River Response due to Discontinuation of Pumping in Zones A, B, C and D.

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Report to the
State Engineer
on the

River Response due to Discontinuing Pumping in Zones A, B, C and D.

As a basis for administration of priorities, the areas in which pumping is practiced has been laid out in zones paralleling the river. These zones are of approximately equal width and are designated as zones A, B, C and D. Zone A is adjacent to the river and zone D is most remote from the river. Details of the zones are shown on maps prepared by the State Engineer.

Purpose of the study

When a well is sunk into water bearing materials, where the ground water is in contact with the waters of a stream, pumping of the well can cause a depletion of the flow of the stream. This possibility becomes of importance where water users, with priorities senior to those of the pumper, depend upon surface diversions to supply their needs and the water in the river becomes insufficient to satisfy them. Time honored custom would hold that in such cases the Junior appropriator, in this case the pumper, should be shut down to give relief to the Senior appropriator but the delay of the river response to cessation of pumping adds complexities which are not present where adjustments can be made by manipulating gates at diversion dams on the river. These complexities increase as the distance of the well from the river increases. In the case of a well more than a mile distant from the stream, response of the river flow to a discontinuation of pumping becomes very sluggish. Furthermore such a well may be depleting the stream this year by the pumping done in previous years. The chart of figure 1 has been prepared, at your request, to permit a quantitative assessment of the effect of pumping changes on the flow of the river. When pumping is initiated the curves give the river depletion due to the pumping as a function of time after pumping begins. If pumping is discontinued the curves will give the restoration of river flow due to the cessation of pumping. In either case, the curves give the changes which will result as a departure from what the flow of the stream would have been if the effect had not been initiated.

Intermittent operation can be treated by superposition. A proper basis is to consider that an effect, once initiated, continues thereafter forever. As an example, the effect of a pump operated at the rate Q for four months and then shut down can be evaluated by estimating the effect of the well for the first four months of pumping. Shut down is then accounted for by superposing a pumping rate of - Q (recharge). Thereafter the effect is estimated as the sum of both effects each counted from the instant of its initiation.

Methods of computation

The expression

$$h = H \sum_{n=1,3,5\cdots}^{n=\infty} A_n e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)} \sin\frac{n\pi x}{L}$$

is a solution of the applicable differential equation

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

which meets the boundary and initial conditions

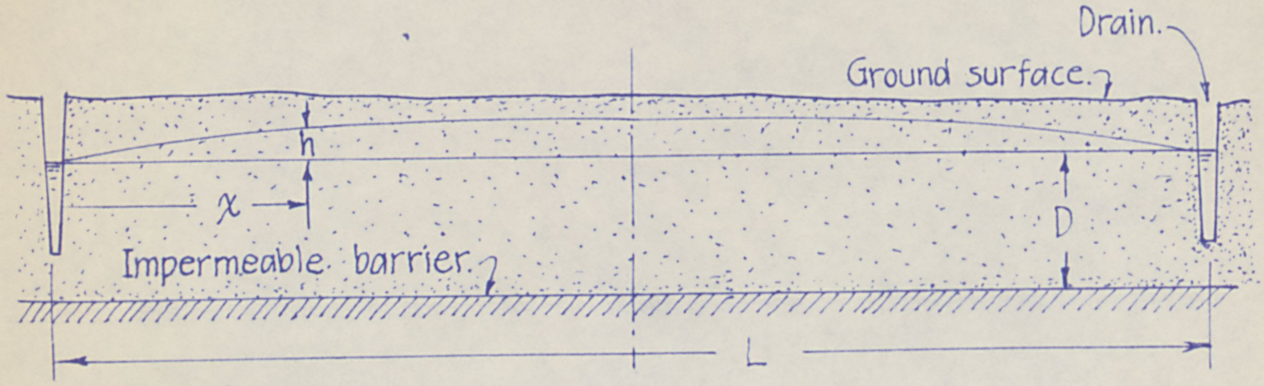
$h = H$ in the appropriate zone when $t = 0$

$h = 0$ for $x = 0$ and $x = L$ for $t > 0$

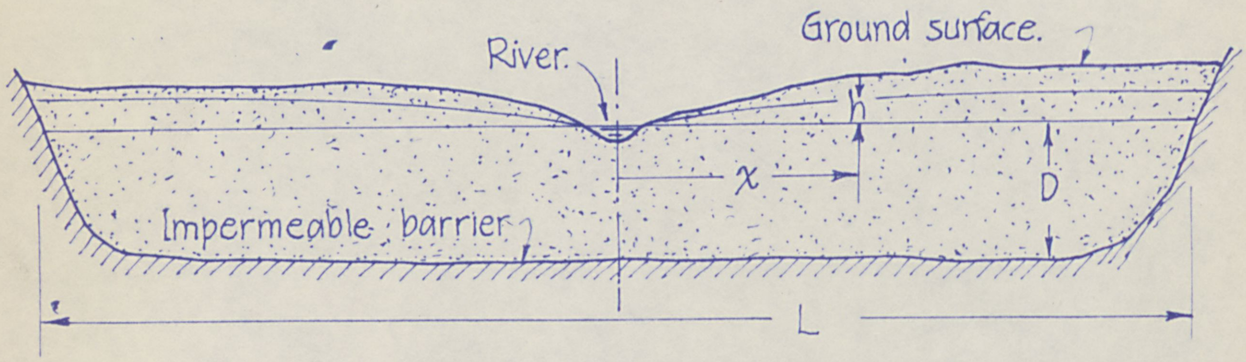
The notation used has the following significance:

A_n	the coefficient of the nth term.
D	the initial saturated depth (ft).
h	a drainable depth.
H	a uniform initial drainable depth.
K	the permeability (ft.sec).
L	distance between drains or a valley width (ft).
t	time.
V	the effective voids ratio (dimensionless).
x	distance measured horizontally from a drain (ft).
α	$= \frac{KD}{V}$. The aquifer constant ft^2/sec.

The above expression will apply to either of the two cases illustrated below.



Parallel drains.



River valley.

In the parallel drains case there is no flow past the center line in either direction because the water table gradient is zero there. If this figure is cut in two along the center line and the two halves shifted to bring the drains into coincidence the representation of a river valley is obtained. Here the condition of no flow at the sides of the valley is imposed by the impermeable barrier. In the case of the South Platte Valley the impermeable barrier is supplied by the Pierre Shale, the Fox Hills and Laramie formations. In the river valley the river becomes the drain.

The flow to a unit length of the river, from one side is given by the expression

$$q_1 = KD\left(\frac{\partial h}{\partial x}\right)_0 = \frac{KDH\pi}{L}\sum_{n=1,3,5\cdots}^{n=\infty} n A_n e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}$$

Where $\left(\frac{\partial h}{\partial x}\right)_0$ represents the gradient at x = 0.

The effect of a continuous infiltration at the rate i can be derived from this expression by letting

$$dH = \frac{i}{V}\,d\xi$$

so that

$$dq_1 = \frac{KDi\pi}{LV}\sum_{n=1,3,5\cdots}^{n=\infty} n A_n\, e^{-n^2\pi^2\left(\frac{\alpha(t-\xi)}{L^2}\right)}\,d\xi$$

Here i represents a steady infiltration rate measured as water applied to the ground. It has the dimensions (ft/sec). The quantity ξ represents a time variable running between 0 and t. It indicates the time at which an event occurs. The time t indicates the time at which the effect of the event is to be computed.

An integration with respect to ξ from 0 to t gives

$$q_1 = iL\sum_{n=1,3,5\cdots}^{n=\infty}\frac{A_n}{n\pi} - iL\sum_{n=1,3,5\cdots}^{n=\infty}\frac{A_n e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n\pi}$$

It remains only to evaluate the A_n quantities for the appropriate zones by the well known procedure of Fourier. If a multiplication and division by 8 is introduced so that q_1 will be expressed in terms of the water supply to the zone the expression will take the form

$$\frac{q_1}{Q} = \sum_{n=1,3,5\cdots}^{n=\infty}\frac{8A_n}{n\pi} - \sum_{n=1,3,5\cdots}^{n=\infty}\frac{8A_n}{n\pi}e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}$$

Where $Q = \frac{iL}{8}$ is the quantity of water applied to the zone of L/8 width.

The required values are shown below.

Table 1. Values of $\left(\frac{8A_n}{n\pi}\right)$

n	Zone A	Zone B	Zone C	Zone D
1	.24680	.70284	1.05187	1.24076
3	.22239	.39260	.07809	- .33283
5	.17932	.04207	- .21152	.11982
7	.12730	-.10792	.07211	- .02532
9	.07701	-.06529	.04362	- .01532
11	.03705	.00869	- .04370	.02476
13	.01184	.02091	.00416	- .01772
15	.00110	.00312	.00467	.00551
17	.00085	.00243	.00364	.00429
19	.00554	.00979	.00195	- .00830
21	.01017	.00239	- .01199	.00679
23	.01179	-.01000	.00668	- .00222
25	.00998	-.00846	.00565	- .00199
Sum	.94114	.99317	1.00523	.99823

The first summation must yield unity. Typical computations of the ratio q_1/Q is shown below:

For zone D with $\left(\frac{\alpha t}{L^2}\right) = 0.01$.

$\pi^2 = 9.86960$ $\quad \pi^2\left(\frac{\alpha t}{L^2}\right) = .0986960$

n	n^2	$n^2\pi^2\left(\frac{\alpha t}{L^2}\right)$	$e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}$	$\left(\frac{8A_n}{n\pi}\right)$
1	1	.09870	.90601	1.24076
3	9	.88826	.41137	- .33283
5	25	2.46740	.08480	.11982
7	49	4.83610	.00794	- .02532
9	81	7.99438	.00034	- .01532
11	121	11.94216	.00001	.02476
13	169	16.67962	.00000	- .01772

$$\sum \frac{8A_n e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n\pi} = 0.99718$$

$$\frac{q_1}{Q} = 1 - 0.99718 = 0.00283$$

For $\left(\frac{\alpha t}{L^2}\right) = 0.05$ $\quad \pi^2\left(\frac{\alpha t}{L^2}\right) = 0.49348$

n	n^2	n^2	$e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}$	$\left(\frac{8A_n}{n\pi}\right)$
1	1	.49348	.61050	1.24076
3	9	4.44132	.01178	- .33283
5	25	12.33700	.00000	.11982

$$\sum \frac{8A_n e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n\pi} = 0.75356$$

$$\frac{q_1}{Q} = 1 - 0.75356 = 0.24644$$

The development has been expressed in terms of a return flow q_1 originating in a supply Q coming from irrigation. Where Q represents pumped water consumed q_1 becomes a stream depletion. The curves will apply equally well in either case.

In the development q and Q represent quantities appropriate to a unit length of the stream. However, if the zone conditions are uniform over a length of reach of the river R, then the ratio becomes (Rq_1/RQ) and its numerical value is the same as for

$\left(\frac{q_1}{Q}\right)$ so that the ratio (q_1/Q) then becomes the ratio of

the total depletion in the reach to the total pumped water consumed in the reach.

<u>Checks</u>

A check was made for zone D by use of the formula for the stream depletion q_1 due to pumping of a single well at the rate Q. This formula is, if the well is at a distance x from the stream.

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x_1}{\sqrt{4\alpha t}}} e^{u^2} du$$

The well was placed at the middle of the zone and the presence of the barrier was accounted for by the method of images. The agreement obtained is shown by the open circles on figure 1.

Checks for zones A snd B were made by a similar procedure but with the pumping distributed across the zone. The effect of the barrier at the outer edge of zone D was neglected in this computation. The values obtained by this method for the interval of $(\alpha t/L^2)$ from 0 to 0.01 are shown in Table 2 below.

Table 2. - Values of (q_1/Q) in zones A and B for small values of the parameter $\left(\frac{\alpha t}{L^2}\right)$

$\left(\frac{\alpha t}{L^2}\right)$	Zone A	Zone B	
0	0	0	
.001	.28490	.00048	
.002	.39454	.00916	
.003	.46654	.02770	
.004	.51816	.05164	
.005	.55737	.07773	
.006	.58842	.10408	
.007	.61378	.13028	
.008	.63497	.15411	
.009	.65303	.17719	
.010	.66866	.19902	
.010	.66867	.19944	Fourier values

The check values for $\left(\frac{\alpha t}{L^2}\right) = .010$ are also shown as open circles on figure 1.

A third check was made by comparing the totals of the four zones with an independent computation for a uniform distributed pumping over all four zones. Some comparative values are shown in the following table:

Table 3. - Comparison of the Fourier values totaled for the four zones with an independent evaluation of the effect of a uniform pumping over all four zones.

$\left(\frac{\alpha t}{L^2}\right)$	$\left(\frac{\text{sum}}{4}\right)$	$1-\frac{8}{\pi^2}\sum_{n=1,3,5\cdots}^{n=\infty}\frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^2}$
0	0	0
.01	.22576	.22568
.02	.31916	.31915
.03	.39088	.39087
.04	.45123	.45124
.05	.50410	.50409
.10	.69788	.69788
.20	.88740	.88740
.30	.95803	.95803
.40	.98437	.98436
.50	.99417	.99417

The division of the sum of the Fourier values by four is needed to put the computations on a comparable basis.

This work was done under the terms of an agreement dated April 29, 1969 between the State of Colorado, for the benefit of the Division of Water Resources, and the writer as Consultant.

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June 13, 1969

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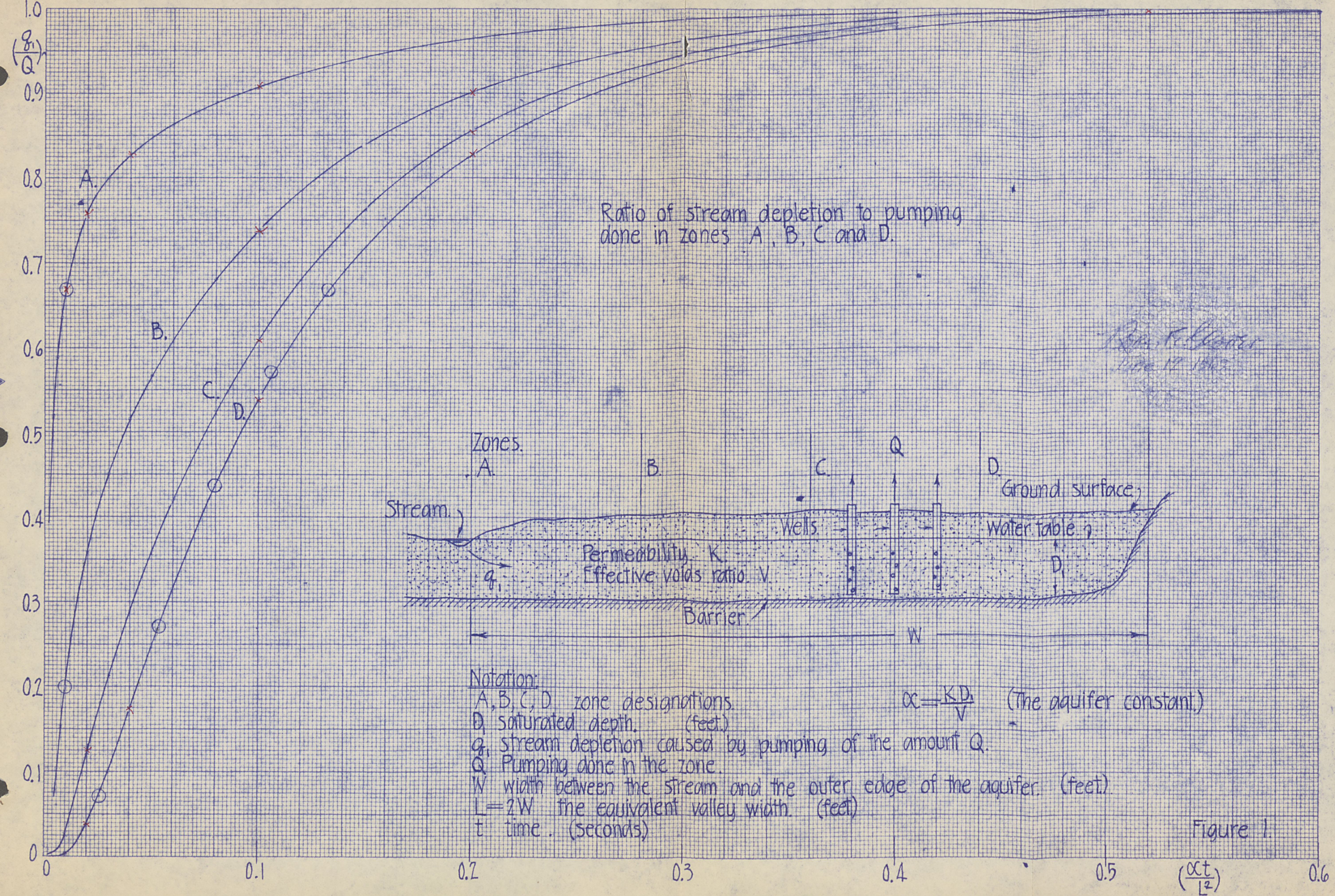

Ratio of stream depletion to pumping done in zones A, B, C and D.
$(\frac{q_1}{Q})$
A.
B.
C.
D.
Zones.
A.
B.
C.
D.
Q
Stream.
Ground surface.
Wells.
Water table.
Permeability K.
Effective voids ratio V.
q_1
D
Barrier.
W
Notation:
A, B, C, D zone designations.
D saturated depth. (feet)
q_1 stream depletion caused by pumping of the amount Q.
Q Pumping done in the zone.
W width between the stream and the outer edge of the aquifer. (feet)
L = 2W the equivalent valley width. (feet)
t time. (seconds)
$\alpha = \frac{KD}{V}$ (The aquifer constant.)
$(\frac{\alpha t}{L^2})$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 1.