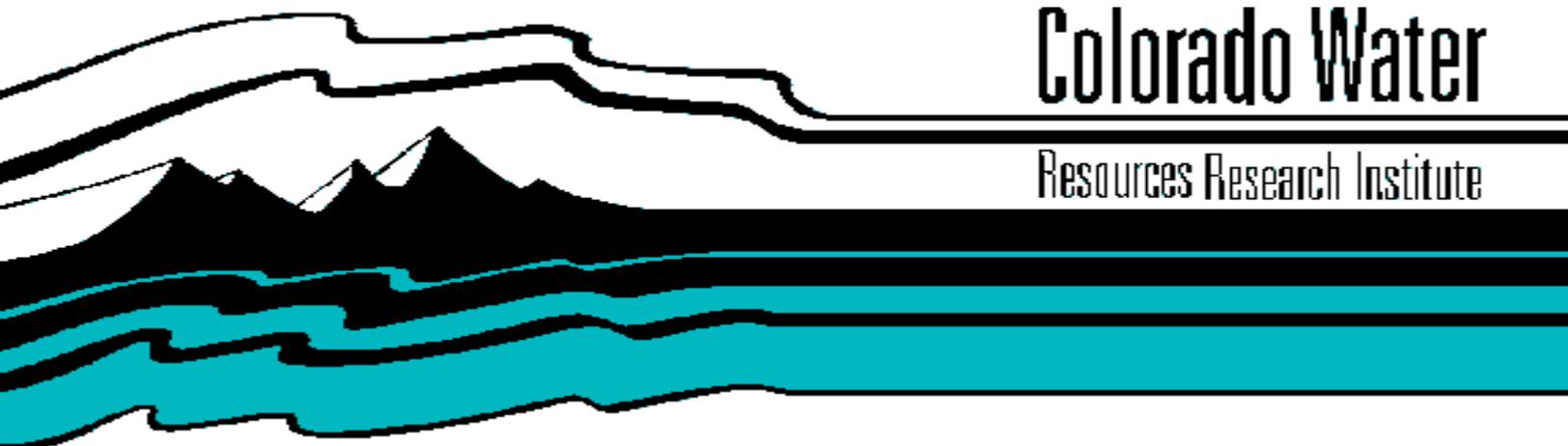


**MODELS DESIGNED TO EFFICIENTLY ALLOCATE  
IRRIGATION WATER USE BASED ON CROP  
RESPONSE TO SOIL MOISTURE STRESS**

by

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and Dr. Robert Young**



**Colorado Water**

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State  
University**

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May 1977

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## INTRODUCTION

This report examines models designed to allocate limited irrigation water to crops throughout the growing season so as to obtain the optimum return from water applied. This is a complex problem involving a great many variables including plant growth over the season, soil moisture status and weather conditions that affect evapotranspiration.

Several models have been developed to estimate the yield effects of various levels of soil moisture available to irrigated crops during the growing season. A number of simulation and linear programming models have been developed to project net returns from various alternative irrigation regimes, ranging from single crops to entire farms or irrigation systems. Central to these models is crop response to situations of soil moisture stress at various periods throughout the growing season. The effect of soil moisture stress on crop yield has long intrigued plant physiologists, agronomists, farmers, and others. Many studies have been conducted to measure reduction in crop growth during periods of soil moisture stress. The results of these experiments are varied due to the large number of factors, other than soil moisture status, that ultimately affect crop yield. Enough has been learned, however, about crop response to soil moisture stress to generally outline the yield response; but variations in the types and varieties of crops, yearly climate, soils, fertility levels, and cultural practices preclude precise definition. Additionally, difficulties in the exact measurement of soil moisture and climatic conditions make the mathematical specification of crop growth response difficult. Thus, a number of ways have been developed to specify crop response. All of the models discussed will have some divergence from the actual response of crops under most circumstances.

Part I reviews a number of models that have been developed to help predict crop response to soil moisture stress and to help plan efficient irrigation water allocation over the season and among crops.

Even with these rather elaborate models designed to determine optimum irrigation patterns, none of them addresses the problem of predicting precisely crop response to soil moisture stress by use of mathematical models. In order to determine what could be done in this area, a detailed analysis was undertaken in the Economics Department at Colorado State University to develop and test various mathematical models for suitability to predict yield response at various soil moisture levels during the growing season for specific crops. Detailed data from irrigation experiments designed to measure soil moisture status and its effect on yield throughout the season were used to test the models.

Two approaches using the agronomic experimental data were tried. Part II reports in detail on efforts to specify yield response to soil moisture stress.

One approach was used by Dr. Habte Neghassi in an effort to predict soil moisture levels by use of models utilizing evapotranspiration data and soil water-holding capacity. Soil moisture status is used to estimate resulting crop yield.

The second approach was developed by Dr. Dan Yaron using several mathematical models to estimate crop yield reductions when soil moisture falls below a predetermined level creating what is termed a "critical day." A critical day is one in which the crop suffers from moisture stress. Various mathematical functions were tested to determine if yield reductions could be predicted with some degree of confidence.

## PART I. REVIEW OF IRRIGATION AND CROP RESPONSE MODELS

Various models of irrigation systems have been proposed with varying purposes in mind. Two basic types of models have been developed: irrigation water scheduling models and crop planning models. Scheduling models attempt to aid the farmer during the season, determining optimal timings and quantities of irrigation. Scheduling models keep track of some state variables related to plant growth and variables measuring water need and availability. These models are generally, but not necessarily, daily models.

Planning models are designed to aid farmers in choosing the best acreages of crops to be grown. The planning model must take into account resources known with certainty at the beginning of the season; these models must also deal in some way with such variables as precipitation, weather conditions including solar radiation, and stream flows which are known only probabilistically. Some form of scheduling model may be implicit in the planning model.

### Simulation Models<sup>1/</sup>

#### Jensen and Heerman

Jensen and Heerman (1970) described an irrigation scheduling program that has been used by the United States Department of Agriculture and the Bureau of Reclamation, United States Department of the Interior, in advising farmers when to irrigate. The combination equation of Penman's evapotranspiration formula forms the basis of the program. Evapotranspiration, ET, is calculated on a daily basis from measured data and available soil moisture is updated by the program throughout the season.

<sup>1/</sup> The basic summarization of the various models were done by Herbert Blank. A more detailed discussion of these and other models can be found in his Ph.D. dissertation, "Optimal Irrigation Decisions With Limited Water," Colorado State University, Oct. 1975.

At any time during the season, the next irrigation can be predicted using the formula:

$$N = \frac{D_0 - D}{\bar{E}_t} \quad (2.1)$$

in which

$D$  = current estimated total depletion of soil moisture (in.)

$D_0$  = maximum allowable depletion for the present stage of growth (in.)

$\bar{E}_t$  = mean daily ET rate for the 3 previous days and 3 forecast days  
(in./day)

$N$  = estimated number of days to next irrigation.

In another paper (Heerman and Jensen, 1970), the  $\bar{E}_t$  value used was obtained from a graph showing  $\bar{E}_t$  as a function of time, normally distributed about the peak ET day. From experiments at Akron, Colorado, better results were obtained by this method than with the previous method, which required a subjective forecast of  $\bar{E}_t$ .

The next refinement in estimating the timing of the next needed irrigation was to add a term to  $N$  due to expected precipitation. The authors concluded that in a relatively dry area such as eastern Colorado, with relatively low precipitation, irrigation dates are not significantly affected by this refinement.

#### Kincaid and Heerman

Kincaid and Heerman (1974) describe a scheduling program for a programmable calculator. Again, the basis for the program is the Penman combination equation and associated crop coefficients and stress factors. As in the two previous papers, the authors assume the lowest soil moisture depletion level

acceptable is 50 percent of the total available moisture within the root zone. At an irrigation, the soil profile is returned to field capacity. The method of forecasting the date of the needed irrigation uses a normally distributed  $\bar{E}_t$  function.

The scheduling programs described have a specific purpose: recommending the timing of the next irrigation based on maintaining the crop within previously determined soil moisture conditions. The assumption, basically, is that water is available as needed and that no crop yield reduction is incurred when moisture depletion is not greater than 50 percent of available moisture.

#### Hanks

Hanks (1974) tested a production function for predicting grain yield from corn and sorghum. The author did not, however, attempt to apply this model in a planning or scheduling sense. The model is limited by data in that it requires daily values of potential evapotranspiration and potential soil evaporation under the crop canopy.

In a later paper, Hill, Hanks, et al. (1974) described a program which predicts corn yield using the production function tested by Hanks. The program was used to predict the effect of supplemental irrigation on an otherwise rainfed site. The conclusion was that a supplemental irrigation system could be economically justified. The program as described in the paper was used as a simulation of an irrigation system, answering a question "what if" irrigation were available.

#### Yaron

Yaron, et al. (1973) developed a soil moisture simulation model using experimental data from wheat. The authors fitted parameters to a Cobb-Douglas type

function, an exponential function, and a Mitscherlich function. The Mitscherlich function was adopted having the following independent variables:

1. Number of days during growth season with soil moisture above about 45 percent of available soil moisture.
2. A variable which measured the quality of the germination period, and
3. A year variable (4 years of data were used in the regression).

Upon obtaining a suitable yield prediction equation, 16 years of rainfall data were used to simulate the effect on yield of two approaches to irrigation scheduling. These were:

1. Irrigation on the basis of a predetermined time schedule, the quantities of water applied being equal to the moisture depletion in the root zone at the time of irrigation, and
2. Irrigating at the date on which the soil moisture is depleted to a predetermined critical level (Yaron, et al., 1973).

Taking into account water costs, the conclusion is that the second policy is slightly better than the first according to three objectives: maximizing expected net return, minimizing variance, and maximizing income during years of low rainfall.

It should be noted that this is still a simulation approach; irrigation times and amounts were chosen according to two arbitrarily chosen rules and tested to determine net return.

#### Stewart, Hagan and Pruitt

Stewart, Hagan and Pruitt (1974b) describe 18 methods of corn production with limited water supply. These methods are derived from data from field trials at Davis, California. Four irrigation times were specified during the season and irrigations were applied in one-inch increments up to field capacity. The irrigations were scheduled to occur when 70 percent of the water applied

previously had been removed from the root zone. A preirrigation to field capacity was made prior to planting. Yields were measured and profits due to water application were calculated, including water and labor costs of irrigating.

The authors recommended that if a fixed quantity of irrigation water per acre is known at the start of the season, the water should be applied according to the tables (see tables 4 and 5, Stewart et al., 1974b) derived by the authors. This model is thus deterministic and examines a single crop and an objective of maximizing return. The model could be adequate for the climatic conditions in the Central Valley of California, but is probably not readily transferable to other sites without repeating the full range of field trials.

#### Crop Optimization Models

The models discussed thus far have dealt with three aspects of the irrigation problem. The first studies were concerned with scheduling and, in particular, predicting date of next irrigation to obtain maximal yield. The second group was concerned with deriving production functions and then proceeding to simulate crop yields under varying conditions, while Stewart and Hagan's main contribution was in generating basic data relating water inputs to yields.

Hall and others have worked from the opposite end of the problem, starting with the optimization formulation and solution techniques, without concentrating on basic data.

#### Hall

Hall and Buras (1961) presented a problem of the optimal crop acreage for a known limited water supply. They dealt with a single crop, for which return as a function of seasonal water input was known. The authors formulated a dynamic program to solve the problem and also developed a graphical

solution technique. This model is limited in that it dealt with a single crop, was deterministic, and dealt only with the seasonal water input.

The model did consider the problem of limited water supply, concluding that, at least in the concave region of the production function (Stage 2) the policy should be to irrigate the selected acreage uniformly. The selected acreage, apparently, depends on the shape of the particular production function.

Hall and Eutcher (1968) introduced additional complexity by considering the effect of time of water application on yields. Again the model dealt with a single crop and again the top-down approach of assuming a production function was used. The form of the return function was

$$Z = P \prod_{i=1}^n a_i(d_i) \cdot Y_{\max} - \sum_{i=1}^n c_i \cdot x_i \quad (2.2)$$

in which

$Z$  = return

$P$  = price per unit of yield (\$/lb.)

$Y_{\max}$  = maximum yield (lbs.)

$d_i$  = soil moisture deficit from field capacity at time  $i$  (in.)

$a_i(d_i)$  = dimensionless yield reduction coefficient for time period  $i$

$x_i$  = quantity of water applied during period  $i$  (acre-inch)

$c_i$  = cost of water application during period  $i$  (\$/acre-inch)

After suggestions by Aron (1969) the model was presented in final form by Hall and Dracup (1970) as a dynamic program having three state variables which are

$q$  = amount of water in storage (acre inch)

w = soil moisture level (in.)

and A = "state of the crop at any time as a result of the possible deficiencies before the time period" (Hall, 1969) (dimensionless).

The model may be classified as a single crop, deterministic, scheduling model. The model assumes a fixed supply of irrigation water to be applied to a known crop acreage. The results of the program are the optimal timings and amounts of irrigation water, determined on the basis of knowledge known at the beginning of the season. Precipitation and other random variables are apparently assumed to take on their mean values. The model is theoretical in that it is not based on actual data and is not applied to an actual site. In addition to the assumption regarding the multiplicative nature of the production function, the model assumes that daily evapotranspiration is a function only of the soil moisture level for that day, not of solar radiation, etc., though a more complicated relation could be adopted. Hall and Dracup (1970) discuss the problems of computation with a three-state variable dynamic program and suggest methods for speeding the program by restricting values of the state variables.

Minhas

Another single crop model was presented by Minhas, et al., (1974). They developed an evapotranspiration ET prediction model for wheat as a function of available soil moisture only. The function was of the form

$$f(x) = (1 - e^{-rx}) / (1 - 2e^{-rx} + e^{-rx}) \quad (2.3)$$

in which

r = parameter fitted from data (1/in.)

x = available soil moisture (ASM) in root zone (in.)

$\bar{x}$  = ASM at field capacity FC (in.)

$f(x)$  = ratio of actual to potential ET for a plant when green cover is fully established.

Actual ET is the product of  $f(x)$ ; potential ET; and a crop weighting function, increasing from planting to full cover, constant until start of senescence, then decreasing to harvest. Parameters were fitted from wheat data from Delhi, India, and tested against results from alfalfa data of Mustonen and McGuinness (1968).

With an adequate ET prediction function, the authors used regression to fit parameters to the multiplicative function

$$Y = a [1 - (1 - x_1)^2]^{b_1} [1 - (1 - x_2)^2]^{b_2} \dots [1 - (1 - x_n)^2]^{b_n} \quad (2.4)$$

in which

$Y$  = yield

$x_j$  = relative (i.e., fraction of maximum) ET in period  $j$

$a, b_1, b_2, \dots, b_n$  are positive parameters fitted from data. The data used were from 21 wheat experiments over 3 years. Dummy variables were introduced "to capture the effects of the differences in experimental designs, varieties used, amounts of fertilizers used, and the climatic factors (nonmoisture) between different years," (Minhas, et al., 1974). The resulting regressions generally had high values of  $R^2$ , but the parameters of interest tended to be nonsignificant.

The authors adopted a production function consisting of two time periods and formulated an optimization problem of maximizing yields subject to meeting a seasonal water availability constraint. The problem was solved via marginal analysis, equating marginal products of water in the two time periods.

Dudley

Dudley, et al., (1971a) formulated a two-state variable dynamic program to determine optimal timing of irrigation for corn with a limited seasonal water supply. The state variables were available soil moisture, average soil moisture, and quantity of water in storage. They assumed an additive growth function with varying dollar values for growth in each time period. A "growth-no-growth" assumption was made, employing a concept similar to the stress-day concept of Flynn and Musgrave (1967). If ASM is high in relation to potential ET, ET occurs at a maximum rate and a growth day occurs, contributing to the dollar value of the crop. If ASM is low, ET occurs at a rate  $E_m$ , "the maximum rate at which water moves into the plant from the soil mass," (Dudley, et al., 1971a) and a no-growth day is recorded, contributing nothing to the value of the crop.

A stochastic dynamic programming model was formulated to make use of 20 years of evaporation and precipitation data. The objective was to maximize expected return as a function of terminal soil moisture TSM, that is the ASM percentage at which an irrigation is to occur. Transition probability matrices of beginning soil moisture are generated for each TSM policy in each time period and for each level of water supply. Similar matrices are generated for beginning water supply and return.

The results of the stochastic dynamic program are employed in a second model described by Dudley, et al. (1971b). While the first model looked at optimal timing for a given acreage, the second looks at the optimal area to be planted to a single crop, adding an additional stochastic variable of reservoir inflow.

The problem solution technique is basically a simulation approach; an acreage is selected and expected return is calculated based on the 20 years of data and the optimal terminal soil moisture policies developed from the previous model. The process continues by varying the acreage and calculating return until an optimal return is achieved assuming return as a function of acreage is a unimodal function.

#### Anderson and Maass

The irrigation system developed by Anderson and Maass (1971, revised 1974) represents the next level of sophistication. This model simulates an irrigation system, including stream diversions and reservoir storage, water distribution rules used to operate the canals, individual farms of varying size, farm water supply and cropping patterns. Crop response to soil moisture conditions are simulated by specifying typical irrigation requirements by periods throughout the growing season. Up to 26 irrigation periods can be specified. Yield reductions are indicated for any missed irrigations. These yield reductions are estimates based on research of agronomists and others of the effects of water shortages on crop yield at various times during the irrigation season. Crop watering sequences are generated by use of one of the formulas specifying typical evapotranspiration demands for particular areas, the type of crop, stage of growth, expected precipitation and soil type. These, together with irrigation efficiency, determine the sequence and amount of water needed throughout the irrigation season.

A variety of rules have been programmed into the model to illustrate the various ways that the water supply of an irrigation system is distributed to farmers. These determine when and how much water a farmer will receive to irrigate his crops.

The model can be run in various ways. The first utilizes water supply data for a single season and runs it through the irrigation season to examine the yield results from a given water supply and fixed crop patterns on the farms. Results are for a particular season. This analysis shows the effects on individual farms and crops of a particular water supply using a particular distribution rule. Various water supplies and distribution rules can be compared this way.

A second way the model can be run is to use what is called the Plan routine of the program. This option allows the program to select within specified limits the optimum crop pattern for each farm given the seasonal water supply, the array of crops, its portion of the system's water supply, and crop yield responses to various irrigation sequences. The Plan routine selects the acres of various crops that can be grown to give the maximum return with water availability throughout the season. This is done by incrementing the highest return crops up to acreage or water limitations before bringing the next crop into the crop pattern.

Another way the program can be run is to use the same data as above but to institute various distribution rules to determine if there is a better way to distribute available water among farms in the system. This type of analysis can aid in estimating the efficiency of distribution rules.

#### Young and Bredehoeft

Young and Bredehoeft (1972) presented a multiple-crop planning model to determine a policy for conjunctive use of groundwater and surface water. Anderson and Maass considered several alternative methods of production for each crop. Young and Bredehoeft used the same idea, considering different amounts and timings of irrigation as different production methods. The

optimal irrigation amount as developed by Anderson and Maass is one method; other methods correspond to skipping certain irrigations. Each method is associated with a certain net benefit per acre.

The model was simplified over Anderson's in that only four irrigation periods were considered. Groundwater was considered as an additional source of supply. A linear program was formulated similar to that of de Lucia except with the added dimension of time.

The irrigation planning problem was solved as a sub-program in a large simulation program. The authors did not consider the stochastic aspect of the problem due to the speed needed in computation. The authors restricted themselves to a site specific model with a single objective of maximizing return and all-or-nothing irrigations.

#### Hall

Hall and others in a report by the R.M. Parsons Co. (Parsons, 1970) applied Hall's work to a study of Indian irrigation. Data were obtained for two crops, wheat and jowar, and graphs were drawn for the coefficients  $a_1(d_1)$  in the multiplicative yield function. For these two crops a dynamic program was developed to determine optimal timings and amounts of irrigation. Fertilizer was also considered, under the assumption that for a given water application, yields were related to relative quantity of fertilizer applied

or

$$Y = a_N(N)a_1(d_1)a_2 \dots a_n(d_n)Y_{\max}$$

in which  $a_N(N)$  is given for maize by a graph. The program differed from that of Hall and Dracup (1970) in that the objective is to maximize yields and returns. Three state variables were considered: quantity of water in storage, soil moisture in the root zone, and available capital. The program

allocates capital over the season between water and fertilizer. The results are optimal irrigation and fertilizer applications for a given level of available capital.

Various methods of production for the two crops are obtained from the dynamic program and these are used as input to a district-wide linear program that considers, deterministically, optimal crop acreages. The objective is to maximize the net value of the output. The constraints considered by this program are water availability in various time periods, land use constraints, fertilizer availability, manpower availability, and a constraint that limits the acreage of nonfood crops.

#### Discussion of Crop Optimization Models

##### Problem Statements

Young and Bredehoeft (1972), Anderson and Maass (1971), Hall (Parsons, 1970), and de Lucia (1969) all consider basically the same problem: maximizing yearly yields or return from a fixed irrigated acreage, considering a given number of feasible crops. Smith (1970) is concerned with maximizing the net present worth of a planned expansion of a presently irrigated area, considering capital investments of the project and capacity dependent operation and maintenance costs, in addition to costs of water.

All of the previously mentioned authors consider linear constraints such as land constraints, water use constraints, etc. Smith (1970) and Hall (Parsons, 1970) consider crops grown in time periods extending throughout the entire year, but none of the studies considers more than one year and possible crop rotation requirements.

### Basic Data

The data used by the authors range from being based on extensive field trials to being based on rather questionable assumptions. Stewart and Hagan (1973b) conducted field trials, growing corn under many different irrigation regimes.

Yaron, et al., (1973) and Minhas, et al., (1974) rely on data from a number of years to establish their respective production functions. A "year" term is often included in the regressions. When the year term accounts for much of the variation in observed yields, the model obviously has not been well constructed. A model of plant growth which includes soil moisture and climatic terms should not require a year term. Another alternative is to use data collected in a single year, thus eliminating complicating effects of climatic variability.

Several of the authors devote little time to discussing the data on which their studies are based. Consumptive use figures for fully watered crops are available for many crops in many locations. These data are adequate for a study such as de Lucia's (1969). In other studies, including Hall's and Anderson and Maass', it appears that data for yields under conditions of less than optimum water supply have been based, in some cases, on judgment resulting from limited observations. This is not meant to be a criticism of the studies, only a reflection on the lack of data and the lack of theory to predict crop yields. These models have turned to substitutes for actual crop response data because of the extreme complexity and interaction of crops, growth stage, soil characteristics, atmospheric conditions and variation in water availability.

### Growth Models and Production Functions

Similar to the diversity in making use of basic data, diversity was noted in the growth models and production functions adopted by the various authors.

Stewart and Hagan (1973b) proposed a growth model linearly relating yields to seasonal ET. Jensen and Heerman (1970) and Hanks (1974) have complicated models for predicting ET. Hanks related ET in various time periods to yields with a multiplicative function. Hall used a multiplicative production function with terms functions of soil moisture during the time periods. Updating soil moisture in Hall's model requires predicting ET. Hall's ET (Hall and Butcher, 1970) is only a function of available soil moisture, ASM.

In the model of Minhas, et al., (1974), ET is a function of ASM, potential ET and a crop factor. Evapotranspiration is related to yields through a multiplicative production function. Dudley, et al., (1971a) predict actual ET from free water evaporation, a crop factor, and a soil factor. Yields are predicted based on the growth-no-growth concept which is based on daily ET values.

All of the previously mentioned authors rely on an ET estimation model. Some authors relate ET to yields while others, such as Hall, require estimates of ET in order to update soil moisture, which in turn is related to the yield coefficients in each time period. In any case, an ET estimation model is needed.

### Additive versus Multiplicative Functions

Multiplicative production functions have been employed by Hanks, Hall, and Minhas. Jensen (1968) proposed using the multiplicative relation for

some crops, but the irrigation scheduling programs of Jensen assume only one method of production. Anderson and Maass and Young and Bredehoeft do not employ continuous production functions.

Smith, in his simulation model, assumes a "linear relationship between crop yield and the water applied during any decision period" (Smith, 1970). An additive function, based on theory by Moore (1961) does not appear to be justified for all crops (Hall and Dracup, 1970, p. 134; and Jensen, 1968). Dudley's growth-no-growth concept is an additive relation with each growth day contributing a dollar value to the crop.

The multiplicative relation implies, for example, that if growth is only 70 percent of potential for a particular growth stage, then the maximum yield attainable by the crop is 70 percent of potential. According to the additive theory 70 percent of potential growth in a particular time period will only result in potential yields being reduced by 30 percent of that particular time period's potential contribution (see Figure 1).

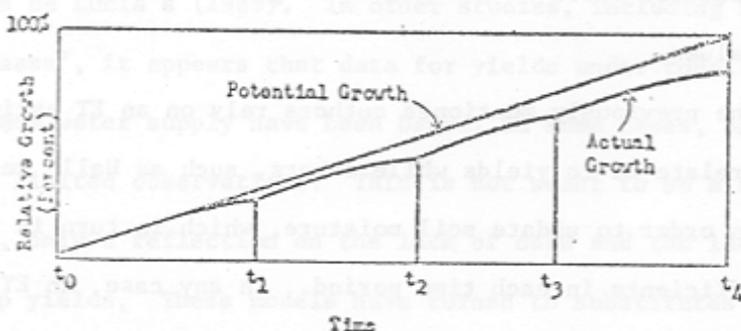


Figure 1

ACTUAL AND POTENTIAL GROWTH BY IRRIGATION CYCLES  
(From Moore, 1961)

Again, the point is made that no adequate theory has been developed and that currently available data are not sufficient to conclusively adopt any of the production functions described. As new data become available there is a need to test them with both approaches.

## PART II. MODELS DESIGNED TO SPECIFY SOIL MOISTURE STATUS AND YIELD RESPONSE

To explore the problems and difficulties of applying experimental water use data to models that are designed to predict or explain yield response to soil moisture stress on crops throughout the growing season, two different approaches were used to test various predictive models on corn and sorghum experiments. The first attempt was by Dr. Habte Neghassi testing several models against observed soil moisture use on corn at the Colorado State University agronomy farm in Fort Collins.

The second attempt was more elaborate and was made by Dr. Dan Yaron and colleagues to predict soil moisture status and yield response of corn from Fort Collins data. These models use data obtained from experiments designed to determine crop response to various levels of soil moisture availability throughout the growing season.

These exercises are presented to show the difficulties encountered when attempting to develop predictive models. The Yaron method does give guidelines for predicting yield reductions in corn.

### CORN WATER USE AND YIELD MODELS (Dr. H. Neghassi and Dr. R. Young)

#### Objective

The broad objectives of this analysis were to simulate water use and response models for various crops using historical data. However, due to limitations and unsuitability for combining the data, only corn grown at Fort Collins, Colorado was studied.

Crop, Soils, and Climatic Data

Corn (*Zea Mays* L.) was grown at the Colorado State University Agronomy Farm. The study was conducted by Twyford (Twyford, 1973) under the supervision of Dr. R.E. Danielson in 1972. The crop, planted May 12, was grown under varying soil moisture regimes. There were 11 treatments, representing three irrigation quotas, involving three schedules each, one irrigation quota involving one schedule, and one control. All irrigation treatments received water during the critical silking period. All irrigation applications were 5 cm (2 inches) by basin irrigation. The schedules refer to length (days) of irrigation delay during silking.

The soil was uniform deep Nunn Clay loam. There were three plant densities of low 54,000, medium 69,000, and high 85,000 plants per hectare. Uniform 47 kg/ha Phosphorous and 107 kg/ha Nitrogen were applied. The ultimate root depth was 195 cm with total water holding capacity of 26.6 cm.

Soil moisture was measured using a neutron probe at intervals during the growing season. Only the medium population density plots were sampled. No measurement of ground water level in the root zone was made but probably did not exist.

Daily climatic records of maximum and minimum air temperatures, precipitation, and minimum relative humidity were recorded at the experimental site. No records of wind speed, saturation vapor pressure, and solar radiation (or percent sunshine) were made. Adaption of solar radiation measurements at the Horticulture Farm, Colorado State University, which is located about 7 miles NNE of the Agronomy Farm, made the climatic data suitable for estimating potential evapotranspiration using the Jensen-Haise method. The solar method malfunctioned many times during the growing season. Measurements indicate obvious overestimation even under clear skies.

Data gathered by Dr. R.E. Danielson in 1968 were also analyzed. The experimental objective and design were not the same as the 1972 experiment.

### Estimation of Evapotranspiration

#### Potential evapotranspiration

The climatic input was incomplete to estimate potential evapotranspiration,  $ET_p$ , by the combination, or Penman, method (Penman, 1963), which would have been preferred. Thus,  $ET_p$  was estimated by the Jensen-Haise method (Jensen and Haise, 1963), which requires average daily temperature and solar radiation as input. The equation is given by

$$ET_p = (p.025T_a + 0.080)R_s \dots \quad (1)$$

where  $T_a$  is the average daily temperature in  $^{\circ}C$ ,  $R$  is the total short wave radiation in  $cal\ cm^{-2}\ day^{-1}$  received from the sun and the sky, and  $ET_p$  is  $cm\ day^{-1}$ .

#### Actual evapotranspiration

Daily evapotranspiration for a given agricultural crop under actual conditions of soils and climate,  $ET$ , is related to daily potential evapotranspiration,  $ET_p$ , as follows:

$$ET = K_c ET_p \quad (2)$$

where  $K_c$  is a dimensionless coefficient. It represents the combined relative effects of the resistance of water movement from the soil to the various evaporating surfaces and the resistance to the diffusion of water

from the surface to the atmosphere, as well as the relative amount of radiant energy available as compared to the reference crop. The crop coefficient derived from conditions of water non-limiting is designated by  $K_{co}$ .

In the USDA irrigation scheduling computer program the crop coefficient is adjusted for soil water availability and soil surface wetness as follows:

$$K_c = K_a K_{co} + K_s \quad (3)$$

where  $K_a$  is the relative coefficient related to percent available soil water, AM, as follows:

$$K_a = \ln(AM + 1) / \ln 101 \quad (4)$$

$K_s$  is the increase in the coefficient when the soil surface is wetted by irrigation or rain. It is approximated by:

$$K_s = (0.90 - K_c)^m \quad (5)$$

in which  $m = 0.8, 0.5, \text{ or } 0.3$ , respectively, for the first, second, or third day after irrigation or rain. In this particular case,  $K_s = 0.8, 0.7, \text{ or } 0.6$  when the rain or irrigation exceeded 1.5 cm for the first, second, and third days.

The mean crop coefficient where soil moisture was not limiting and normal irrigation stands are used,  $K_{co}$ , varies with type of crop. For corn,  $K_{co}$ , is given by

$$K_{co} = 0.23 - 0.4276P + 2.756P^2 - 1.583P^3 \quad (6)$$

where  $P$  is the fraction of days from planting to time of heading. After heading,  $K_{co}$  is given by

$$K_{co} = 0.915 + 1.195 - 4.688D^2 + 2.75D^3 \quad (7)$$

in which D is the number of days after heading divided by 100.

For this case,  $K_{co}$  was kept at 1.00 for the first 40 days after heading, or until  $D \geq 0.40$ .

#### Soil moisture depletion

The major dependent variable is soil moisture depletion and the major components are:

$$DSW = \sum_{i=1}^n (ET - R_e - I + W_d) \quad (8)$$

where DSW is soil moisture depletion (after a thorough irrigation  $D = 0$ ),  $R_e$  is effective rainfall (excluding surface runoff), I is irrigation water applied, and  $W_d$  is drainage from the root zone. The terms to the right of the equal sign are daily totals, expressed in cm, in the present computer program.

The amount of water available in the root zone (holding capacity = 26.6 cm) at any time during the growing season is given by:

$$ASW = 26.6 - DSW \quad (9)$$

where ASW is available soil water.

#### Comparison of estimated and measured water use

Available soil water was selected as a criterion for comparing the estimated and measured water use. Microfilm plats of the measured available soil water (points) and estimated available soil water are presented in figures 2 and Appendix B, figures 1-10, one for each irrigation treatment. Soil water measurements were first made on June 22 (Julian day 173). This measurement is taken as the initial soil water

level for the simulation, and thus is implicitly assumed correct. The estimate of soil water between planting (May 12, Julian day 132) was made by reading in an initial value for available soil water which would, after considering the various components of depletion, give close correspondence to the measurement of June 22.

The measured and estimated soil water availability compare well for the drier treatments, 0, 1A, 1B, and 1C (fig. 2 and Appendix B figures 1-3). Treatment 0 received no irrigation and 1A, 1B, and 1C received one 5 cm irrigation. Treatment 3C (fig. A6) also gave close agreement.

As shown in Appendix figures A4, A5, A6, A7, A8 and A9, treatments 3A-B, 4A-C, and 5 compare very poorly. The measured available soil water level is consistently lower than that estimated. Some possible causes for the discrepancies are:

1. Error in measurement (Neutron probe). Some of the measurements were obviously in error and reasonable adjustments were made in such cases.
2. High advective energy causing water losses much higher than a normal field would experience. The plots were separated by dry boundaries, which would increase advective loss.
3. Lateral and vertical movements of soil water from the root zone. These were not measured.
4. The solarimeter obviously malfunctioned occasionally during the season. It was overestimating solar radiation indicating higher values than would be expected on clear days at this