.73	.27
1911	10.89	- .93	1946	14.11	- .12
1912	19.61	1.24	1947	17.95	.83
1913	15.85	.30	1948	10.45	-1.04
1914	14.31	- .07	1949	18.79	1.04
1915	22.79	2.03	1950	12.70	- .48
1916	13.15	- .36	1951	22.52	1.97
1917	13.72	- .22	1952	12.74	- .47
1918	21.79	1.78	1953	11.42	- .80
1919	10.92	- .92	1954	7.98	-1.66
1920	11.65	- .74	1955	12.97	- .41
1921	14.83	.05	1956	12.19	- .60
1922	9.98	-1.16	1957	19.56	1.23
1923	27.57	3.23	1958	17.44	.70
1924	10.64	- .99	1959	14.67	.01
1925	14.46	- .04			

APPENDIX I (continued)

In the preceding Table, P means the total amount of annual precipitation in inches, and S is the total amount of annual precipitation in standard measure, i.e., $S = \frac{P-\mu}{\sigma}$ where $\mu = 14.62$ and $\sigma = 4.00$.

Precipitation Stations in Colorado



ANNUAL PRECIPITATION
STATIONS OF COLORADO

Station Name	Station No.	**	N	μ	σ	r	E (x_{11}^*)	E (x_{21}^*)	Var (x_{11}^*)	Var (x_{21}^*)	$E(N_1)$	$E(N_2)$	Var (N_1)	Var (N_2)	$E(S_1)$	$E(S_2)$	Var (S_1)	Var (S_2)	$E(I_1)$	$E(I_2)$	Var (I_1)	Var (I_2)
Burlington	5.1121	1	67	17.08	4.81	12.1	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.735	0.735	0.240	0.570	2.235	1.810	2.758	1.466	1.5950	1.355	1.880	1.840	0.735	0.735	0.160	0.423
		3					0.819	0.816	0.236	0.413	1.944	1.882	1.599	1.046	1.5917	1.537	1.275	1.262	0.736	0.853	0.149	0.329
Byers	5.1179	1	25	14.04	4.51	9.7	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.725	0.725	0.234	0.572	2.129	1.886	2.405	1.669	1.5950	1.340	1.875	1.830	0.816	0.832	0.154	0.428
		3					0.816	0.832	0.243	0.273	1.667	2.143	2.221	1.265	1.3600	1.783	1.641	1.508	0.797	0.880	0.192	0.241
Canon City	5.1294	1	36	12.68	3.24	15.3	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.747	0.747	0.250	0.560	2.149	1.871	2.468	1.628	1.5955	1.368	1.882	1.852	0.740	0.740	0.162	0.416
		3					0.659	0.993	0.338	0.629	3.167	2.429	4.803	1.957	2.3050	2.127	2.378	1.311	0.875	0.943	0.318	0.167
Cedaredge	5.1440	1	33	11.80	2.67	19.5	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.765	0.765	0.265	0.550	2.080	1.926	2.246	1.783	1.5955	1.385	1.892	1.870	0.750	0.750	0.168	0.410
		3					0.715	0.889	0.166	0.931	2.667	2.429	3.554	3.957	2.0267	2.031	0.659	2.560	0.823	1.060	0.019	1.085
Cheesman	5.1528	1	54	15.37	3.13	24.1	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.778	0.778	0.280	0.540	2.117	1.895	2.364	1.677	1.5957	1.408	1.903	1.888	0.759	0.759	0.174	0.400
		3					0.860	0.788	0.413	0.318	1.786	1.933	1.310	3.131	1.5364	1.523	0.936	3.209	1.010	0.723	0.411	0.178
Cheyenne Wells	5.1564	1	58	16.14	4.81	11.2	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.730	0.730	0.238	0.570	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.730	0.730	0.158	0.424
		3					0.722	0.934	0.261	0.296	2.357	2.000	3.802	2.000	1.7333	1.468	4.853	0.962	0.575	0.934	0.185	0.232
Cortez	5.1886	1	27	15.13	4.29	9.4	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.725	0.725	0.234	0.572	2.071	1.934	2.217	1.806	1.5950	1.340	1.875	1.830	0.727	0.727	0.154	0.428
		3					0.511	0.966	0.202	0.954	2.000	1.222	1.250	0.104	1.0212	1.181	0.502	1.673	0.484	0.881	0.072	0.884
Del Norte	5.2184	1	32	8.53	2.58	11.0	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.730	0.730	0.238	0.570	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.730	0.730	0.158	0.424
		3					0.748	0.873	0.202	0.703	1.778	1.778	1.283	1.061	1.3300	1.552	0.807	1.875	0.690	0.836	0.343	0.169
Dillon	5.2281	1	44	18.23	4.09	19.9	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.765	0.765	0.265	0.550	2.004	1.996	2.011	1.989	1.5955	1.385	1.893	1.871	0.750	0.750	0.168	0.410
		3					0.625	0.612	0.173	0.326	1.357	1.667	0.659	2.088	0.8936	1.049	0.671	1.356	0.645	0.613	0.084	0.279
Durango	5.2432	1	63	19.13	5.49	12.2	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.735	0.735	0.240	0.570	2.178	1.849	2.566	1.569	1.5900	1.355	1.880	1.840	0.735	0.735	0.160	0.423
		3					0.757	0.833	0.219	0.565	2.286	2.067	4.060	2.861	1.7307	1.721	2.327	2.321	0.767	0.856	0.162	0.606
Eads	5.2446	1	32	13.78	4.19	10.8	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.730	0.730	0.235	0.570	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.730	0.730	0.158	0.424
		3					0.682	1.017	0.363	0.457	2.857	2.000	7.267	3.733	1.9486	2.035	5.393	3.587	0.559	1.000	0.171	0.578
Edgewater	5.2557	1	47	15.53	4.37	12.6	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.737	0.737	0.240	0.568	2.446	1.692	3.735	1.170	1.5950	1.357	1.880	1.841	0.736	0.736	0.160	0.424
		3					0.647	1.066	0.203	0.504	3.200	1.667	2.360	2.665	2.0700	1.777	1.359	3.481	0.659	1.006	0.052	0.584
Fort Collins	5.3005	1	69	14.62	4.00	13.4	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.740	0.740	0.245	0.565	2.166	1.857	2.527	1.760	1.5950	1.360	1.880	1.845	0.738	0.738	0.160	0.420
		3					0.670	0.910	0.243	0.502	1.850	1.524	0.928	0.725	1.2390	1.386	0.816	1.027	0.670	0.910	0.123	0.586
Fort Morgan	5.3038	1	68	13.54	3.50	15.0	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.745	0.745	0.249	0.559	2.198	1.835	2.633	1.532	1.5955	1.368	1.881	1.851	0.740	0.740	0.162	0.416
		3					0.751	0.783	0.287	0.524	1.833	1.842	0.807	0.870	1.3767	1.443	1.006	2.814	0.736	0.685	0.216	0.215
Greeley	5.3546	1	38	12.16	3.43	12.6	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.737	0.737	0.240	0.568	2.446	1.692	3.535	1.170	1.5950	1.357	1.880	1.841	0.736	0.736	0.160	0.424
		3					0.743	0.790	0.267	0.218	2.000	1.333	1.727	1.890	1.7090	1.077	2.313	1.453	0.659	0.784	0.113	0.199
Hayden	'5.3867	1	28	16.13	3.22	25.0	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.780	0.780	0.282	0.540	2.112	1.899	2.347	1.707	1.5957	1.411	1.908	1.890	0.760	0.760	0.175	0.398
		3					0.690	0.828	0.306	0.496	2.571	1.250	2.247	0.437	2.1860	1.064	2.765	0.728	0.768	0.874	0.187	0.593
Hermit	5.3951	1	38	15.43	4.00	14.9	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.745	0.745	0.249	0.559	2.198	1.835	2.633	1.532	1.5955	1.368	1.881	1.851	0.740	0.740	0.162	0.416
		3					0.711	0.823	0.226	0.642	2.222	1.800	4.619	1.560	1.5800	1.480	2.279	1.058	0.752	1.035	0.050	0.748
Holyoke	5.4082	1	32	18.12	4.64	15.3	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.747	0.747	0.250	0.560	2.102	1.908	2.315	1.731	1.5955	1.368	1.882	1.852	0.741	0.741	0.163	0.415
		3					0.801	1.035	0.242	0.207	2.125	1.667	0.909	0.665	1.7012	1.726	0.948	1.232	0.910	0.991	0.090	0.240
Ignacio	5.4250	1	41	15.30	4.24	13.0	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.738	0.738	0.243	0.567	2.149	1.871	2.468	1.728	1.5950	1.360	1.880	1.844	0.737	0.737	0.160	0.420
		3					0.668	0.843	0.272	0.656	2.200	1.900	5.560	1.090	1.3964	1.380	1.822	1.490	0.672	0.894	0.114	0.653
Julesburg	5.4413	1	46	16.82	5.07	11.0	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.730	0.730	0.238	0.570	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.730	0.730	0.158	0.424
		3					0.586	0.810	0.228	0.754	1.833	2.000	2.640	1.667	1.122	1.553	1.6950	1.124	0.540	1.0341	0.168	1.110
Las Animas	5.4834	1	87	12.34	3.69	11.2	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.730	0.730	0.238	0.570	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.730	0.730	0.158	0.424
		3					0.701	0.836	0.367	0.435	1.885	1.462	2.409	0.786	1.3219	1.222	3.889	1.309	0.624	0.820	0.264	0.397
Leadville	5.4884	1	51	18.07	4.77	14.3	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.742	0.742	0.248	0.553	2.160	1.862	2.506	1.605	1.5955	1.367	1.880	1.851	0.740	0.740	0.161	0.417
		3					0.571	0.968	0.140	0.379	2.000	2.077	3.667	5.148	1.1417	2.010	1.283	8.928	0.617	0.747	0.134	0.167
Longmont	5.5116	1	47	13.35	3.93	11.5	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.732	0.732	0.239	0.569	1.973	2.028	1.921	2.084	1.5950	1.348	1.878	1.836	0.731	0.731	0.159	0.423
		3					0.910	0.776	0.172	0.418	1.833	1.923	2.140	1.764	1.6692	1.493	2.163	1.555	0.877	0.878	0.130	0.226
Monument	5.5730	1	48	19.06	4.81	15.7	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.749	0.749	0.252	0.559	2.170	1.840	2.606	1.546	1.5955	1.368	1.882	1.852	0.741	0.741	0.163	0.415
		3					0.772	0.810	0.309	0.437	1.600	1.500	1.173	0.500	1.2353	1.215	1.546	0.854	0.729	0.810	0.284	0.318
Ordway	5.6131	1	37	11.37	3.69	9.5	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.725	0.725	0.234	0.572	2.071	1.934	2.217	1.806	1.5950	1.340	1.875	1.830	0.727	0.727	0.154	0.428
		3					0.678	1.036	0.248	0.615	2.750	2.143	9.437	2.979	2.2290	2.417	6.126	3.885	0.520	1.160	0.117	0.276
Shoshone	5.7618	1	49	17.72	3.64	23.7	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.778	0.778	0.280	0.540	2.554	1.866	2.486	1.617	1.5957	1.405	1.901	1.885	0.758	0.758	0.172	0.401
		3					0.710	0.855	0.213	0.563	2.500	2.182	4.050	3.603	1.6773	1.639	3.310	2.995	0.638	0.868	0.160	0.207
Yuma	5.9295	1	69	17.14	4.51	14.5	0.798	0.798	0.363	0.363	2.000	2.000	2.000	2.000	1.5958	1.5958	2.000	2.000	0.798	0.798	0.252	0.252
		2					0.743	0.⁷⁷3	0.249	0.553	1.899	2.112	1.707	2.350	1.5955	1.368	1.881	1.851	0.740	0.740	0.161	0.417
		3					0.78ʳ	^.ɹ16	0.216	0.421	1.945	1.889	1.828	1.540	1.5317	1.541	1.587	0.834	0.751	0.889	0.006	0.339

**Hypothesis No.

In the preceding table:

Hypothesis 1 corresponds to the case where the original random variable X_i is a sumed to be normally distributed.

In hypothesis 2 the original random variable is assumed to be gamma distributed.

In hypothesis 3 the expected values and variances of all random variables were estimated from the available data,

i.e.,

$$E(Y) = \frac{1}{N} \sum_{i=1}^{N} Y_i$$

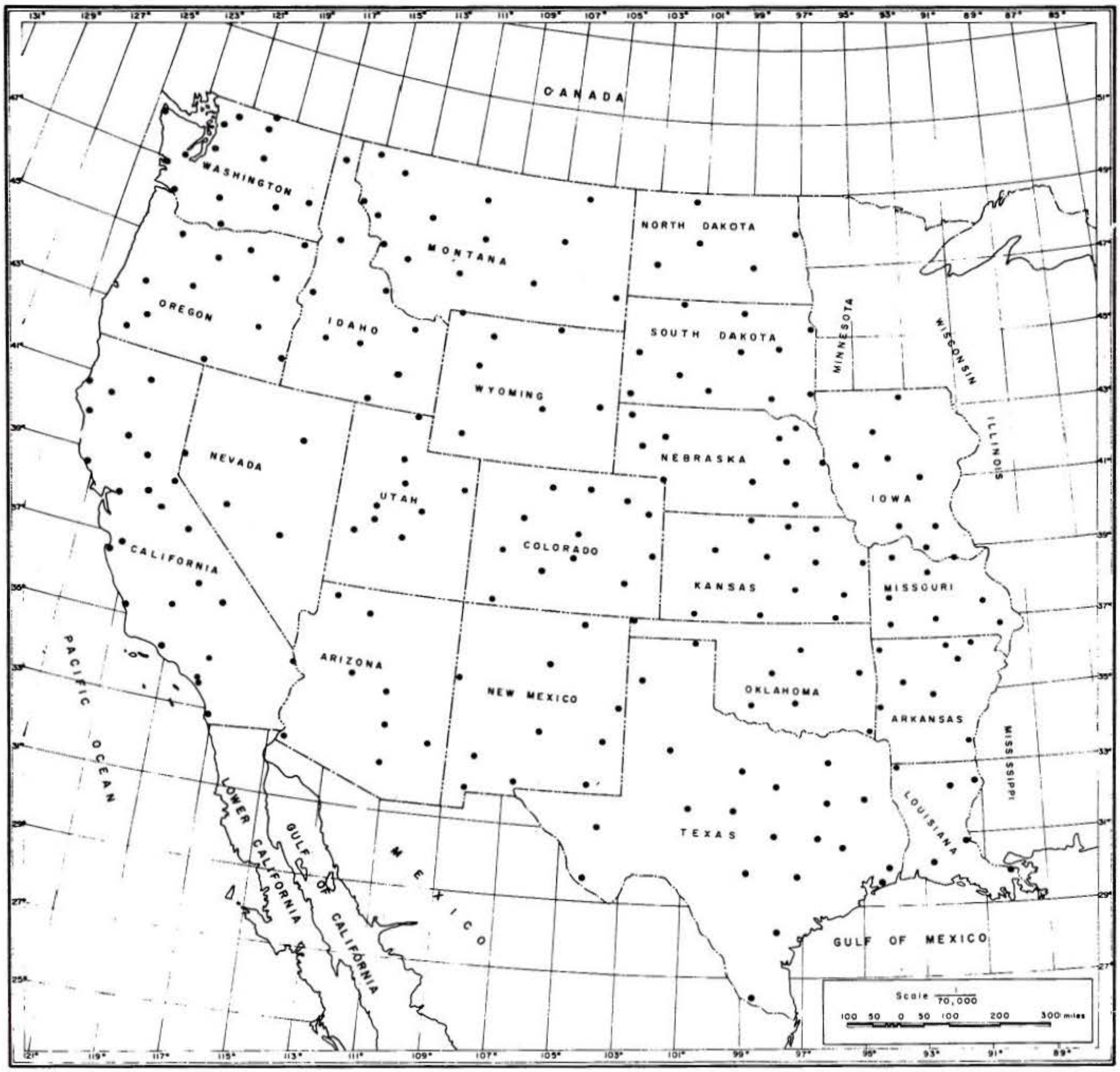
$$E(Y)^2 = \frac{1}{N} \sum_{i=1}^{N} Y_i^2$$

$$\text{Var }(Y) = E(Y)^2 - [E(Y)]^2$$

where N is the number of observations.

APPENDIX II

ANALYSIS OF MONTHLY DATA



Areal distribution of precipitation stations (After Roesner and Yevjevich)

APPENDIX II (continued)

STATION NAME	STATION NO.	$E(x_1^*)$	$E(x_1^*)^2$	$E(x_2^*)$	$E(x_2^*)^2$	α	β	π_0	π_1	$E(N_1)$	$Var(N_1)$
CLIFTON	2.1849	.6570	.5344	1.0371	1.6894	.3639	.5726	.3886	.6114	2.7462	4.8044
GRAND CANYON NATIONAL	2.3591	.6658	.5974	.9163	1.4946	.4101	.5644	.4208	.5792	2.4386	3.5082
MOUNT TRUMBULL	2.5744	.6671	.5843	.9025	1.4839	.4058	.5519	.4237	.5763	2.4643	3.6084
PAYSON RS	2.6320	.9565	.5608	.9571	1.5972	.4215	.6145	.4069	.5931	2.3725	3.2564
PINAL RANCH	2.6561	.6232	.4962	.9917	1.7619	.3716	.5914	.3859	.6141	2.6910	4.5505
PRESCOTT	2.6796	.6339	.5109	.9675	1.6980	.3740	.5726	.3951	.6049	2.6738	4.4752
TUSCON UNIVERSITY OF A	2.8815	.3753	.4216	1.0462	2.0092	.3327	.6050	.3548	.6452	3.0059	6.0294
YUMA CITRUS STATION	2.9652	.4593	.2417	1.2767	3.0122	.2493	.6953	.2639	.7361	4.0114	12.0797
ARKANSAS CITY	3.0234	.6200	.5546	.9401	1.6361	.3938	.5884	.4009	.5991	2.5392	3.9084
BATESVILLE LAND D. NO.	3.0460	.9823	.6453	.9236	1.4416	.3919	.5305	.4249	.5751	2.5515	3.9587
CONWAY	3.1596	.6817	.6343	.9129	1.4594	.4234	.5660	.4280	.5720	2.3616	3.2156
FAYETTEVILLE EXP. STA	3.2444	.0734	.6084	.8892	1.4840	.4372	.5762	.4315	.5685	2.2871	2.9437
MENA	3.4755	.5659	.6042	.9070	1.4919	.4335	.5889	.4240	.5760	2.3067	3.0140
MOUNTAIN HOME 1 NNW	3.5036	.6735	.6094	.9728	1.5086	.3974	.5721	.4099	.5901	2.5161	3.8148
POCAHONTAS	3.5820	.0646	.6022	.9282	1.5197	.4240	.5863	.4197	.5803	2.3586	3.2043
SUBIACO	3.6928	.0733	.6212	.9224	1.4813	.4165	.5705	.4220	.5780	2.4011	3.3642
ANTIOCH F. HILLS	4.0227	.4976	.3390	.9998	2.2910	.3097	.6223	.3323	.6677	3.2289	7.1967
AUBURN	4.0383	.3351	.3984	1.0768	2.1653	.3067	.6173	.3320	.6680	3.2600	7.3676
BIG CREEK POWER HOUSE	4.0755	.3739	.4369	.9680	1.6991	.3481	.5891	.3714	.6286	2.8729	5.3806
BIG SUR STATE PARK	4.0790	.3062	.3688	1.0375	2.2276	.3208	.6593	.3273	.6727	3.1176	6.6021
CHESTER	4.1700	.6227	.4956	.9809	1.7430	.3515	.5556	.3875	.6125	2.8450	5.2488
FORT BRAGG	4.3161	.3649	.4517	1.0195	1.9435	.3397	.6145	.3560	.6440	2.9437	5.7219
FORT ROSS	4.3191	.3170	.3694	1.0115	2.1990	.3215	.6279	.3386	.6614	3.1106	6.5652
HOLLISTER	4.4022	.4723	.3336	1.0402	2.4348	.2981	.6575	.3119	.6881	3.3551	7.9018
LYTLE CREEK POWER HOUS	4.5215	.4925	.3145	1.1167	2.4968	.3122	.7094	.3056	.6944	3.2028	7.0551
MC CLOUD	4.5449	.6062	.4849	.9549	1.7579	.3760	.5923	.3883	.6117	2.6594	4.4131
NEEDLES	4.6118	.4633	.2456	1.1484	2.8197	.2780	.6904	.2871	.7129	3.5976	9.3449
NEWPORT BEACH HARBOR	4.6175	.4770	.3007	1.1742	2.6090	.2695	.6667	.2879	.7121	3.7101	10.0550
OJAI	4.6399	.4777	.2888	1.1602	2.6659	.3004	.7296	.2917	.7083	3.3287	7.7514
SAN DIEGO WB APT.	4.7740	.4629	.2866	1.0941	2.6560	.2895	.6843	.2973	.7027	3.4539	8.4754
SAN LUIS OBISPO POLY	4.7851	.4567	.2945	1.0778	2.0281	.2894	.6831	.2976	.7024	3.4550	8.4818
SCOTIA	4.8045	.6152	.5065	.9413	1.6869	.3937	.6024	.3952	.6048	2.5400	3.9116
SONORA	4.8353	.6428	.4003	1.0163	2.0833	.3398	.6373	.3477	.6523	2.9433	5.7197
TOPANGA PATROL STATION	4.8967	.4889	.3049	1.1715	2.5564	.2874	.6916	.2936	.7064	3.4795	8.6271
TRONA	4.9035	.4980	.2952	1.1033	2.4832	.3127	.6948	.3104	.6896	3.1981	7.0298
TUSTIN IRVIN RANCH	4.9087	.4685	.2910	1.0850	2.0064	.2912	.6743	.3016	.6984	3.4341	8.3592
TWIN LAKES	4.9105	.6218	.5062	.9892	1.7174	.3750	.5989	.3851	.6149	2.6667	4.4444
WASCO	4.9452	.4810	.3195	1.1050	2.5093	.2686	.6171	.3033	.6967	3.7226	10.1353
WEAVERVILLE RS.	4.9490	.6194	.5008	.9797	1.7532	.3582	.5680	.3868	.6132	2.7914	5.0007
WILLOWS	4.9699	.3365	.3835	1.0633	2.1653	.2920	.5775	.3359	.6641	3.4241	8.3003
CANNON CITY	5.1294	.6242	.5107	.9247	1.6878	.3833	.5679	.4030	.5970	2.6087	4.1966
CHEESMAN	5.1528	.9401	.5727	.8707	1.5327	.4612	.6182	.4272	.5728	2.1685	2.5338
CHEYENNE WELLS	5.1564	.6092	.4809	.9504	1.7677	.3782	.5806	.3912	.6088	2.6441	4.3470
DEL NORTE	5.2184	.9414	.5213	.9543	1.6370	.3689	.5460	.4032	.5968	2.7111	4.6390
DURANGO	5.2432	.5453	.5519	.9369	1.6135	.3881	.5635	.4078	.5922	2.5769	4.0636
FORT COLLINS	5.3005	.6101	.4907	.9376	1.7424	.3734	.5724	.3948	.6052	2.6784	4.4953
FORT MORGAN	5.3038	.6237	.5143	.8899	1.6481	.3760	.5393	.4120	.5880	2.6458	4.3546
JULESBURG	5.4413	.9466	.5491	.9245	1.5951	.3726	.5331	.4116	.5884	2.6822	4.5119
LAS ANIMAS	5.4834	.6189	.4776	.9822	1.8016	.3613	.5734	.3865	.6135	2.7680	4.8938
MONTROSE NO. 2	5.5722	.6773	.6095	.9208	1.4972	.4091	.5562	.4238	.5762	2.4444	3.5309
SHOSHONE	5.7618	.6910	.6604	.8811	1.3885	.3965	.5056	.4395	.5605	2.5221	3.8387
STEAMBOAT SPRINGS	5.7936	.7034	.6964	.8876	1.3317	.4047	.5110	.4419	.5581	2.4710	3.6349
YUMA	5.9295	.6642	.5602	.9254	1.5771	.3931	.5465	.4184	.5816	2.5436	3.9263
ABERDEEN EXPT. STATION	10.0010	.6321	.5068	.9118	1.6503	.4110	.5929	.4094	.5906	2.4328	3.4859
ARROWROCK DAM	10.0448	.9512	.5739	.9310	1.5577	.3699	.5309	.4107	.5893	2.7031	4.6038
CAMBRIDGE	10.1408	.9408	.5341	.9693	1.6613	.3842	.5827	.3974	.6026	2.6025	4.1704
DUBOIS EXPT. STATION	10.2707	.6471	.5451	.8981	1.5701	.3824	.5330	.4177	.5823	2.6154	4.2249
HAILEY RS	10.3942	.6517	.5432	.9556	1.6265	.3747	.5512	.4047	.5953	2.6691	4.4548
KOOSKIA	10.5011	.7115	.6826	.8913	1.3542	.4150	.5216	.4431	.5569	2.4097	3.3970
OAKLEY	10.6542	.0604	.5823	.9534	1.5666	.3853	.5562	.4092	.5908	2.5956	4.1417
SALMON	10.8076	.7055	.6471	.8872	1.3846	.4567	.5743	.4430	.5570	2.1897	2.6040
SANDPOINT EXPT. STATIO	10.8137	.7000	.6673	.8849	1.3723	.4000	.5057	.4417	.5583	2.5000	3.7500
ATLANTIC 1 NE	13.0366	.0875	.6328	.9347	1.4605	.3935	.5322	.4232	.5768	2.5638	3.9971
DES MOINES WB CITY	13.2208	.0717	.6002	.9174	1.5175	.3930	.5368	.4227	.5773	2.5442	3.9289

STATION NAME	STATION NO.	$E(x_1^*)$	$E(x_1^*)^2$	$E(x_2^*)$	$E(x_2^*)^2$	α	β	π_0	π_1	$E(N_1)$	$Var(N_1)$
MASON CITY 3 N	13.5230	.0701	.6248	.9136	1.4676	.4250	.5743	.4253	.5747	2.3529	3.1834
OTTUMWA	13.6391	.7114	.6590	.9238	1.4890	.4143	.5380	.4350	.5650	2.4136	3.4119
ROCKWELL CITY	13.7161	.0666	.6080	.9213	1.5036	.3916	.5396	.4205	.5795	2.5537	3.9676
CONCORDIA WB CITY	14.1769	.0585	.5598	.9713	1.6292	.3952	.5900	.4011	.5989	2.5305	3.8730
COUNCIL GROVE	14.1866	.0317	.5386	.9387	1.6316	.3861	.5720	.4030	.5970	2.5903	4.1193
ELLSWORTH	14.2693	.0493	.5727	.9777	1.6799	.3843	.5843	.4077	.5923	2.7448	4.7893
HOLTON	14.3759	.0562	.5680	.9454	1.5715	.3794	.5466	.4097	.5903	2.6357	4.3110
LA CYGNE	14.4421	.0301	.5496	1.0137	1.6317	.3649	.5839	.3846	.6154	2.7407	4.7709
MEDICINE LODGE	14.5173	.0348	.5215	.9591	1.6860	.3494	.5804	.3988	.6012	2.5707	4.0377
PHILLIPSBURG	14.6374	.0603	.5776	.9314	1.5506	.4037	.5652	.4167	.5833	2.4769	3.6582
PLAINS	14.6427	.0321	.5103	.9731	1.7040	.3558	.5496	.3930	.6070	2.8106	5.0889
QUINTER	14.6637	.0458	.5271	.9606	1.6290	.3935	.5874	.4012	.5988	2.5412	3.4164
SEDAN	14.7305	.0443	.5581	.9268	1.6036	.4291	.6043	.4101	.5899	2.3865	3.2864
SEDGWICK	14.7313	.0471	.5478	.9201	1.5880	.3871	.5464	.4138	.5862	2.5833	4.0903
TORONTO	14.8186	.0227	.5284	.9452	1.6758	.4082	.6197	.3971	.6029	2.4497	3.5515
CALHOUN EXP. STATION	16.1411	.0561	.5445	.9125	1.5343	.3921	.5449	.4185	.5815	2.5503	3.9536
JENNINGS	16.4700	.0239	.5303	.9225	1.6591	.3858	.5705	.4034	.5966	2.5940	4.1263
MELVILLE	16.6117	.0598	.5942	.8984	1.5247	.4395	.5986	.4234	.5766	2.2756	2.9026
NEW ORLEANS WB CITY	16.6659	.0395	.5726	.9159	1.5894	.4075	.5844	.4108	.5892	2.4542	3.5689
PLAIN DEALING	16.7344	.0406	.5577	.8833	1.5744	.4013	.5519	.4210	.5790	2.4920	3.7180
TALLULAH DELTA LAB.	16.8923	.0943	.4452	.9864	1.7760	.3851	.6342	.3760	.6240	2.5968	4.1465
CAPRINGER MILLS	23.1304	.0107	.5355	.9564	1.6519	.3655	.5723	.3897	.6103	2.7363	4.7509
CHILLICOTHE 2S	23.1588	.0650	.6051	.8806	1.4670	.4150	.5495	.4302	.5698	2.4098	3.3975
DEXTER	23.2235	.0316	.5548	.9177	1.5896	.3878	.5611	.4087	.5913	2.5784	4.0699
ELDON	23.2503	.0357	.5516	.9117	1.6094	.3697	.5266	.4116	.5884	2.7047	4.6107
FAYETTE	23.2823	.0745	.5424	.8868	1.4397	.4035	.5305	.4320	.5680	2.4785	3.8643
FREDERICKTOWN	23.3036	.0502	.5782	.9367	1.5400	.3882	.5593	.4097	.5903	2.5758	4.0588
HERMANN	23.3793	.0716	.6143	.9824	1.4749	.4164	.5471	.4322	.5678	2.4016	3.3662
NEOSHO	23.5976	.0305	.5613	.9216	1.6048	.4007	.5812	.4081	.5919	2.4955	3.7320
SHELBINA	23.7720	.0649	.6103	.9846	1.4837	.4031	.5378	.4285	.5715	2.4811	3.6747
WARRENSBURG	23.8712	.0460	.5820	.9165	1.5617	.3635	.5157	.4134	.5866	2.7513	4.8182
WILLOW SPRINGS	23.8995	.0793	.6110	.8834	1.4436	.3944	.5104	.4359	.5641	2.5354	3.8927
AUGUSTA	24.0364	.0744	.5918	.9441	1.5099	.3950	.5556	.4155	.5845	2.5319	3.8787
BALLANTINE	24.0432	.0411	.5433	.9051	1.5894	.4097	.5784	.4146	.5854	2.4407	3.5162
BIG SANDY	24.0770	.0311	.5288	.9887	1.6671	.3675	.6069	.3896	.6104	2.5810	4.0804
BOZEMAN AGRI. COLLEGE	24.1044	.0533	.6016	.8675	1.4804	.4156	.5518	.4296	.5704	2.4061	3.3831
EAST ANACONDA	24.2604	.0649	.6071	.9036	1.4872	.3984	.5374	.4258	.5742	2.5099	3.7898
EKALAKA	24.2689	.0682	.6006	.8984	1.4940	.4078	.5495	.4278	.5722	2.4522	3.5612
FORTINE 1 NNE	24.3139	.0731	.6150	.9260	1.4607	.3838	.5279	.4209	.5791	2.6056	4.1843
HAMILTON	24.3885	.0835	.6105	.9569	1.4765	.3593	.5030	.4167	.5833	2.7831	4.9627
HAUGAN	24.3984	.0478	.6625	.8905	1.3833	.3994	.5118	.4383	.5617	2.5039	3.7655
JORDAN	24.4522	.0523	.5435	.9344	1.5728	.4104	.5850	.4123	.5877	2.4368	3.5011
LUSTRE 4 NNW	24.5285	.0445	.5507	.9859	1.6265	.3710	.5699	.3943	.6057	2.6952	4.5691
MOCCASIN EXPT. STATION	24.5761	.0934	.5415	.9170	1.3907	.4537	.6000	.4306	.5694	2.2043	2.6546
SAINT IGNATIUS	24.7286	.0955	.6407	.9001	1.4145	.4176	.5421	.4351	.5649	2.3946	3.3393
WEST GLACIER	24.8809	.0808	.6632	.9142	1.3597	.4208	.5611	.4286	.5714	2.3762	3.2703
BLAIR	25.0930	.0465	.5949	.9289	1.5296	.4205	.5842	.4185	.5815	2.3783	3.2779
BRIDGEPORT	25.1145	.0388	.5514	.9395	1.6174	.3844	.5694	.4048	.5952	2.6012	4.1649
CRETE	25.2020	.0400	.5653	.9047	1.5392	.4199	.5756	.4218	.5782	2.3814	3.2895
EWING	25.2805	.0729	.5982	.9232	1.5156	.3941	.5427	.4216	.5784	2.5376	3.9020
FORT ROBINSON	25.3015	.0339	.5356	.9239	1.6449	.3965	.5660	.4074	.5926	2.5607	3.9967
GENOA	25.3185	.0665	.5959	.9358	1.5375	.4105	.5859	.4167	.5833	2.3896	3.3204
HARTINGTON	25.3630	.0955	.6346	.9222	1.4564	.4492	.5955	.4300	.5700	2.2264	2.7305
PURDUM	25.4970	.0355	.5623	.9220	1.5912	.3908	.5669	.4080	.5920	2.5570	3.9895
RAVENNA	25.7046	.0738	.5943	.9203	1.5236	.4087	.5562	.4227	.5773	2.4468	3.5401
ADAVEN	26.0044	.0310	.5051	.9513	1.6803	.3626	.5771	.3988	.6012	2.6121	4.2108
ELKO WB APT.	26.2573	.0660	.4034	1.0148	2.0381	.3424	.6138	.3581	.6419	2.9206	5.6104
KING	26.5166	.0702	.3565	1.1007	2.1937	.3310	.6452	.3391	.6609	3.0216	6.1084
RENO WB APT.	26.6779	.0592	.3746	1.0619	2.1774	.3301	.6386	.3407	.6593	3.0298	6.1498
CARRIZOZO	29.1515	.0178	.4743	.9181	1.7275	.3700	.5498	.4022	.5978	2.7029	4.6028
CIMARRON	29.1813	.0141	.4830	.9651	1.7673	.3708	.5811	.3895	.6105	2.6968	4.5758
CLOVIS	29.1939	.0866	.4281	1.0287	1.9434	.3503	.6103	.3646	.6354	2.8550	5.2958
ELIDA	29.2854	.0010	.4560	1.0299	1.8744	.3460	.5909	.3693	.6307	2.8898	5.4613
FORT BAYARD	29.3265	.0200	.4484	.8372	1.7434	.3692	.5571	.3986	.6014	2.7083	4.6267
HACHITA	29.3775	.0032	.4524	1.0014	1.8512	.3717	.6157	.3765	.6235	2.6901	4.5497
LAKE AVALON	29.4736	.0647	.3832	.9939	2.0236	.3665	.6432	.3630	.6370	2.7287	4.7170

STATION NAME	STATION NO.	$E(x_1^*)$	$E(x_1^*)^2$	$E(x_2^*)$	$E(x_2^*)^2$	α	β	π_0	π_1	$E(N_1)$	$Var(N_1)$
PECOS RS	29.6676	.6591	.5660	.9313	1.5401	.4154	.5847	.4153	.5847	2.4274	3.3882
STATE UNIVERSITY	29.8535	.5914	.4231	.9786	1.9292	.3610	.5965	.3770	.6230	2.7704	4.9046
ZUNI FAA AP.	29.9897	.6638	.5633	.9433	1.5679	.3765	.6371	.4121	.5879	2.6557	4.3972
DICKENSON EXPT. STATIO	32.2188	.6389	.5344	.9565	1.6574	.3740	.5663	.4010	.5990	2.6383	4.3223
GRAND FORKS U.	32.3621	.6476	.5584	.9066	1.5824	.4203	.5872	.4172	.5828	2.3793	3.2818
JAMESTOWN ST. HOSP.	32.4418	.6375	.5537	.9730	1.5451	.3793	.5776	.3964	.6036	2.6364	4.3140
MAX	32.5636	.6915	.6392	.8841	1.3803	.4455	.5669	.4401	.5599	2.2444	2.7931
MOHALL	32.6025	.6701	.6041	.9073	1.5245	.4144	.5934	.4112	.5888	2.4133	3.4106
GEARY	34.3497	.6618	.5831	.9524	1.5595	.4179	.6000	.4105	.5895	2.3931	3.3338
IDABEL	34.4451	.6815	.6281	.9073	1.4296	.4206	.5575	.4300	.5700	2.3774	3.2752
KENTON	34.4786	.5803	.4336	1.0083	1.9306	.3414	.5932	.3653	.6347	2.9295	5.5524
PAULS VALLEY	34.6926	.6829	.5668	.9081	1.5549	.4051	.5561	.4232	.5768	2.4668	3.6260
PERRY	34.7012	.6409	.5481	.9287	1.5731	.4272	.6106	.4094	.5906	2.3407	3.1380
WEDDER FALLS	34.9445	.6601	.5880	.9242	1.5301	.4078	.5896	.4173	.5827	2.4520	3.5602
WICHITA MT. WLR	34.9629	.6769	.5924	.9393	1.5173	.4095	.5733	.4167	.5833	2.4419	3.5208
ANTELOPE 1 N	35.0197	.6712	.5656	.8962	1.4804	.4130	.5514	.4282	.5718	2.4216	3.4424
BEND	35.0694	.5245	.4102	.9915	1.7290	.3924	.6197	.3477	.6123	2.5486	3.9468
COTTAGE GROVE 1 S	35.1897	.6625	.6124	.9203	1.4952	.3974	.5560	.4145	.5814	2.5164	3.8158
DANNER	35.2135	.6735	.6070	.9429	1.4792	.3667	.5133	.4167	.5833	2.7273	4.7107
ESTACHADA 2 SE	35.2691	.6415	.6700	.9306	1.3991	.4450	.5468	.4255	.5745	2.4490	3.4268
GRANTS PASS	35.3445	.6044	.4931	.9490	1.7478	.3728	.6000	.3819	.6181	2.6970	4.5767
HEPPNER	35.3827	.6751	.6194	.9752	1.5093	.4026	.5815	.4091	.5909	2.4841	3.5866
LAKEVIEW	35.4470	.6240	.5250	.9671	1.6774	.3811	.5859	.3941	.6059	2.6241	4.2616
MINAM 7 NE	35.5610	.6986	.6585	.9148	1.4019	.3833	.5038	.4321	.5679	2.6090	4.1980
PROSPECT 2 SW	35.6907	.6325	.5751	.9206	1.5761	.3820	.5500	.4072	.5928	2.6101	4.2362
ROCK CREEK	35.7250	.7130	.6908	.9036	1.3305	.4109	.5207	.4411	.5589	2.4336	3.4889
WARM SPRINGS RESERVOIR	35.9046	.6675	.5673	.9238	1.5175	.3971	.5497	.4194	.5806	2.5181	3.8226
ARMOUR	39.0296	.6671	.5873	.9442	1.5408	.3792	.5387	.4140	.5860	2.6369	4.3164
COTTON WOOD	39.1972	.6244	.5077	.9418	1.6933	.3614	.5469	.3979	.6021	2.7669	4.8849
EUREKA	39.2797	.6274	.5342	.9386	1.6414	.3770	.5622	.4014	.5986	2.6525	4.3832
HIGHMORE 1 W	39.3832	.6510	.5620	.9672	1.6078	.4063	.6022	.4029	.5971	2.4615	3.5976
HOT SPRINGS	39.4007	.6187	.5201	.9372	1.6868	.3447	.5221	.3977	.6023	2.9014	5.5168
LAUELLE 7 NE	39.4661	.6168	.5257	.9312	1.6709	.3896	.5882	.3984	.6016	2.5667	4.0211
LEMMON	39.4864	.6194	.5171	1.0241	1.7381	.3507	.5930	.3769	.6231	2.7881	4.9856
MILBANK	39.5536	.6496	.5769	.9183	1.5641	.3928	.5552	.4143	.5857	2.5459	3.9358
SIOUX FALL WB AP	39.7667	.6702	.6255	.9448	1.4876	.4049	.5629	.4184	.5816	2.4697	3.6297
VALE	39.8552	.6148	.5051	.9198	1.6942	.3957	.5904	.4013	.5987	2.5270	3.8588
WOOD	39.9442	.6255	.5323	.9901	1.6895	.3796	.5991	.3879	.6121	2.6343	4.3054
ALBANY	41.0120	.6456	.5312	.9400	1.6515	.3950	.5740	.4076	.5924	2.5315	3.8771
BALMORHEA EXP. STATION	41.0498	.6008	.4415	1.0062	1.8680	.3345	.5576	.3750	.6250	2.9892	5.9464
BEAUMONT	41.0611	.6343	.5504	.9295	1.6265	.4021	.5879	.4061	.5939	2.4872	3.6989
BROWNWOOD	41.1136	.6320	.5209	1.0000	1.7240	.3700	.5854	.3873	.6127	2.7027	4.6019
CORSICANA	41.2019	.6592	.5908	.9313	1.5499	.3852	.5442	.4144	.5856	2.5961	4.1435
FLATONIA	41.3183	.6423	.5350	.9411	1.6349	.4206	.6148	.4062	.5938	2.3774	3.2745
GALVESTON WB CITY	41.3430	.6309	.5160	.9620	1.7096	.3767	.5745	.3961	.6039	2.6543	4.3911
GEORGE WEST	41.3506	.6049	.4599	1.0366	1.8693	.3754	.6414	.3692	.6308	2.6641	4.4332
GREENVILLE 2 SW	41.3734	.6310	.5380	.9532	1.6584	.3850	.5801	.3989	.6011	2.5976	4.1498
HENDERSON	41.4081	.6703	.6200	.9021	1.4663	.4274	.5736	.4270	.5730	2.3399	3.1351
KERRVILLE	41.4780	.6260	.5016	.9192	1.6948	.3750	.5506	.4051	.5949	2.6667	4.4444
LAMPASAS	41.5018	.6438	.5404	.9447	1.6370	.3800	.5576	.4053	.5947	2.6313	4.2924
MISSION	41.5972	.5693	.4137	1.0477	2.0080	.3408	.6250	.3529	.6471	2.9340	5.6742
PERRYTON	41.6950	.6347	.5214	.9211	1.6435	.3959	.5745	.4080	.5920	2.5259	3.8544
POST	41.7206	.5900	.4504	1.0046	1.8665	.3837	.6429	.3728	.6262	2.6063	4.1865
PRESIDIO	41.7262	.5718	.4037	1.1018	2.0515	.3291	.6371	.3406	.6594	3.0385	6.1938
RIVERSIDE	41.7651	.6613	.5968	.9314	1.5297	.4098	.5669	.4159	.5841	2.4691	3.6275
STERLING CITY	41.8630	.6351	.5075	1.0097	1.8987	.3756	.5971	.3861	.6139	2.6627	4.4271
VALLEY JUNCTION	41.9280	.6421	.5521	.9151	1.5995	.3643	.5192	.4124	.5876	2.7450	4.7899
VEGA	41.9330	.6102	.4774	.9367	1.7176	.3670	.5603	.3958	.6042	2.7250	4.7006
WEATHERFORD	41.9532	.6257	.5245	.9222	1.6634	.3674	.5401	.4049	.5951	2.7216	4.4855
DESSERT	42.2101	.6535	.5387	.9305	1.6171	.4163	.5927	.4126	.5874	2.4022	3.3685
FORT DUCHESNE	42.2998	.6441	.5184	.9559	1.6778	.3752	.5509	.4026	.5974	2.6649	4.4369
HIAWATHA	42.3896	.6240	.5357	1.0155	1.6865	.3564	.5754	.3825	.6175	2.8058	5.0668
KANAB POWER HOUSE	42.4508	.6502	.5431	.9103	1.5701	.4322	.6051	.4167	.5833	2.3136	3.0390
LOA	42.5148	.5917	.4555	.9648	1.8417	.3608	.5845	.3801	.6199	2.7712	4.9085
MILFORD WB APT.	42.5694	.6526	.5618	.9651	1.5917	.4190	.6215	.4027	.5973	2.3864	3.3086
RICHMOND	42.7271	.7090	.6779	.9068	1.3694	.4242	.5444	.4380	.5620	2.3571	3.1990

STATION NAME	STATION NO.	$E(x_1^*)$	$E(x_1^*)^2$	$E(x_2^*)$	$E(x_2^*)^2$	α	β	π_0	π_1	$E(N_1)$	$Var(N_1)$
SPANISH FORK POWER HOU	42.8119	.7038	.6732	.8957	1.3704	.4286	.5472	.4392	.5608	2.3333	3.1111
TOOELE	42.8771	.7066	.6702	.8802	1.3797	.4531	.5643	.4453	.5547	2.2073	2.6647
BROOKLYN	45.0917	.7238	.7046	.8846	1.2869	.4242	.5215	.4486	.5514	2.3571	3.1990
CEDAR LAKE	45.1223	.7214	.7351	.8331	1.2687	.4638	.5370	.4634	.5366	2.1561	2.4926
CHELAN	45.1350	.6446	.5335	.9679	1.6642	.3783	.5680	.3998	.6002	2.6436	4.3451
COLFAX 1 NW	45.1586	.6209	.5511	.9370	1.6410	.3855	.5818	.3986	.6014	2.5938	4.1338
GOLDENDALE	45.3222	.6310	.5333	.9665	1.6641	.3912	.5992	.3950	.6050	2.5563	3.9785
HATTON 8 E	45.3546	.6279	.5304	.9291	1.6506	.3840	.5699	.4026	.5974	2.6039	4.1764
LONGVIEW	45.4769	.7467	.7454	.8582	1.2329	.4589	.5297	.4642	.5358	2.1792	2.5699
NEWHALEM	45.5840	.7471	.7634	.8586	1.2123	.4502	.5198	.4641	.5359	2.2212	2.7124
RIMROCK TETON DAM	45.7038	.6392	.5382	.9256	1.6207	.3812	.5520	.4085	.5915	2.6232	4.2579
SEDRO WOLLEY 1 E	45.7507	.7057	.7074	.8790	1.3294	.4437	.5526	.4453	.5547	2.2540	2.8264
SHELTON	45.7584	.7031	.6724	.8789	1.3345	.4600	.5750	.4444	.5556	2.1739	2.5520
SUNNYSIDE	45.8207	.6276	.5108	1.0021	1.7418	.3655	.5850	.3845	.6155	2.7360	4.7495
TATOOSH ISLAND WB	45.8332	.7063	.7033	.8894	1.3493	.4350	.5488	.4421	.5579	2.2991	2.9868
WINTHROP 1 WSW	45.9376	.6599	.5520	.9320	1.5709	.3832	.5412	.4145	.5855	2.6095	4.2001
BUFFALO BILL DAM	48.1175	.6341	.5135	.9335	1.6647	.3965	.5855	.4038	.5962	2.5221	3.8387
DUBOIS	48.2715	.6449	.5385	.9291	1.6140	.3706	.5359	.4088	.5912	2.6984	4.5830
GREEN RIVER	48.4065	.6373	.5488	.9644	1.6198	.3841	.5812	.3979	.6021	2.6036	4.1751
LUSK	48.5830	.6589	.5874	.8876	1.5179	.3958	.5316	.4268	.5732	2.5266	3.8572
PATHFINDER DAM	48.7105	.6567	.5662	.9038	1.5573	.4245	.5842	.4208	.5792	2.3559	3.1945
SHERIDAN FIELD STATION	48.8160	.6465	.5651	.9345	1.5718	.4033	.5849	.4081	.5919	2.4797	3.6691
YELLOWSTONE PARK	48.9905	.6173	.5314	.9109	1.6571	.3612	.5330	.4039	.5961	2.7688	4.8975

APPENDIX II (continued)

STATION NAME	STATION NO.	EN_2	Var N_2	ES_1	Var S_1	ES_2	Var S_2	EI_1	Var I_1	EI_2	Var I_2
CLIFTON	2.1849	1.7464	1.3035	1.8056	2.3561	1.8111	2.4670	.6570	.0594	1.0371	.4555
GRAND CANYON NATIONAL	2.3591	1.7719	1.3978	1.6237	1.9311	1.6237	2.3090	.6658	.0955	.9163	.4854
MOUNT TRUMBULL	2.5744	1.8118	1.4707	1.6439	1.9491	1.6351	2.4108	.6671	.0858	.9025	.4901
PAYSON HS	2.6320	1.6275	1.0211	1.5577	1.6640	1.5577	2.0440	.6565	.0691	.9571	.5287
PINAL RANCH	2.6561	1.6910	1.1685	1.6769	2.0574	1.6769	2.4655	.6232	.0632	.9917	.5918
PRESCOTT	2.6796	1.7465	1.3037	1.6949	2.0898	1.6898	2.5510	.6339	.0641	.9675	.5691
TUSCON UNIVERSITY OF A	2.8815	1.6529	1.0793	1.7292	2.2678	1.7292	2.6932	.5753	.0497	1.0462	.7041
YUMA CITRUS STATION	2.9652	1.4382	.6302	1.8425	2.6717	1.8361	3.0167	.4593	.0147	1.2767	1.1472
ARKANSAS CITY	3.0234	1.6995	1.1088	1.5945	1.9482	1.5977	2.3224	.6280	.0970	.9401	.5673
BATESVILLE LAND O. NO.	3.0460	1.8848	1.6078	1.7409	2.3016	1.7409	2.5320	.6823	.1085	.9236	.4215
CONWAY	3.1596	1.7668	1.3348	1.6098	1.8948	1.6129	2.2351	.6817	.1071	.9129	.4646
FAYETTESVILLE EXP. STA	3.2444	1.7356	1.2767	1.5401	1.6891	1.5432	2.2128	.6734	.0996	.8892	.5197
MENA	3.4756	1.6980	1.1852	1.5359	1.7074	1.5401	2.1113	.6659	.1029	.9070	.5076
MOUNTAIN HOME 1 NNW	3.5036	1.7480	1.3074	1.6946	2.1224	1.7005	2.2201	.6735	.0948	.9728	.4198
POCAHONTAS	3.5820	1.7056	1.2034	1.5798	1.7998	1.5831	2.1525	.6698	.0970	.9282	.4950
SUBIACO	3.6928	1.7527	1.3194	1.6168	1.9283	1.6168	2.2275	.6733	.1049	.9224	.4700
ANTIOCH F. HILLS	4.0227	1.6070	.9754	1.6067	2.0770	1.6067	3.0502	.4976	.0480	.9998	1.0092
AUBURN	4.0383	1.6200	1.0944	1.7444	2.4683	1.7444	2.7940	.5351	.0576	1.0768	.7827
BIG CREEK POWER HOUSE	4.0755	1.6975	1.1840	1.6488	2.0810	1.6431	2.7272	.5739	.0606	.9680	.7231
BIG SUR STATE PARK	4.0790	1.5167	.7836	1.5700	2.0426	1.5735	2.5894	.5062	.0604	1.0375	.9280
CHESTER	4.1700	1.8000	1.4900	1.7717	2.3422	1.7656	2.7910	.6227	.0611	.9809	.5737
FORT BRAGG	4.3161	1.6273	1.0209	1.6631	2.2163	1.6591	2.5324	.5649	.0736	1.0195	.7018
FORT ROSS	4.3191	1.5926	.9438	1.6082	2.0724	1.6110	2.8383	.5170	.0549	1.0115	.9234
HOLLISTER	4.4022	1.5209	.7923	1.5647	2.1337	1.5821	2.9086	.4723	.0568	1.0402	1.0856
LYTLE CREEK POWER HOUS	4.5215	1.4097	.5776	1.5775	1.9418	1.5743	2.4793	.4925	.0380	1.1167	1.0458
MC CLOUD	4.5449	1.6884	1.1923	1.6122	1.9348	1.6122	2.4917	.6062	.0692	.9549	.6453
NEEDLES	4.6118	1.4485	.6496	1.6666	2.1171	1.6635	3.0306	.4633	.0153	1.1484	1.2399
NEWPORT BEACH HARBOR	4.6175	1.5000	.7500	1.7698	2.5593	1.7513	2.8750	.4770	.0354	1.1742	.9953
OJAI	4.6399	1.3706	.5080	1.5902	1.9707	1.5902	2.4929	.4777	.0313	1.1602	1.1229
SAN DIEGO WB APT.	4.7740	1.4613	.6740	1.5987	2.0056	1.5987	2.9388	.4629	.0365	1.0941	1.1998
SAN LUIS OBISPO POLY	4.7851	1.4640	.6792	1.5778	2.0069	1.5778	2.9358	.4567	.0434	1.0778	1.2047
SCOTIA	4.8045	1.6600	1.0956	1.5625	1.8055	1.5625	2.2935	.6152	.0775	.9413	.6119
SONORA	4.8353	1.5692	.8933	1.5978	1.9964	1.5948	2.5711	.5428	.0587	1.0163	.8315
TOPANGA PATROL STATION	4.8967	1.4459	.6448	1.7010	2.2912	1.6939	2.5913	.4889	.0331	1.1715	.9759
TRONA	4.9035	1.4393	.6322	1.5926	1.9942	1.5880	2.5915	.4980	.0250	1.1033	1.0493
TUSTIN IRVIN RANCH	4.9087	1.4829	.7161	1.6090	2.0005	1.6090	2.9564	.4685	.0362	1.0850	1.1628
TWIN LAKES	4.9105	1.6698	1.1185	1.6580	2.0371	1.6517	2.3283	.6218	.0704	.9892	.5657
WASCO	4.9452	1.6204	1.0054	1.7907	2.6730	1.7907	3.3192	.4810	.0425	1.1050	1.0022
WEAVERVILLE RS.	4.9490	1.7606	1.3392	1.7289	2.2456	1.7249	2.6823	.6194	.0672	.9797	.5900
WILLOWS	4.9699	1.7316	1.2668	1.8371	2.7169	1.8412	3.2586	.5365	.0486	1.0633	.7916
CANNON CITY	5.1294	1.7609	1.3398	1.6283	1.9510	1.6283	2.6120	.6242	.0722	.9247	.6192
CHEESMAN	5.1528	1.6175	.9788	1.4055	1.3953	1.4084	2.0116	.6481	.1011	.8707	.6039
CHEYENNE WELLS	5.1564	1.6989	1.1873	1.6108	1.9037	1.6146	2.5445	.6092	.0849	.9504	.6571
DEL NORTE	5.2184	1.8315	1.5228	1.7389	2.2065	1.7477	2.7207	.6414	.0641	.9543	.5301
DURANGO	5.2432	1.7747	1.3749	1.6628	2.0412	1.6628	2.5126	.6453	.0814	.9369	.5447
FORT COLLINS	5.3005	1.7471	1.3052	1.6339	1.9905	1.6380	2.6557	.6101	.0696	.9376	.6448
FORT MORGAN	5.3038	1.8542	1.5838	1.6501	2.0255	1.6501	2.8416	.6237	.0741	.8899	.6188
JULESBURG	5.4413	1.8760	1.6433	1.7344	2.2379	1.7344	2.7934	.6466	.0768	.9245	.5317
LAS ANIMAS	5.4834	1.7440	1.2775	1.7130	2.1361	1.7130	2.7113	.6189	.0545	.9822	.6256
MONTROSE NO. 2	5.5722	1.7980	1.4348	1.6555	1.9882	1.6555	2.3840	.6773	.0933	.9208	.4774
SHOSHONE	5.7618	1.9779	1.9343	1.7428	2.2942	1.7428	2.7124	.6910	.1111	.8811	.4269
STEAMBOAT SPRINGS	5.7936	1.9568	1.8724	1.7430	2.3000	1.7369	2.5511	.7054	.1223	.8876	.3858
YUMA	5.9295	1.8299	1.5186	1.6894	2.0348	1.6933	2.6230	.6642	.0720	.9254	.5263
ABERDEEN EXPT. STATION	10.0010	1.6866	1.1579	1.5378	1.6338	1.5378	2.3573	.6321	.0666	.9118	.6296
ARROWROCK DAM	10.0448	1.8837	1.6047	1.7602	2.3572	1.7538	2.7481	.6512	.0876	.9310	.4965
CAMBRIDGE	10.1408	1.7160	1.2688	1.6677	2.0340	1.6634	2.3931	.6408	.0737	.9693	.5443
DUBOIS EXPT. STATION	10.2707	1.8762	1.6939	1.6925	2.0996	1.6850	2.7585	.6471	.0791	.8981	.5484
HAILEY RS	10.3942	1.8143	1.4773	1.7394	2.2082	1.7338	2.6359	.6517	.0697	.9556	.5188
KOOSKIA	10.5011	1.9172	1.7986	1.7146	2.1449	1.7089	2.4703	.7115	.1100	.8913	.3972
OAKLEY	10.6542	1.7978	1.4343	1.7141	2.1856	1.7141	2.4861	.6604	.0874	.9534	.4835
SALMON	10.8076	1.7414	1.2710	1.5449	1.6236	1.5449	2.0562	.7056	.0984	.8872	.4469
SANDPOINT EXPT. STATIO	10.8137	1.9776	1.9333	1.7499	2.2807	1.7499	2.6851	.7000	.1083	.8849	.4131
ATLANTIC 1 NE	13.0364	1.8789	1.6515	1.7604	2.2994	1.7562	2.5549	.6875	.0965	.9347	.4247
DES MOINS WB CITY	13.2208	1.8628	1.6073	1.7090	2.1519	1.7090	2.6118	.6717	.0901	.9174	.4873

STATION NAME	STATION NO.	EN_2	Var N_2	ES_1	Var S.	ES_2	Var S_2	EI_1	Var I_1	EI_2	Var I_2
MASON CITY 3 N	13.5230	1.7412	1.2705	1.5908	1.8479	1.5908	2.1792	.6761	.1055	.9136	.4736
OTTUMWA	13.6391	1.6586	1.5959	1.7170	2.0958	1.7170	2.3946	.7114	.0953	.9238	.4011
ROCKWELL CITY	13.7161	1.6531	1.5809	1.7073	2.1846	1.7073	2.5553	.6064	.0971	.9213	.4730
CONCORDIA WB CITY	14.1789	1.6948	1.1776	1.6462	1.9463	1.6462	2.2849	.6505	.0828	.9713	.5169
COUNCIL GROVE	14.1866	1.7483	1.3081	1.6362	2.0053	1.6411	2.4755	.6317	.0835	.9367	.5849
ELLSWORTH	14.2459	1.6897	1.6811	1.6097	2.4605	1.8097	2.7888	.6593	.0748	.9577	.4720
HULTON	14.3759	1.6295	1.5175	1.7295	2.2184	1.7295	2.5962	.6562	.0814	.9454	.4936
LA CYGNE	14.4421	1.7125	1.2402	1.7271	2.3123	1.7360	2.2986	.6301	.0883	1.0137	.4605
MEDICINE LODGE	14.5173	1.7053	1.2927	1.6318	1.9317	1.6338	2.4120	.6348	.0712	.9591	.5794
PHILLIPSBURG	14.6374	1.7692	1.3909	1.6478	1.9534	1.6478	2.3990	.6653	.0829	.9314	.5111
PLAINS	14.6427	1.8195	1.4912	1.7766	2.3446	1.7706	2.7847	.6321	.0632	.9731	.5530
QUINTER	14.6637	1.7024	1.1757	1.6410	1.9130	1.6490	2.2927	.6458	.0866	.9666	.5204
SEDAN	14.7305	1.6549	1.0837	1.5337	1.7046	1.5317	2.1831	.6443	.0899	.9268	.5729
SEDGWICK	14.7313	1.6235	1.5817	1.6714	2.0461	1.6779	2.6233	.6471	.0774	.9201	.5408
TORONTO	14.8186	1.6138	.9955	1.5286	1.7228	1.5236	2.1474	.6227	.0973	.9452	.6120
CALHOUN EXP. STATION	16.1411	1.6351	1.5225	1.6786	2.1143	1.6746	2.3846	.6351	.0949	.9125	.5071
JENNINGS	16.4780	1.7529	1.3187	1.6171	1.9714	1.6171	2.6326	.6239	.0844	.9225	.5994
MELVILLE	16.6117	1.6711	1.1615	1.5014	1.6251	1.5014	2.0976	.6598	.1026	.8904	.5459
NEW ORLEANS WB CITY	16.6659	1.7110	1.2166	1.5695	1.8612	1.5671	2.2979	.6395	.1010	.9159	.5839
PLAIN DEALING	16.7344	1.6118	1.4709	1.5965	1.8929	1.6003	2.5065	.6406	.0901	.8833	.5914
TALLULAH DELTA LAB.	16.8923	1.5665	.8832	1.5432	1.6032	1.5432	2.1157	.5943	.0849	.9864	.6367
CAPPINGLER MILLS	23.1304	1.7477	1.3495	1.6711	2.2168	1.6711	2.4824	.6107	.0942	.9564	.5506
CHILLICOTHE 2S	23.1580	1.6197	1.4715	1.6026	1.9948	1.6025	2.4186	.6650	.1016	.8606	.5065
DEXTER	23.2235	1.7622	1.3940	1.6285	2.0254	1.6365	2.4690	.6316	.0936	.9177	.5436
ELDON	23.2503	1.6919	1.6874	1.7194	2.2621	1.7249	2.6573	.6357	.0961	.9117	.5449
FAYETTE	23.2823	1.6852	1.6947	1.6717	2.1316	1.6717	2.5436	.6745	.1151	.8668	.4680
FREDERICKTOWN	23.3038	1.7879	1.4986	1.6744	2.1168	1.6744	2.4824	.6502	.0933	.9357	.4887
HERMANN	23.3793	1.6279	1.5632	1.6129	1.9103	1.6129	2.4601	.6716	.1080	.8826	.5109
NEOSHO	23.6976	1.7207	1.2402	1.6858	1.9001	1.6858	2.3932	.6395	.0963	.9216	.5689
SHELBINA	23.7720	1.6602	1.6902	1.6497	2.0419	1.6456	2.5269	.6640	.1039	.8846	.5059
WARRENSBURG	23.8712	1.9391	1.8610	1.7772	2.4937	1.7772	2.9290	.6464	.0952	.9165	.5084
WILLOW SPRINGS	23.8995	1.9592	1.8792	1.7222	2.1753	1.7308	2.7862	.6793	.0906	.8834	.4651
AUGUSTA	24.0364	1.8000	1.4400	1.7074	2.1109	1.6994	2.3806	.6744	.0831	.9441	.4478
BALLANTINE	24.0432	1.7248	1.2000	1.5647	1.7061	1.5647	2.3977	.6411	.0814	.9051	.5759
BIG SANDY	24.0770	1.6478	1.0979	1.6289	1.9614	1.6289	2.1926	.6311	.0781	.9887	.5333
BOZEMAN AGRI. COLLEGE	24.1044	1.7121	1.1717	1.5720	1.6844	1.5720	2.4418	.6533	.1091	.8675	.5390
EAST ANACONDA	24.2004	1.6809	1.6521	1.6815	2.0992	1.6815	2.5563	.6689	.0955	.9036	.4439
EKALAKA	24.2689	1.8333	1.5278	1.6386	1.9579	1.6433	2.4934	.6982	.0952	.8966	.5023
FORTINE 1NNE	24.3139	1.6962	1.6939	1.7540	2.3177	1.7540	2.6103	.6731	.0966	.9260	.4367
HAMILTON	24.3885	1.9680	1.9040	1.9023	2.7174	1.9023	2.4056	.6835	.0923	.4569	.3873
HAUGAN	24.3986	1.9538	1.8537	1.7464	2.2728	1.7398	2.6313	.6974	.1075	.8905	.4146
JORDAN	24.4522	1.7093	1.2124	1.5496	1.7774	1.5973	2.2545	.6523	.0731	.9344	.5287
LUSTRE 4 NNW	24.5265	1.7591	1.4643	1.7370	2.2625	1.7290	2.4286	.6446	.0742	.9450	.4867
MOCCASIN EXPT. STATION	24.5761	1.6667	1.1111	1.5244	1.6306	1.5244	1.8606	.6931	.1055	.9170	.4258
SAINT IGNATIUS	24.7286	1.8446	1.5279	1.6953	2.0105	1.6953	2.3692	.6995	.1033	.9001	.4352
WEST GLACIER	24.8809	1.7822	1.3742	1.6293	2.0438	1.6293	2.0917	.6854	.1340	.9142	.3841
BLAIR	25.0930	1.7116	1.2180	1.5898	1.8288	1.5898	2.1923	.6685	.0962	.9289	.5036
BRIDGE PORT	25.1185	1.7608	1.3998	1.6617	2.0734	1.6617	2.5644	.6388	.0858	.9395	.5470
CRETE	25.2020	1.7373	1.2809	1.5717	1.7594	1.5717	2.3005	.6800	.0980	.9067	.5394
EWING	25.2805	1.6495	1.5710	1.7074	2.1357	1.7074	2.5673	.6728	.0881	.9232	.4807
FORT ROBINSON	25.3015	1.7606	1.3290	1.4223	1.5446	1.6266	2.5368	.6339	.0806	.9239	.5985
GENOA	25.3185	1.7068	1.2464	1.5973	1.6399	1.5973	2.1980	.6685	.0935	.9358	.5005
HARTINGTON	25.3630	1.5792	1.1106	1.5485	1.6867	1.5485	1.9782	.6955	.0984	.9222	.4582
PURDUM	25.6970	1.7540	1.3476	1.6294	2.0193	1.6264	2.4830	.6355	.0961	.9220	.5506
RAVENNA	25.7040	1.7415	1.4179	1.6467	1.9507	1.6467	2.4166	.6734	.0868	.9203	.4998
AUSTIN	26.0046	1.7328	1.2847	1.6483	1.9561	1.6483	2.5029	.6318	.0537	.9513	.5862
ELKO WB APT.	26.2573	1.6292	1.0650	1.6933	2.0414	1.6533	2.5988	.5660	.0466	1.0148	.7821
MINA	26.5188	1.5500	.8925	1.5660	1.9226	1.5660	2.1640	.5702	.0172	1.1087	.7701
RENO WB APT.	26.6779	1.5860	.8963	1.7229	2.0807	1.7185	2.5460	.5599	.0339	1.0819	.7990
CARRIZOZO	29.1515	1.8108	1.4893	1.6941	2.1104	1.6941	2.6143	.6178	.0864	.9181	.6462
CIMARRON	29.1813	1.7208	1.2403	1.6563	2.0112	1.6607	2.5937	.6141	.0619	.9651	.6293
CLOVIS	29.1939	1.9365	1.0961	1.6826	2.0678	1.6854	2.5575	.5886	.0462	1.0287	.6847
ELIDA	29.2854	1.6923	1.1716	1.7369	2.2466	1.7429	2.6156	.6010	.0932	1.0299	.6180
FORT BAYARD	29.3295	1.7950	1.4670	1.6792	2.0802	1.6823	2.6098	.6200	.0607	.9372	.6381
HACHITA	29.3775	1.6241	1.0136	1.6228	1.8927	1.6272	2.4036	.6032	.0519	1.0019	.6632
LAKE AVALON	29.4736	1.5547	.8824	1.5409	1.6796	1.5452	2.4653	.5847	.0373	.9939	.8256

STATION NAME	STATION NO.	EN_2	Var N_2	ES_1	Var S_1	ES_2	Var S_2	EI_1	Var I_1	EI_2	Var I_2
PECOS RS	29.6676	1.7103	1.2148	1.5867	1.7886	1.5929	2.2179	.6591	.0821	.9313	.5143
STATE UNIVERSITY	29.8535	1.6766	1.1343	1.6383	1.9185	1.6408	2.7134	.5914	.0422	.9786	.7412
ZUNI FAA AP.	29.9897	1.8618	1.6045	1.7629	2.2634	1.7562	2.6901	.6638	.0724	.9433	.4891
DICKENSON EXPT. STATIO	32.2188	1.7660	1.3026	1.6856	2.0974	1.6856	2.5534	.6389	.0747	.9545	.5556
GRAND FORKS U.	32.3621	1.7030	1.1271	1.5408	1.7072	1.5439	2.2807	.6476	.0874	.9066	.5766
JAMESTOWN ST. HOSP.	32.4418	1.7312	1.2858	1.6806	2.1415	1.6844	2.4056	.6375	.0873	.9730	.5234
MAX	32.5638	1.7640	1.3478	1.5520	1.6970	1.5545	2.1184	.6915	.1046	.8841	.4485
MOHALL	32.6025	1.6853	1.1049	1.6171	1.9056	1.6138	2.0914	.6701	.0967	.9576	.4665
GEARY	34.3497	1.6667	1.1111	1.5837	1.8074	1.5881	2.0797	.6618	.0909	.9528	.4924
IDABEL	34.4451	1.7938	1.4640	1.6202	1.9101	1.6276	2.2546	.6815	.1029	.9073	.4442
KENTON	34.4766	1.6859	1.1364	1.6999	2.1671	1.6999	2.7298	.5803	.0540	1.0083	.7020
PAULS VALLEY	34.6926	1.8113	1.4096	1.6366	1.9078	1.6412	2.5281	.6629	.0783	.9061	.5343
PERRY	34.7012	1.6222	1.0494	1.5002	1.6105	1.5066	2.0234	.6409	.0871	.9287	.5525
WEBBER FALLS	34.9445	1.7557	1.3667	1.6186	1.9244	1.6225	2.3341	.6601	.0940	.9242	.5095
WICHITA MT. WLR	34.9629	1.7442	1.2780	1.6363	1.9322	1.6383	2.2528	.6709	.0881	.9393	.4747
ANTELOPE 1 N	35.0197	1.8137	1.4159	1.6254	1.4760	1.6254	2.4282	.6712	.0840	.8962	.5014
BEND	35.0694	1.6138	.9905	1.6043	1.8544	1.6001	2.1630	.6298	.0689	.9915	.5746
CUTTAGE GROVE 1 S	35.1897	1.8115	1.4700	1.6671	2.1113	1.6671	2.3993	.6625	.1056	.9203	.4665
DANNER	35.2135	1.9481	1.8469	1.8367	2.5551	1.8367	2.7740	.6735	.0891	.9429	.4088
ESTACHAUA 2 SE	35.2693	1.8288	1.5456	1.7072	2.2078	1.7019	2.2873	.6915	.1180	.9306	.3982
GRANTS PASS	35.3445	1.6667	1.1111	1.6301	2.0166	1.6301	2.4417	.6044	.0747	.9781	.6339
HEPPNER	35.3827	1.7197	1.2378	1.6770	2.0867	1.6770	2.1304	.6751	.1003	.9752	.4176
LAKEVIEW	35.4670	1.7068	1.2063	1.6506	2.0257	1.6506	2.3948	.6290	.0769	.9671	.5513
MINAM 7 NE	35.5610	1.9851	1.9554	1.6226	2.4934	1.8154	2.7582	.6986	.1014	.9148	.3934
PROSPECT 2 SW	35.6907	1.7986	1.4364	1.6558	2.1530	1.6558	2.5206	.6325	.1042	.9206	.5325
ROCK CREEK	35.7250	1.9204	1.7074	1.7351	2.2176	1.7351	2.4419	.7130	.1132	.9036	.3488
WARM SPRINGS RESERVOIR	35.9046	1.8193	1.4705	1.6807	2.0097	1.6807	2.4837	.6675	.0741	.9238	.4864
ARMOUR	39.0296	1.8631	1.6980	1.7592	2.2460	1.7592	2.6526	.6671	.0842	.9442	.4717
CUTTON WOOD	39.1972	1.8284	1.5145	1.7278	2.2322	1.7219	2.8176	.6244	.0678	.9418	.5874
EUREKA	39.2797	1.7786	1.3847	1.6649	2.1115	1.6694	2.5723	.6274	.0859	.9386	.5623
HIGHMORE 1 W	39.3832	1.6607	1.0973	1.6025	1.8650	1.6063	2.1430	.6510	.0852	.9672	.5161
HOT SPRINGS	39.4007	1.9155	1.7536	1.7952	2.6103	1.7952	3.0811	.6187	.0769	.9372	.5711
LAUELLE 7 NE	39.4661	1.7000	1.1900	1.5830	1.9026	1.5830	2.4084	.6168	.0874	.9312	.5138
LEMMON	39.4864	1.6864	1.1376	1.7270	2.2847	1.7270	2.3767	.6194	.0765	1.0241	.5249
MILBANK	39.5536	1.8010	1.4427	1.6538	2.0592	1.6538	2.5148	.6494	.0936	.9183	.5295
SIOUX FALL WB AP	39.7667	1.7766	1.3798	1.6749	2.0783	1.6786	2.2887	.6782	.1018	.9448	.4402
VALE	39.8552	1.6939	1.1753	1.5537	1.7800	1.5541	2.4279	.6148	.0772	.9198	.6429
WOOD	39.9442	1.6692	1.1170	1.6477	2.0580	1.6526	2.2730	.6255	.0836	.9901	.5407
ALBANY	41.0120	1.7421	1.2928	1.6344	1.9056	1.6376	2.4801	.6456	.0693	.9400	.5744
BALMORHEA EXP. STATION	41.0498	1.7935	1.4631	1.7980	2.3873	1.8045	2.9662	.6008	.0444	1.0062	.6262
BEAUMONT	41.0611	1.7010	1.1925	1.5777	1.8565	1.5811	2.3206	.6343	.0907	.9295	.5749
BROWNWOOD	41.1138	1.7081	1.2045	1.7042	2.1666	1.7062	2.4394	.6320	.0709	1.0000	.5443
CORSICANA	41.2019	1.8374	1.5387	1.7112	2.2062	1.7112	2.5815	.6595	.0934	.9313	.4930
FLATONIA	41.3183	1.6266	1.0192	1.5270	1.6419	1.5307	2.1213	.6429	.0770	.9411	.5817
GALVESTON WB CITY	41.3430	1.7407	1.2894	1.6746	2.0609	1.6746	2.5584	.6309	.0694	.9620	.5869
GEORGE WEST	41.3508	1.5591	.8716	1.6115	1.8725	1.6161	2.1094	.6049	.0553	1.0366	.6282
GREENVILLE 2 SW	41.3734	1.7239	1.2480	1.6390	2.0102	1.6432	2.4260	.6316	.0824	.9532	.5641
HENDERSON	41.4081	1.7434	1.2761	1.5683	1.8041	1.5727	2.1923	.6703	.1083	.9021	.4879
KERRVILLE	41.4780	1.8161	1.4821	1.6693	2.0343	1.6693	2.7940	.6260	.0846	.9192	.6207
LAMPASAS	41.5018	1.7933	1.4426	1.6941	2.1105	1.6941	2.6048	.6438	.0746	.9447	.5481
MISSION	41.5972	1.6000	.9800	1.6704	2.1019	1.6764	2.5102	.5693	.0498	1.0477	.7130
PERRYTON	41.6950	1.7407	1.2894	1.6033	1.8522	1.6033	2.4780	.6347	.0719	.9211	.5950
POST	41.7206	1.5556	.8642	1.5586	1.7390	1.5630	2.1993	.5980	.0553	1.0048	.6784
PRESIDIO	41.7262	1.5696	.8741	1.7374	2.2582	1.7294	2.4000	.5718	.0418	1.1018	.6629
RIVERSIDE	41.7651	1.7578	1.3320	1.6329	1.9801	1.6373	2.3124	.6613	.0981	.9314	.4899
STERLING CITY	41.8630	1.6747	1.1299	1.6910	2.0629	1.6910	2.2861	.6351	.0613	1.0097	.5175
VALLEY JUNCTION	41.9280	1.9262	1.7940	1.7626	2.3986	1.7526	2.9561	.6421	.0809	.9151	.5373
VEGA	41.9330	1.7848	1.4407	1.6627	2.0364	1.6719	2.7290	.6109	.0611	.9367	.6203
WEATHERFORD	41.9532	1.8514	1.5764	1.7030	2.1963	1.7074	2.8467	.6257	.0773	.9222	.5885
DESSERT	42.2101	1.6872	1.1363	1.5698	1.7064	1.5698	2.2712	.6535	.0698	.9305	.5719
FORT DUCHESNE	42.2996	1.7458	1.4691	1.7166	2.1166	1.7166	2.6815	.6441	.0609	.9559	.5636
HIAWATHA	42.3896	1.7379	1.2823	1.7048	2.3977	1.7648	2.4542	.6290	.0800	1.0155	.4876
KANAB POWER HOUSE	42.4508	1.6525	1.0784	1.5043	1.5633	1.5043	2.1320	.6502	.0768	.9103	.5764
LOA	42.5148	1.6993	1.1884	1.6396	2.0106	1.6396	2.6541	.5917	.0607	.9648	.6906
MILFORD WB APT.	42.5654	1.6090	.9799	1.5573	1.7333	1.5528	1.9752	.6526	.0853	.9651	.5157
RICHMOND	42.7271	1.8369	1.5372	1.6711	2.0211	1.6657	2.2618	.7090	.1107	.9068	.3946
SPANISH FORK POWER HOU	42.8119	1.8276	1.5125	1.6421	1.9561	1.6370	2.2518	.7038	.1131	.8957	.4140
TOOELE	42.8771	1.7720	1.3980	1.5597	1.7078	1.5597	2.1247	.7066	.1121	.8802	.4454
BROOKLYN	45.0917	1.9176	1.7397	1.7060	2.1019	1.6963	2.3443	.7238	.1142	.8846	.3579
CEDAR LAKE	45.1223	1.8621	1.6052	1.5554	1.7601	1.5513	2.1842	.7214	.1427	.8331	.4144
CHELAN	45.1350	1.7606	1.3492	1.7040	2.1173	1.7040	2.5353	.6446	.0698	.9679	.5410
COLFAX 1 NW	45.1586	1.7188	1.2354	1.6105	2.0232	1.6105	2.3961	.6209	.0990	.9370	.5750
GOLDENDALE	45.3222	1.6690	1.1166	1.6131	1.9297	1.6131	2.2614	.6310	.0815	.9665	.5589
HATTON 8 E	45.3546	1.7548	1.3246	1.6350	2.0009	1.6304	2.5253	.6279	.0812	.9291	.5867
LONGVIEW	45.4769	1.8879	1.6761	1.6273	1.8421	1.6202	2.1717	.7467	.1240	.8582	.3553
NEWHALEM	45.5840	1.9238	1.7772	1.6594	1.9697	1.6518	2.2241	.7471	.1341	.8586	.3365
RIMROCK TETON DAM	45.7038	1.8116	1.4703	1.6768	2.0798	1.6768	2.6436	.6392	.0770	.9256	.5593
SEDRO WOLLEY 1 E	45.7507	1.8095	1.4649	1.5906	1.8795	1.5906	2.1393	.7057	.1357	.8790	.4079
SHELTON	45.7584	1.7391	1.2954	1.5286	1.6486	1.5286	1.9704	.7031	.1177	.8789	.4208
SUNNYSIDE	45.8207	1.7095	1.2129	1.7171	2.1906	1.7131	2.4789	.6276	.0678	1.0021	.5574
TATOOSH ISLAND WB	45.8332	1.8222	1.4783	1.6239	1.9599	1.6207	2.1934	.7063	.1310	.8894	.4038
WINTHROP 1 WSW	45.9376	1.8476	1.5061	1.7220	2.1331	1.7220	2.6578	.6599	.0895	.9320	.5086
BUFFALO BILL DAM	48.1175	1.7080	1.2093	1.5993	1.8244	1.5944	2.4089	.6341	.0677	.9335	.5999
DUBOIS	48.2715	1.8661	1.6163	1.7402	2.2370	1.7338	2.7962	.6449	.0717	.9291	.5408
GREEN RIVER	48.4065	1.7207	1.2402	1.6594	2.0674	1.6594	2.3403	.6373	.0851	.9644	.5195
LUSK	48.5830	1.8810	1.6570	1.6649	2.0620	1.6695	2.6786	.6589	.0930	.8876	.5236
PATHFINDER DAM	48.7105	1.7119	1.2186	1.5471	1.6957	1.5471	2.2631	.6567	.0853	.9038	.5592
SHERIDAN FIELD STATION	48.8160	1.7097	1.2133	1.6030	1.8983	1.5976	2.2538	.6465	.0903	.9345	.5279
YELLOWSTONE PARK	48.9905	1.8763	1.6443	1.7092	2.2825	1.7092	2.9168	.6173	.0866	.9109	.5941

Key Words: Run-Length, Run-Sum, Run-Intensity, Gamma and Normal Distributions, Moments

Abstract: Three quantitative measures are introduced for the concepts of "surplus" and "deficit" in hydrologic series. These are: run-length, run-sum, and run-intensity. Positive and negative runs of a series are defined in terms of a fixed value, say c, of the variable under consideration, namely precipitation. The distribution function, moments, and other statistical properties of the three variables, run-length, run-sum, and run-intensity, are obtained analytically under the following alternative assumptions on the sequence of annual precipitations:

1. It is independent and normally distributed.
2. It is independent and gamma distributed.

For monthly precipitations, z_t, the series was first standardized by the transformation

$$x_t = \frac{z_t - \mu_\tau}{\sigma_\tau} ,$$

where t is of the form $t = 12(n-1) + \tau$, $\tau = 1, \ldots, 12$, $n = 1,2,\ldots$, and where μ_τ and σ_τ are mean and standard deviation of the series corresponding to the month τ. Calling "$x_t \le c$" as state "0" and "$x_t > c$" as state "1", the series is then analyzed as a two-state Markov chain with stationary transition probabilities.

Annual precipitation from 27 stations in Colorado, and monthly precipitation from 219 stations in the Western United States are analyzed.

References: Jose Llamas and M. M. Siddiqui, Colorado State University Hydrology Paper No. 33 (May 1969), "Runs of Precipitation Series."

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