APPENDIX A

MATHEMATICAL BASIS FOR THE STREAM-AQUIFER INTERACTION MODEL

Background

Isolated aquifer

At the heart of the mathematical modeling of the physical (hydrologic) interaction of an alluvial water-table aquifer with a hydraulically connected stream is the Boussinesq equation, namely:

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x}\left(T \frac{\partial s}{\partial x}\right) - \frac{\partial}{\partial y}\left(T \frac{\partial s}{\partial y}\right) = \sum_{p=1}^{P} q_p \quad (1)$$

where ϕ is the effective porosity (specific yield), s is drawdown at the point of horizontal coordinates x and y and at time t, T is the transmissivity of the aquifer at point of coordinate x and y, q_p is the net pumping (withdrawal) rate per unit area at well (withdrawal) point p and P is the total number of well (withdrawal) points.

For an aquifer which is not mined and relatively deep, the transmissivity can be considered to be constant. Then Eq. (1) is a linear equation and it is known then that the solution of Eq. (1) for drawdown at a point w for week n is of the form:

$$s_w(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp} (n-\nu+1)\, Q_p(\nu) \quad (2)$$

where $\delta_{wp}(\ )$ is the discrete kernel function (influence function) of drawdown at point w due to pumping at point p and $Q_p(\nu)$ is the net withdrawal volume from the well p. The coefficients $\delta_{wp}(\ )$ are calculated for a given system by a numerical solution of a finite-difference approximation to the Boussinesq Eq. (1). These procedures are discussed in detail in various publications (Morel-Seytoux and Daly, 1975; Morel-Seytoux et al., 1975). Figure A-1 displays a typical grid system superimposed on the area of interest with one well at the center. Figure 6 shows the discrete kernel function of drawdown 350 m (about 1000 feet) away from the center well due to pumping at the center well. As an example let us assume that the well pumped water volumes during 10 weeks according to the schedule shown in Table 1.

TABLE 1

Week	1	2	3	4	5	6	7	8	9	10
Volume (million m^3)	0.2	0	0	0.1	0	0	0.2	0	0.1	0
$\delta(\ )$ m/million m^3	3.6	3.3	2.6	2.0	1.5	1.3	1.1	1.0	0.9	0.8
Drawdown (m)	0.72	0.66	0.52	0.76	0.66	0.52	1.14	1.01	1.19	1.00

The values of δ (in m/million m^3) in Table 1 were read from Figure A-2 From Eq. (2) one can calculate drawdown 350 m away from the well at the end of one week, two weeks, three weeks, etc., namely:

$$s(1) = \delta(1)\ Q(1) = 3.6 \times 0.2 = 0.72 \text{ m}$$

$$s(2) = \delta(2)\ Q(1) + \delta(1)\ Q(2) = 3.3 \times 0.2 = 0.66 \text{ m}$$

Note that during week 2 the water table recovers at point w. Proceeding similarly for the other weeks:

$$s(3) = \delta(3)Q(1) + \delta(2)Q(2) + \delta(1)Q(3) = 2.6 \times 0.2 = 0.52 \text{ m}$$

$$s(4) = \delta(4)Q(1) + \delta(3)Q(2) + \delta(2)Q(3) + \delta(1)Q(4) = 2.0 \times 0.2 + 3.6 \times 0.1 = 0.76 \text{ m}$$

The other values are shown in Table 1, and the drawdowns plotted on Figure A-3.

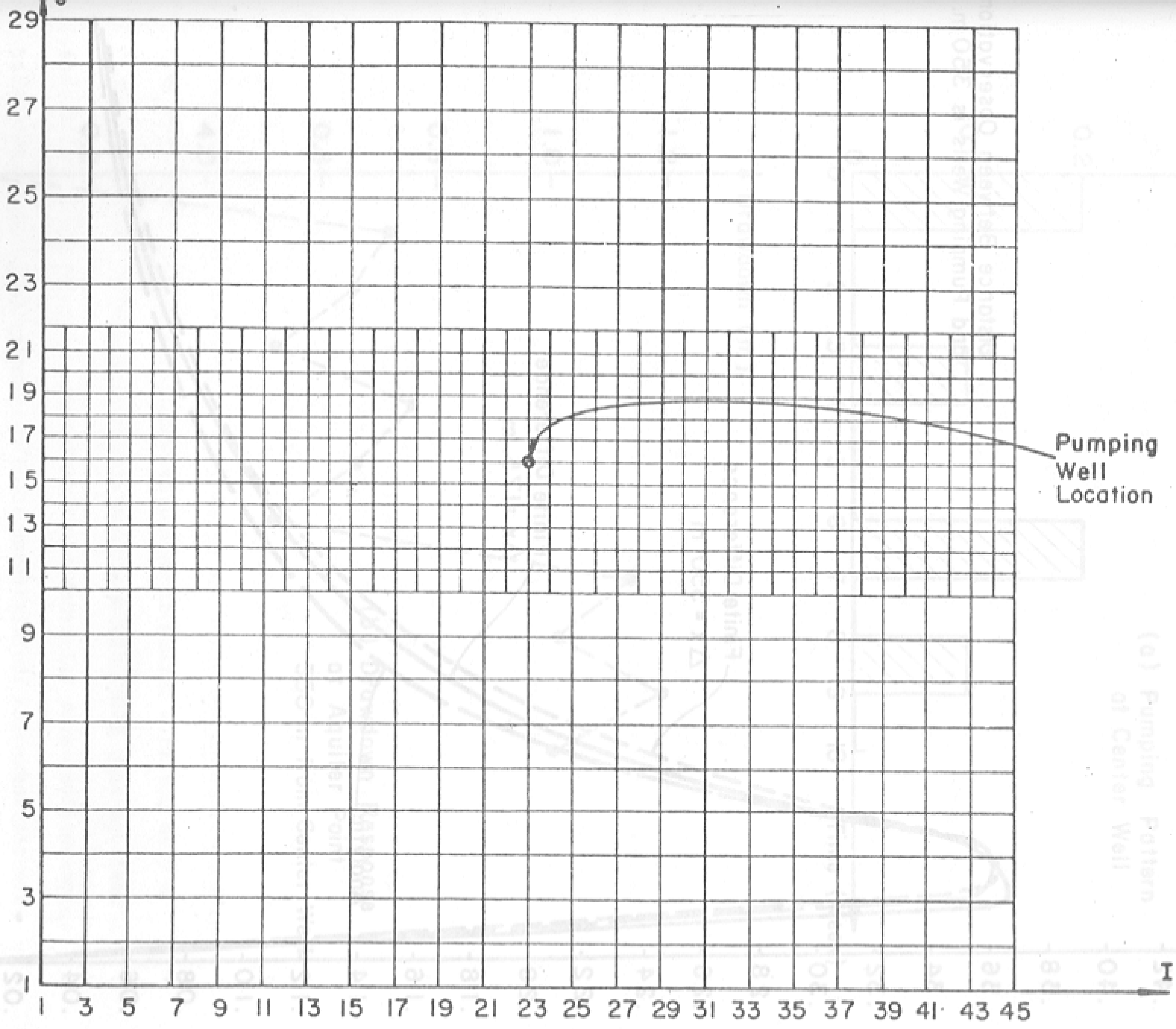
Pumping
Well
Location
I
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29

Figure A-1

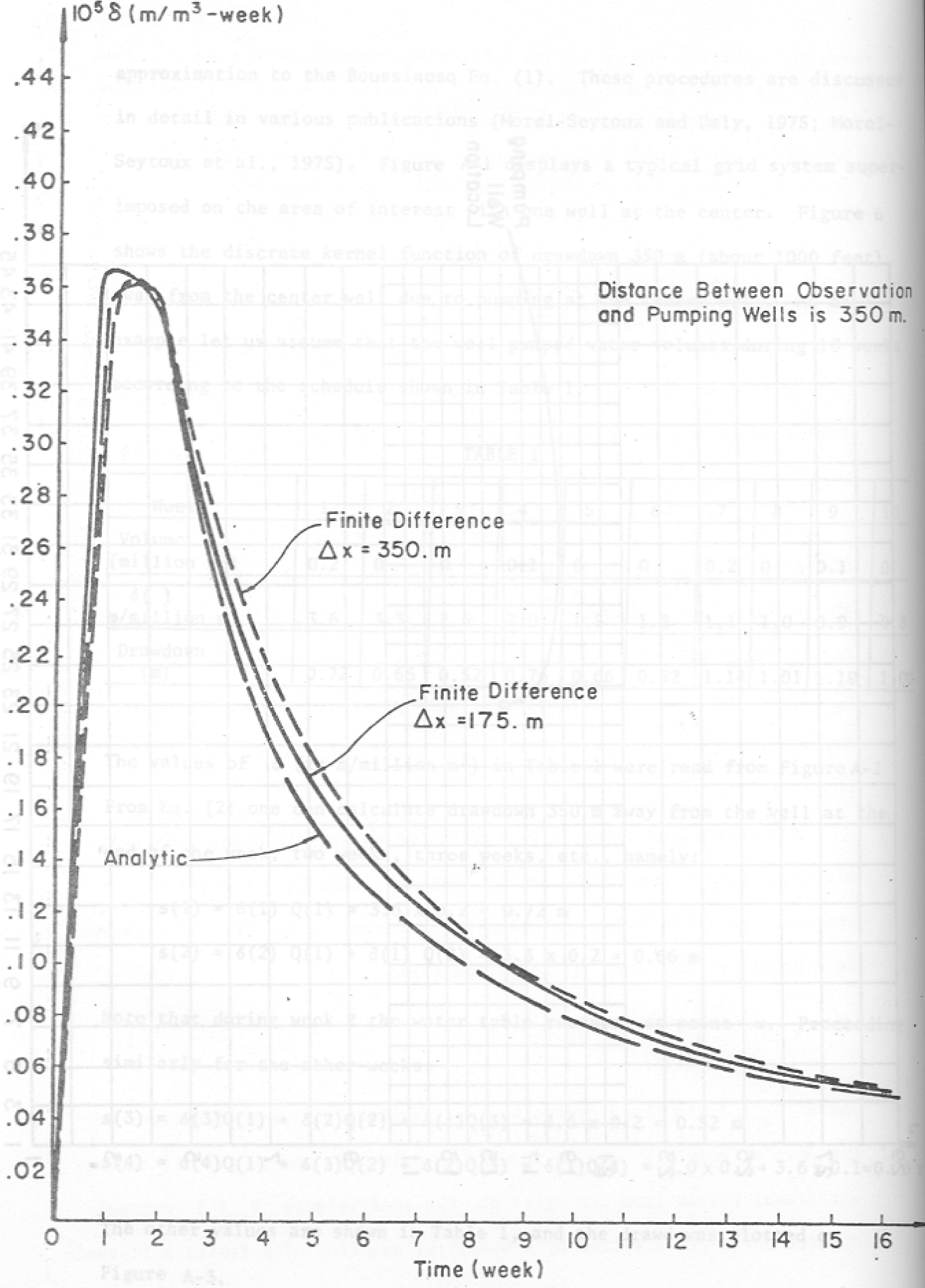
$10^5 \delta$ (m/m^3-week)
.44
.42
.40
.38
.36
.34
.32
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Time (week)
Distance Between Observation and Pumping Wells is 350 m.
Finite Difference Δx = 350. m
Finite Difference Δx =175. m
Analytic

Figure A-2

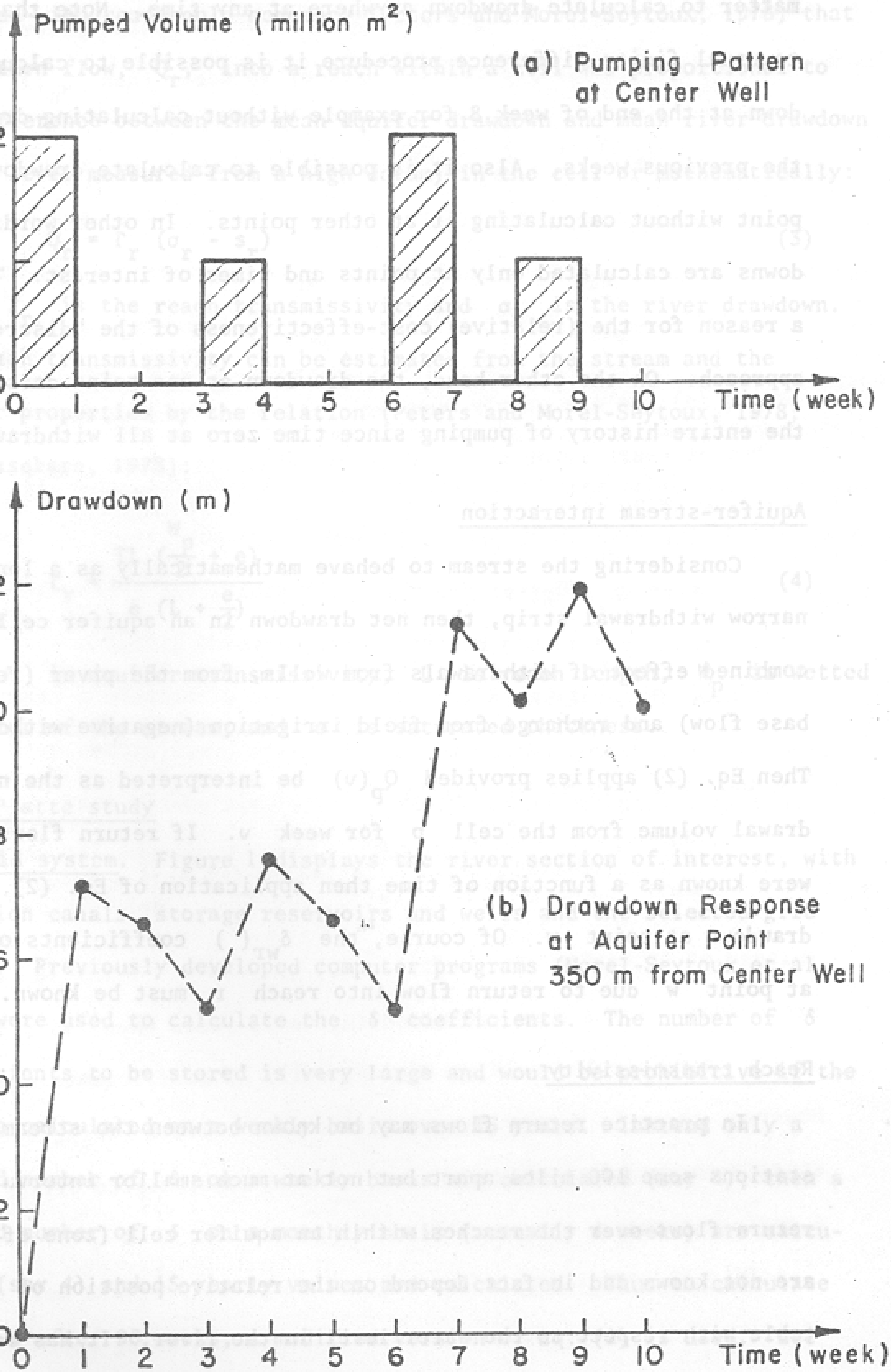
Pumped Volume (million m²)
(a) Pumping Pattern at Center Well
0.2
0.1
0 1 2 3 4 5 6 7 8 9 10 Time (week)
Drawdown (m)
1.2
1.0
0.8
0.6
4.0
0.2
0
(b) Drawdown Response at Aquifer Point 350 m from Center Well
0 1 2 3 4 5 6 7 8 9 10 Time (week)

Figure A-3

Once the δ coefficients have been calculated then it is an easy matter to calculate drawdown anywhere at any time. Note that contrary to usual finite difference procedure it is possible to calculate drawdown at the end of week 8 for example without calculating drawdowns for the previous weeks. Also it is possible to calculate drawdown at one point without calculating it at other points. In other words, drawdowns are calculated only at points and times of interest. Therein lies a reason for the (relative) cost-effectiveness of the "discrete kernel" approach. On the other hand, the drawdown at one point does depend upon the entire history of pumping since time zero at all withdrawal points.

Aquifer-stream interaction

Considering the stream to behave mathematically as a long and narrow withdrawal strip, then net drawdown in an aquifer cell is the combined effect of withdrawals from wells, from the river (return flow, base flow) and recharge from field irrigation (negative withdrawal). Then Eq. (2) applies provided $Q_p(\nu)$ be interpreted as the net withdrawal volume from the cell p for week ν. If return flows in a reach were known as a function of time then application of Eq. (2) would yield drawdown at point w. Of course, the $\delta_{wr}(\ )$ coefficients of drawdown at point w due to return flow into reach r must be known.

Reach transmissivity

In practice return flows may be known between two stream gauging stations some 100 miles apart but not at much smaller intervals. The return flows over the reaches within an aquifer cell (zone of influence) are not known and in fact depend on the relative position of the water table with respect to the water level in the river. It has been postulated (Morel-Seytoux and Daly, 1975; Morel-Seytoux, 1975a,b) and verified with reasonable accuracy (Peters and Morel-Seytoux, 1978) that the return flow, Q_r, into a reach within a cell was proportional to the difference between the mean aquifer drawdown and mean river drawdown (river level measured from a high datum) in the cell or mathematically:

$$Q_r = \Gamma_r \, (\sigma_r - s_r) \tag{3}$$

where Γ_r is the reach transmissivity and σ_r is the river drawdown. The reach transmissivity can be estimated from the stream and the aquifer properties by the relation (Peters and Morel-Seytoux, 1978; Illangasekare, 1978):

$$\Gamma_r = \frac{TL \, (\frac{W_p}{2} + e)}{e \, (L + \frac{e}{2})} \tag{4}$$

where T is aquifer transmissivity, L is reach length, W_p is wetted perimeter of the stream, and e is saturated thickness.

<u>South Platte study</u>

<u>Grid system</u>. Figure 1 displays the river section of interest, with diversion canals, storage reservoirs and wells and the selected grid system. Previously developed computer programs (Morel-Seytoux et al., 1975) were used to calculate the δ coefficients. The number of δ coefficients to be stored is very large and would be prohibitive if the δ were calculated on a weekly basis over 15 years. Instead only a limited number of δ on a weekly basis are calculated (say 4), then a limited number of δ on a monthly basis (actually 4 weeks) are calculated (say 4) and 15 yearly values are calculated. Thus to calculate drawdown after 90 weeks, a modified version of Eq. (2) is used. Let

$Q_p(\nu)$ be the weekly pumping rates. Let $Q_m(\lambda)$ be the monthly pumping rates occurring at integer multiple number of months prior to the date of interest. For example for the date of 90 weeks then $Q_m(1)$ is:

$$Q_m(1) = \sum_{\nu=83}^{86} Q_p(\nu)$$

and similarly:

$$Q_m(2) = \sum_{\nu=79}^{82} Q_p(\nu); \quad Q_m(3) = \sum_{\nu=75}^{78} Q_p(\nu); \quad Q_m(4) = \sum_{\nu=71}^{74} Q_p(\nu)$$

Let $Q_y(\alpha)$ be the yearly pumping rates occurring at integer multiple number of years prior to the date of interest. For example again for the date of 90 weeks, then:

$$Q_y(1) = \sum_{\nu=19}^{70} Q_p(\nu)$$

$$Q_y(2) = \sum_{\nu=1}^{18} Q_p(\nu)$$

The modified form of Eq. (2) is:

$$s(90) = \sum_{\nu=87}^{90} \delta_w(90-\nu+1)Q_p(\nu) + \sum_{\lambda=1}^{4} \delta_m(\lambda+1)Q_m(\lambda) + \sum_{\alpha=1}^{2} \delta_y(\alpha+1)Q_y(\alpha)$$

Instead of 90 weekly δ coefficients, only $4 + 4 + 2 = 10$ coefficients are needed. For a date corresponding to 15 years when $15 \times 52 = 780$ weekly coefficients would be needed, still 23 coefficients suffice. However, there is a computer price for this storage saving because the monthly and yearly pumping volumes must be recalculated every week (like a moving average).

<u>Initial conditions.</u> The original computer programs were developed to solve aquifer operational management problems using optimization on

APPENDIX B

DESCRIPTION OF CALIBRATION PROCEDURES AND RESULTS OF SOUTH PLATTE COMPUTER MODEL

Available Data

The entire area under study was divided into 35 subareas referred to as "*service areas*". A *service area* consists of a set of farms (and associated land area) supplied by a *common* ditch bringing to the area water diverted from the stream and/or by a *common* outlet from a reservoir and/or from wells. For each of the service areas the following data were gathered and compiled on a weekly basis for the period 1947-1961:

(a) surface water made available,

(b) total amount of ground water pumped from wells,

(c) amount of precipitation received,

(d) an average irrigation efficiency for the farms.

For the same period the following South Platte streamflow data were compiled on a weekly basis:

(a) Stream flow at a point upstream of Balzac gaging station (upper boundary of the study area).

(b) Stream flow at a point downstream of Julesburg gage at the Colorado-Nebraska state line (lower boundary of the study area).

(c) Return flow to the stream between diversions.

In addition the following information was also gathered:

(a) Phreatophyte losses,

(b) Seepage from canals under average flow conditions,

(c) Seepage from reservoirs, and

(d) Stage discharge relationships at Balzac and Julesburg gages.

Calibration Criterion

Historical data for two types of state variables were available for the stream-aquifer system for the purpose of calibration, namely: the aquifer return flows and the stream outflow measured at the downstream boundary of the system. The observed return flows in the subreaches are functions of the stages in the stream (state of stream) and the aquifer water table elevations (state of the aquifer). The observed downstream flows show the aggregated effect of the state of the total system. The observed outflows were selected as a measure of calibration, taking into consideration the fact that more reliable weekly outflow data compiled from *daily* flow data were available as compared to the weekly return flows compiled from *monthly* estimates. Also, the outflow which is also the flow into Nebraska becomes an important parameter to be analysed in drawing major conclusions from the study.

Calibration Runs

A simulation period of 350 weeks, starting from the first week of July, 1951 to the last week of February, 1958, was selected for calibration. An important consideration given in the selection of this period was that it includes the drought years 1953 through 1956. These drought years with fairly low observed downstream flows are bounded by the wet years of 1952 and 1957 with observed high flows. The main objective was to calibrate the model for the low flow drought years for which the overall project objective of studying the impact of different physical and/or managerial strategies was addressed. The 46-weeks wet period

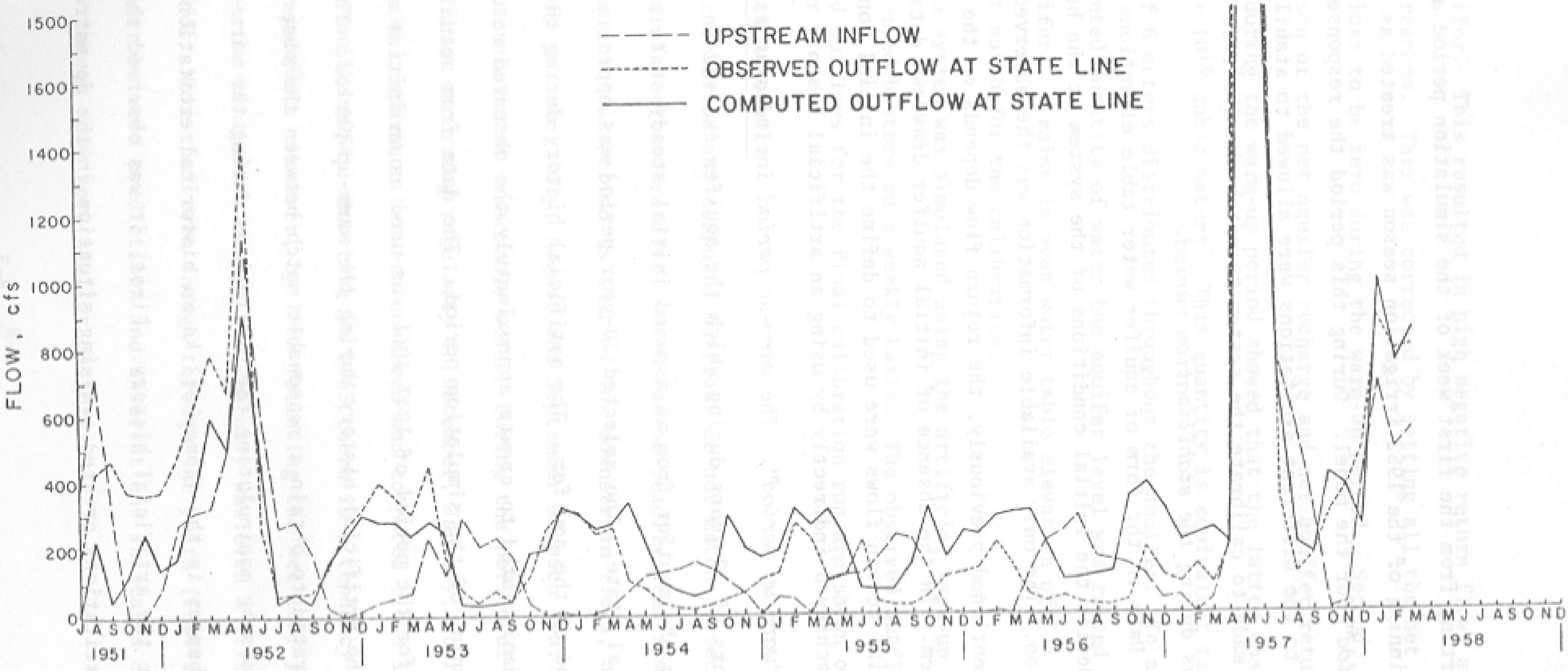

Figure B-1 Comparison of monthly observed and calculated flows at stateline starting from the first week of the simulation period and ending at the beginning of the 1952 irrigation season was treated as a "*stabilization period*" for the model. During this period the responses of the system due to the different excitations were allowed to stabilize. No attempt was made to calibrate the system to match the observed and computed outflows during the *stabilization period*.

Data in the form of aquifer water table elevations were not available to define the initial conditions of the system at the beginning of simulation. The only available information was the observed return flows. As mentioned previously, the return flow depends on the state of the system. In the absence of initial aquifer drawdown data the observed initial return flows were used to define the initial conditions. This was achieved indirectly by using an artificial history referred to as the "*warm-up period*". The *warm-up period* includes an assumed excitation history of 20 years during which the aquifer excited in such a way to bring the aquifer from an assumed initial steady state to the observed initial state. The selected 20-year period was approximately the memory period of the aquifer. The artificial history during the warm-up period was manipulated to create approximately the observed return flows at the beginning of the simulation period. The data from actual observed history for the period of 1947-1961 was used as an initial approximation for the artificial history during the warm-up period. The system was calibrated to obtain a reasonable match between the observed and computed outflows by manipulating the excitations during the warm-up period. A few changes in the (somewhat) known historical excitations were made to create the artificial history. First, it was observed that the historical excitations created a mining situation in the downstream section of the aquifer. This resulted in high negative return flows at the downstream reaches. This was corrected by setting all the net withdrawal excitations to be zero during the warm-up period. Second, a mass balance comparison of the net aquifer recharge and net aquifer return flow to stream during the warm-up period showed that the latter was in excess by $.17 \times 10^{10}$ cubic meters. This quantity is equivalent to an average depth of 6 meters distributed throughout the aquifer. To allow for this unaccounted quantity of water the aquifer level was raised throughout the aquifer. A raise in mean water table elevation of 7.8 meters gave the best match for the calibration.

The system was simulated using the artificial warm-up period and the observed history on a weekly basis. The observed outflows and the computed outflows for the final calibration run are shown on Figure B-1.

Calibration Results

The observed and computed outflows do not match perfectly but there are several positive results, which can be readily seen from the figure. The most critical period is that of low downstream flows, occurring regularly in June, July, August and September of each year. These low flows are predicted reasonably well. During the period of high downstream flows occurring regularly in December, January, February and March, the comparison between observed and computed flows is quite good in 1953 and in 1954. Starting in 1955 there is a tendency for the model to *over-predict*. However, by 1958 following the wet 1957 year, the agreement for the months of January and February was even a little better than 1953. Note that even though through the years (1954-1955-1956) the difference between predictions and observations increases *cumulatively*, the two curves show very similar patterns.

One interpretation of the results is that the downstream low flows in August and September occur when return flows are small because: (1) the aquifer level has been lowered as a result of pumping, (2) upstream inflows are fairly high, and (3) surface diversions are extensive. The downstream outflows are conditioned mostly by the diversions. The small return flows do not affect significantly the streamflow. On the other hand, in December, January, February and March the upstream inflow is quite low. The river stage is at its lowest through the stream and the downstream flows are conditioned by the return flows and diversions, diversions being possible only from the existence of return flows. The drift in the predictions during these months may be caused either by overprediction of water-table elevations in the aquifer, thus causing higher return flows from the aquifer to the stream or by underprediction of diversions from the stream. Overprediction of water-table elevations in the aquifer could be due to underestimation of pumped volumes (Bureau of Reclamation data are used in the study) or underestimation of irrigation efficiency (i.e., overprediction of recharge) or underestimation of evapotranspiration losses (or losses from phreatoph tes). The purpose of the calibration runs was mostly to check the possibility of errors in the programs and to check that the values of reach transmissivities and of the stage-discharge relations in the stream. The results indicate that the reach transmissivities and the stage-discharge curves require no adjustments.

As indicated previously, a possible cause of apparent drift in the prediction of the high downstream flows could be that the Bureau of Reclamation data on pumping, diversions and return flows are somewhat in error, but it could be due to errors in estimation of irrigation efficiency. One could have played on these factors to obtain a better fit but it is difficult to time the impact of a correction and many trial and error runs would have been required. It was too costly to do with the resources allocated to this study. However, with *hindsight* looking at the results of the Series IV run it is clear that by progressively increasing values of pumping for the historical period 1953-1956 over the estimated Bureau of Reclamation values one could have obtained a better match between predicted and observed outflows in 1955 and 1956.