

Fundamentals of Investment Analysis

Quick Notes...

When analyzing potential investments, adjust future year income and expenses to reflect when they will occur.

Net present value analysis is an appropriate method of comparing investments which have lives greater than a single year.

Capital investments for periods greater than one year are common in agriculture.

Examples include the investment in land, machinery, irrigation equipment, breeding livestock, and orchards. Appropriate analysis of multi-year investments must adjust future years income and expenses to reflect when they will occur (the time value of money).

Time Value of Money

A \$1000 amount of income earned one year from today is not the same as \$1000 of income received today. If the current interest rate is 10%, the \$1000 we would get in one year would have a value today of \$909.09. That is to say, if we deposited \$909.09 at 10% interest for one year we would earn \$90.91 of interest and the

principal (\$909.09) plus the interest (\$90.91) would total \$1000 after one year.

Compounding is the term used to define computing a future value. **Discounting** is the term used to define computing a present value. We use the **Discount Rate or Compound Rate** (read interest rate) to determine the present value or future value of a fixed amount of money (**sum**) or a series or stream of payments (**annuity**).

Future value of a sum (FV_{sum}) is the value at a specified future time of a given sum of money invested today at a selected interest rate. Equation (3) is the formula to compute (FV_{sum}):

$$FV_{sum} = SUM * (1+i)^n \quad (3)$$

where: SUM = is the amount of money invested

i = is the compound (interest) rate

n = number of years the money is invested

EXAMPLE 1: Rancher Roy invested \$10,000 at 8% interest for 3 years. What is the value of the investment at the end of the third year?

$$\$10,000 * (1+.08)^3 = \$10,000 * 1.2597 = \$12,597$$

The 1.2597 is known as the **compound factor for a sum**--if the compound factor is known it can be multiplied times any sum to determine the future value of that sum or we could view this as:

\$10,000 * 1.08 = \$10,800 (End of year 1 value)
 \$10,800 * 1.08 = \$11,664 (End of year 2 value)
 \$11,664 * 1.08 = \$12,597 (End of year 3 value)

Table 1 shows the compound factor (for a sum) for selected interest rates and time periods.

Present Value of a sum (PV_{sum}) is the value today of a given sum of money that we would receive at a specified future date at a selected interest rate. Equation (4) is the formula to compute PV_{sum}.

$$PV_{sum} = SUM * [1/(1+i)^n] \quad (4)$$

where: SUM = is the amount of money to be received in the future
 i = is the discount (interest) rate
 n = number of years until you receive the money

EXAMPLE 2: Rancher Ray will receive \$10,000 three years from now from his Uncle Bill C. Year. If the interest rate is 6%, what is the Present Value of this future income?

$$\$10,000 * [1/(1+.06)^3] = \$10,000 * 0.8396 = \$8,396$$

The 0.8396 is known as the **discount factor**. If the discount factor is known, it can be multiplied times any sum to determine the present value of the future sum or we could view this as:

$$\$10,000 * [1/1.06] = \$10,800 * .9434 = \$9,434 \text{ (Value discounted 1 yr.)}$$

$$\$9,434 * [1/1.06] = \$9,434 * .9434 = \$8,900 \text{ (Value discounted 2 yrs.)}$$

$$\$8,900 * [1/1.06] = \$8,900 * .9434 = \$8,396 \text{ (Value discounted 3 yrs.)}$$

Table 2 shows the discount factors (for a sum) for selected interest rates and time periods.

The future value of an annuity (FV_{ann}) at the time the last payment is made is shown in equation (5):

$$FV_{ann} = PMT * [(1+i)^n - 1]/i \quad (5)$$

where: PMT = periodic payment
 i = periodic interest or compound rate
 n = number of payments

EXAMPLE 3: Farmer Fred started an Individual Retirement Account (IRA) when he was 26. He makes a \$2000 deposit each year for 40 years and earns 9% interest on his IRA. What will be the value of the IRA after he makes his 40th deposit?

$$\begin{aligned} FV_{ann} &= \$2,000 * [(1+.09)^{40} - 1]/.09 \\ &= \$2,000 * [31.4094 - 1]/.09 \\ &= \$2,000 * 337.8824 = \$675,765 \end{aligned}$$

The 337.8824 is the compound factor for an annuity; these factors are shown in Table 3 for selected interest rates and time periods.

The present value of an annuity (PV_{ann}) at the time of the first deposit is shown in equation (6):

$$PV_{ann} = PMT * ([1 - (1+i)^{-n}]/i) \quad (6)$$

where: PMT = periodic payment
 i = periodic interest or compound rate
 n = number of payments

EXAMPLE 4: Farmer Fredia has \$600,000 in her retirement account on her 65th birthday. She expects to live an additional 20 years. If the interest rate is 8% what would be her annual income if she does NOT spend any of the principal?

$$\$600,000 * .08 = \$48,000$$

What would be her annual income if she spends all the principal during the 20 years of retirement?

The present value of \$1 of retirement income in each of the next 20 years would be:

$$\begin{aligned}
 PV_{\text{ann}} &= \$1 * (([1.08^{19} - 1]/.08)/[1.08]^{19}) + 1 \\
 &= \$1 * (([4.3157 - 1]/.08)/4.3157) + 1 \\
 &= \$1 * 10.60 = \$10.60
 \end{aligned}$$

Thus if Fredia had \$10.60 she could withdraw \$1 each year for the next 20 years. She has more than \$10.60 (she has \$600,000) thus she has 56,604 (\$600,000/\$10.60) \$10.60 units each, providing \$1 for the next 20 years.

At the end of the 20 years she would have a retirement account balance of zero!

Net Present Value

The principles of discounting and compounding can be applied to enterprises which have a life greater than a single year. For example, if you planted Christmas trees you would have a major investment at the time of planting, annual expenses for fertilizer, trimming, and pest control and in the year they are harvested, the expenses of harvest and marketing. This analysis is known as capital budgeting. **Net present value** is the discounted value of expenses and revenue over the life of the enterprise. Comparing the net present value of two enterprises over the same time period is the appropriate method of evaluation.

EXAMPLE 5. Budgets for one acre of Christmas trees marketed in 7 years and one acre of corn with a discount rate of 10%.

Year	Expenses	Revenue	Net Income	Present Value Factor	Present Value	Net Income	Present Value
Christmas Trees						Corn	
1	500	0	-500	1.0000	-500	80	80
2	50	0	-50	0.9091	-45	80	73
3	60	0	-60	0.8264	-50	80	66
4	70	0	-70	0.7513	-53	80	60
5	80	0	-80	0.6830	-55	80	55
6	90	0	-90	0.6209	-56	80	50
7	700	2500	1800	0.5645	1016	80	45
Total	1650	3000	1350		258	560	429

Several points are important to the above analysis. First, we have assumed that income and expenses occur at the same time each year. Second the present value of the \$500 of Christmas tree expenses in year 1 is \$500. In year two, the \$50 of Christmas tree expenses have a present value of only \$45 because they have been discounted for one year at 10% using the discount or present value factor (see Table 2). Second, the net present value of the Christmas trees of \$258 is the return for seven years. If we want to decide between planting continuous corn with a return of \$80/ac/yr vs

Christmas trees, we should compare the income streams or net present values of the two alternatives. At a ten percent discount rate, Christmas trees are \$171 less profitable than corn over the seven years when net present values are compared. If a lower discount rate was used, Christmas trees would show a higher net present value. Finally, because the Christmas tree revenue is in year seven and a relatively high discount rate (10%) is used, that caused the net present value of Christmas trees to be less than corn.

*Notes...
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Table 1. Value of \$1 compounded annually at the end of an interest-bearing period of n years.

Year	Rate of Interest (% per Year)											
	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%
1	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.1300	1.1400	1.1500	1.2000
2	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544	1.2769	1.2996	1.3225	1.4400
3	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049	1.4429	1.4815	1.5209	1.7280
4	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735	1.6305	1.6890	1.7490	2.0736
5	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623	1.8424	1.9254	2.0114	2.4883
6	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738	2.0820	2.1950	2.3131	2.9860
7	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107	2.3526	2.5023	2.6600	3.5832
8	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760	2.6584	2.8526	3.0590	4.2998
9	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731	3.0040	3.2519	3.5179	5.1598
10	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058	3.3946	3.7072	4.0456	6.1917
15	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736	6.2543	7.1379	8.1371	15.4070
20	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463	11.5231	13.7435	16.3665	38.3376
30	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	22.8923	29.9599	39.1159	50.9502	66.2118	237.3763
40	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593	65.0009	93.0510	132.7816	188.8835	267.8635	1469.7716

$$FV_{\text{sum}} = (1+i)^n$$

Table 2. Value at the beginning of the period, of \$1 received after an interest-bearing period of n years.

Year	Rate of Interest (% per Year)											
	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%
1	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8333
2	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.6944
3	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.5787
4	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.4823
5	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4019
6	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.3349
7	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	0.2791
8	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.2326
9	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.1938
10	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.1615
15	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.0649
20	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0261
30	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0042
40	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0007

$$PV_{\text{sum}} = 1/(1+i)^n$$

Table 3. Value of a finite series of n annual payments of \$1 compounded annually at the date of last payment.

Year	Rate of Interest (% per Year)												
	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.050	2.060	2.070	2.080	2.090	2.100	2.110	2.120	2.130	2.140	2.150	2.150	2.200
3	3.153	3.184	3.215	3.246	3.278	3.310	3.342	3.374	3.407	3.440	3.473	3.473	3.640
4	4.310	4.375	4.440	4.506	4.573	4.641	4.710	4.779	4.850	4.921	4.993	4.993	5.368
5	5.526	5.637	5.751	5.867	5.985	6.105	6.228	6.353	6.480	6.610	6.742	6.742	7.442
6	6.802	6.975	7.153	7.336	7.523	7.716	7.913	8.115	8.323	8.536	8.754	8.754	9.930
7	8.142	8.394	8.654	8.923	9.200	9.487	9.783	10.089	10.405	10.730	11.067	11.067	12.916
8	9.549	9.897	10.260	10.637	11.028	11.436	11.859	12.300	12.757	13.233	13.727	13.727	16.499
9	11.027	11.491	11.978	12.488	13.021	13.579	14.164	14.776	15.416	16.085	16.786	16.786	20.799
10	12.578	13.181	13.816	14.487	15.193	15.937	16.722	17.549	18.420	19.337	20.304	20.304	25.959
15	21.579	23.276	25.129	27.152	29.361	31.772	34.405	37.280	40.417	43.842	47.580	47.580	72.035
20	33.066	36.786	40.995	45.762	51.160	57.275	64.203	72.052	80.947	91.025	102.444	102.444	186.688
30	66.439	79.058	94.461	113.283	136.308	164.494	199.021	241.333	293.199	356.787	434.745	434.745	1181.882
40	120.800	154.762	199.635	259.057	337.882	442.593	581.826	767.091	1013.704	1342.025	1779.090	1779.090	7343.858

$$FV_{ann} = [((1+i)^n) - 1] / i$$

Table 4. Value of a finite series of n annual payments of \$1 compounded annually at the date of first payment.

Year	Rate of Interest (% per Year)												
	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.952	1.943	1.935	1.926	1.917	1.909	1.901	1.893	1.885	1.877	1.870	1.870	1.833
3	2.859	2.833	2.808	2.783	2.759	2.736	2.713	2.690	2.668	2.647	2.626	2.626	2.528
4	3.723	3.673	3.624	3.577	3.531	3.487	3.444	3.402	3.361	3.322	3.283	3.283	3.106
5	4.546	4.465	4.387	4.312	4.240	4.170	4.102	4.037	3.974	3.914	3.855	3.855	3.589
6	5.329	5.212	5.100	4.993	4.890	4.791	4.696	4.605	4.517	4.433	4.352	4.352	3.991
7	6.076	5.917	5.767	5.623	5.486	5.355	5.231	5.111	4.998	4.889	4.784	4.784	4.326
8	6.786	6.582	6.389	6.206	6.033	5.868	5.712	5.564	5.423	5.288	5.160	5.160	4.605
9	7.463	7.210	6.971	6.747	6.535	6.335	6.146	5.968	5.799	5.639	5.487	5.487	4.837
10	8.108	7.802	7.515	7.247	6.995	6.759	6.537	6.328	6.132	5.946	5.772	5.772	5.031
15	10.899	10.295	9.745	9.244	8.786	8.367	7.982	7.628	7.302	7.002	6.724	6.724	5.611
20	13.085	12.158	11.336	10.604	9.950	9.365	8.839	8.366	7.938	7.550	7.198	7.198	5.843
30	16.141	14.591	13.278	12.158	11.198	10.370	9.650	9.022	8.470	7.983	7.551	7.551	5.975
40	18.017	15.949	14.265	12.879	11.726	10.757	9.936	9.233	8.627	8.100	7.638	7.638	5.996

$$PV_{ann} = [({(1+i)^{n-1}} - 1) / i] / (1+i)^{n-1} + 1$$